MAT280: Longest increasing subsequences and combinatorial probability

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Course description. If σ is a permutation, let $L(\sigma)$ denote the maximal length of an increasing subsequence of σ. A famous problem studied since the 1960's, known as *Ulam's problem*, concerns the likely behavior of $L(\sigma)$ if σ is chosen at random from among all permutations of given order n. An early and already highly nontrivial result in this direction was proved in 1977, and states that as $n \to \infty$, with high probability $L(\sigma)$ grows approximately like $2\sqrt{n}$. In the late 1990's, a big breakthrough in the understanding of $L(\sigma)$ came with the discovery of the limiting law for the fluctuations of $L(\sigma)$ from its typical value of $2\sqrt{n}$, which turned out to be the famous Tracy-Widom distribution, originally discovered in connection with the study of random matrices. Many later and equally remarkable developments tied this problem to other natural problems in combinatorial probability, such as the question of the limiting shape of an area in the first quadrant of $\mathbb{R}^2$ formed by randomly stacking unit squares against the corner of the quadrant.

Syllabus. The goal of the course is to give an introduction to the fascinating combinatorial and probabilistic theory behind the discoveries described above. Specific topics to be covered include:

(1) The Erdös-Szekeres theorem.
(2) The patience sorting algorithm.
(3) Young tableaux and the Robinson-Schensted correspondence.
(4) Plancherel measure on Young diagrams.
(5) The limit shape of Plancherel measure and the solution to Ulam's problem.
(6) Discrete determinantal point processes.
(7) The Tracy-Widom distribution and an introduction to the Baik-Deift-Johansson theorem.
(8) Square Young tableaux and extremal Erdös-Szekeres permutations.
(9) The corner growth model.

Prerequisites. The material will be self-contained, but I will assume familiarity and a reasonable comfort level with the following concepts: permutations; binomial coefficients; probability, expectation and variance; basic concepts in real analysis (integrals, power series, trigonometric functions); linear algebra and determinants.

Textbook. There will be no textbook, but I am preparing and will make available detailed lecture notes covering the course material.