Homework 5
due Thursday July 23rd

1. Section 4.2 # 1(g), 3(e), 14(d).

2. Let $A = \mathbb{N} \cup \{0\}$. Define the equivalence relation $R_n \subseteq A \times A$ by $(x, y) \in R_n$ iff $x \mod n \equiv y \mod n$. Denote the set of equivalence classes $A/R_n = \mathbb{Z}_n$.

   (a) Let $f : \mathbb{Z}_4 \to \mathbb{Z}_2$ be the map given by $f(x/R_4) = (x \mod 2)/R_2$. Prove f is a well-defined function. Is f injective and/or surjective? Prove your claims.

   (b) Let $f : \mathbb{Z}_4 \to \mathbb{Z}_4$ be the map given by $f(x/R_4) = (x+1 \mod 4)/R_4$. Prove f is a well-defined function. Is f injective and/or surjective? Prove your claims.

3. Recall the kernel of a map $f : A \to B$ (where $0 \in B$) is the set

$$ker(f) = \{x \in A \mid f(x) = 0\}.$$

   (a) Prove: if f is a bijection, then the kernel of f contains exactly one element.

   (b) Give a counterexample to show the converse of 3a does not hold.

   (c) Use the contrapositive of 3a to prove that $f(x) = x^2 - 5x + 6$ is not a bijection.

4. Let $A = \{1, \ldots n\}$, and $f : A \to A$ be a surjective function. Prove f is also an injective function. Give a counterexample to show that surjectivity does not imply injectivity when A contains infinitely many elements.

5. Section 4.3 # 8(a)(e), 9(a), 16(a)(e)(g).

6. Give an example of a function $f : A \to B$ with the property that there exists a subset $C \subseteq B$ with $f(f^{-1}(C)) \neq C$.

7. Section 4.4 # 5(a)(c), 10, 14(b)(e).