Sample Exam

Important notes:

1. This is a sample exam. **Disclaimer:** I have tried to approximate the length, style and difficulty level of the final exam, but I cannot guarantee that you will find both exams equally challenging. Also, you should not assume that the precise choice of subject matter in the sample exam is representative of the final exam.
2. The final exam time is 2 hours. The sample exam is designed for a similar time frame, or maybe slightly longer.
3. Calculators and written notes are not allowed on the final exam.
4. Numerical answers should be expressed in terms of known constants such as $e, \pi, \ln(2)$, etc., when necessary. Try to simplify expressions – e.g., write 0 instead of $\ln(1)$.

Question 1 (24 points) Compute the following integrals:

(a) $\displaystyle\int_0^\pi (x^2 + \sin(x))\,dx$

(b) $\displaystyle\int \sec^2(x)\,dx$

(c) $\displaystyle\int \cos(2x-1)\,dx$

(d) $\displaystyle\int \frac{(1+\sqrt{x})^7}{\sqrt{x}}\,dx$

(e) $\displaystyle\int_1^b (e^y + e^{-y})\,dy$ (note: the answer is a function of b)

(f) $\displaystyle\int_0^2 x e^{x^2}\,dx$

Question 2 (6 points) Compute the arc length of the curve $y = x^3/6 + 1/(2x)$ from $x = 2$ to $x = 3$.

Question 3 (36 points) Compute the following integrals using integration by parts or any other technique.

(a) $\displaystyle\int \ln x \, dx$

(b) $\displaystyle\int x \ln x \, dx$

(c) $\displaystyle\int_0^\pi x^2 \sin(x) \, dx$

(d) $\displaystyle\int \frac{x^2 - 2}{x(x-1)(x-2)} \, dx$

Question 4 (8 points) Estimate the value of the definite integral $\int_0^2 3x^2 \, dx$ in two ways: (a) Using the trapezoidal rule, and (b) using Simpson's rule. In both cases use a partition of the interval $[0, 2]$ into $n = 4$ sub-intervals of equal length. (Note: there are 4 partition intervals, and therefore 5 partition points x_0, x_1, x_2, x_3, x_4.) Compare the results to the correct value of the integral $\int_0^2 3x^2 \, dx$ and determine which one is closer to this value.

Question 5 (6 points) Compute the volume of the solid formed by revolving around the x-axis the region bounded between the x-axis and the curve $y = x - x^2/4$.

Question 6 (8 points) A computer scientist is modelling the spread of a computer virus on the internet. She assumes that if $y(t)$ represents the number of infected computers t days after the virus originated, then $y(t)$ can be approximated by a differentiable function satisfying

$$\frac{dy}{dt} = 0.25\, y.$$

Furthermore it is known that at time $t = 0$ there were 350 infected computers. How many computers were infected 30 days later? (You do not need to compute the precise numerical answer – a formula would be enough.)

Question 7 (6 points) Compute the area of the two-dimensional region bounded between the y-axis, the line $y = 3$ and the curve $y = \sqrt{x}$.

Question 8 (6 points) Compute the center of mass of a thin rod extending between $x = 0$ and $x = 2$ with a linear mass density given by

$$\delta(x) = x + 1$$