Chapter 6 part 2 outline

Appendix B.1 and 6.8: Inner product spaces and orthonormal bases

- Review of perpendicular vectors in $\mathbb{R}^2$; relationship to dot product.
- Definition of inner product space.
- Example: $\mathbb{R}^n$ is an inner product space where $\langle u, v\rangle = u \cdot v$ (i.e. the inner product is the dot product).
- Example: show $\mathbb{R}^2$ is an inner product space where $\langle u, v\rangle = u_1v_1 + 3u_2v_2$.
- Example: suppose V is an inner product space with basis $\{e_1, e_2\}$. Assume $\langle e_1, e_1\rangle = \langle e_2, e_2\rangle = 1$, and $\langle e_1, e_2\rangle = 0$. Use the linearity and symmetry properties of the inner product to find $\langle 7e_2, e_1 - 2e_2\rangle$.
- Example: M_n is an inner product space with inner product $\langle A, B\rangle = \text{tr}(A^tB)$. For example, if
$$A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}, \ B = \begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix},$$
then $\langle A, B\rangle = 5$.
- $P_n(x)$ is an inner product space with inner product
$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)dx.$$
- Definitions: vectors v and w in an inner product space are orthogonal; the length of a vector v; when a basis is orthonormal.
- Algorithm: Gram-Schmidt procedure for making a basis orthogonal.
- Make $\{(1, 1), (1, 2)\}$ into an orthonormal basis for $\mathbb{R}^2$.
- Consider
$$W = \text{span}\left\{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right\}$$
as a subspace of M_2. Use the Gram-Schmidt algorithm to find an orthonormal basis for W.
- Find an orthonormal basis for the subspace $\{(a, a + b, b) \mid a, b \in \mathbb{R}\}$ of $\mathbb{R}^3$.

6.9: Orthogonal complements

- Definition: orthogonal complement $W^\perp$ of W.
- Example: find the orthogonal complement of $\text{span}\{(1,1)\}$ in $\mathbb{R}^2$.
- Theorem: let V be an inner product space and W a subspace of V. Then $W^\perp$ is a subspace of V.
- Example: find the orthogonal complement of the subspace $W = \{(a, a+b, b) \mid a, b \in \mathbb{R}\}$ in $\mathbb{R}^3$.
- Example: show that the orthogonal complement of the symmetric matrices ($A^t = A$) in M_2 is the anti-symmetric matrices ($A^t = -A$).