

Homework 6

due Wednesday September 10th

1. Consider the set $S = [0, 3) \cup (3, 5)$. What is the interior of S ? What are the limit points of S ? What is the closure of S ?
2. If A is open and B is closed, prove that $A \setminus B$ is open.
3. Let S be a bounded infinite set and let $x = \sup S$. Prove that either $x \in S$ or x is a limit point of S .
4. Use the definition of compactness to prove that $[1, 3)$ is not compact.
5. Let $S = \mathbb{R} \times \mathbb{R}$ and let $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ be the Euclidean metric. Define $d^*(x, y) = \min\{1, d(x, y)\}$. Verify that d^* is a metric on S . Draw neighborhoods around the origin of radius $1/2$, 1 , and 2 .
6. If A and B are compact subsets of a metric space (X, d) , prove that $A \cup B$ is also compact.
7. Let x be a point in the metric space (X, d) . Prove that the singleton set $\{x\}$ is closed.