

# Homework 1

due Thursday August 7th

1. Construct a truth table for the statement:

$$[p \Rightarrow (q \wedge \sim q)] \Leftrightarrow \sim p.$$

2. Consider the following statement:

*A function  $f$  is uniformly continuous on a set  $S$  if and only if for every  $\epsilon > 0$  there is a  $\delta > 0$  such that  $|f(x) - f(y)| < \epsilon$  whenever  $x$  and  $y$  are in  $S$  and  $|x - y| < \delta$ .*

- (a) Rewrite the statement using the logical symbols  $\Rightarrow$ ,  $\Leftrightarrow$ ,  $\forall$ ,  $\exists$ , etc.  
(b) Write the negation of the statement in words and in logical symbols.

3. Prove or give a counterexample:

*For every positive integer  $n$ ,  $n^2 + 3n + 8$  is even.*

4. Prove:

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

5. An *open cover* of a set  $A$  is a collection of sets  $\mathcal{B}$  satisfying:

- (a) Each  $B \in \mathcal{B}$  is open.  
(b)  $A$  is contained in the union of sets in  $\mathcal{B}$ , i.e.  $A \subseteq \bigcup_{B \in \mathcal{B}} B$ .

Show that

$$\mathcal{B} = \left\{ \left( \frac{1}{n}, \frac{n-1}{n} \right) \text{ for } n = 0, 1, 2, \dots \right\}$$

is an open cover of  $(0, 1)$ . Is there a finite subset of  $\mathcal{B}$  that is an open cover of  $(0, 1)$ ?

6. Prove that when  $A$  and  $B$  are countable,  $A \times B$  is countable.  
7. Let  $f : A \rightarrow B$  and  $C, D \subseteq B$ . Prove  $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$ . What about  $f^{-1}(C \cap D)$ ?