

Final practice problems

- Use properties from section 3.2 to find the determinant of the following types of matrices:
 - An orthogonal matrix, $AA^t = I$.
 - An idempotent matrix $A^2 = A$.
 - An anti-symmetric matrix $A^t = -A$ (this depends on the parity of the dimensions of A).
 - A matrix satisfying $A^2 = 3A$ (this, again, depends on the dimensions of A).
- Suppose that $\{e_1, e_2\}$ is an orthonormal basis for the 2-dimensional inner product space V . Let $v = ae_1 + be_2$ be an arbitrary vector in V . Prove that the following equation holds for the length of v :

$$\|v\| = \sqrt{a^2 + b^2}.$$

- Use the Gram-Schmidt algorithm to orthogonalize the following bases. Then, normalize to make the bases orthonormal.
 - $\{(1, 1, 1), (0, 1, 1), (1, 2, 3)\}$ of \mathbb{R}^3 (where the inner product is the dot product).
 - $\{2, x + 1, x^2 + x - 1\}$ of $P_2(x)$ (where the inner product is the integral $\int_{-1}^1 p(x)q(x)dx$).
- Find an orthonormal basis for the kernel of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & -6 \end{pmatrix}.$$

- Find a basis for the orthogonal complement of

$$\text{span} \left\{ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right\}$$

in M_2 , where the inner product of two matrices is $\text{tr}(A^tB)$.

- Suppose the characteristic equation of a matrix A is given by $\lambda(\lambda - 2)(\lambda + 4)^2$.
 - What are the eigenvalues of A ?
 - Compute $\det(A)$ and $\text{tr}(A)$.

- (c) Write down the 2 possible Jordan forms of A .
7. If B is a 2×2 matrix with eigenvalues 1 and -5 , then what is its characteristic polynomial?
8. Find the change-of-basis matrix between the following two bases of the indicated space:
- (a) The monomial basis $S = \{1, x, x^2\}$ and the basis $T = \{2, x + 1, x^2 + x - 1\}$ of $P_2(x)$.
- (b) The basis $S = \{(1, 2), (0, 1)\}$ and the basis $T = \{(1, 1), (2, 3)\}$ of \mathbb{R}^2 .
9. Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the matrix transformation which takes a vector and reflects it across the line $y = x$.
- (a) Write a matrix representation of L with respect to the standard basis.
- (b) Find the eigenvalues and a basis for the eigenspaces of your matrix. Diagonalize the matrix.
10. Chapter 10 test, p. 554: # 2(a), 3(a), 4, 5, 6(a)(b).