# Midterm 1 Solutions

1.

$$\int \frac{\sqrt{y}}{5} + \frac{3}{\sqrt{y}} dy = \frac{2}{15} y^{3/2} + 6y^{1/2} + c.$$

2.

$$\int_5^6 \frac{7}{x^9} dx.$$

3.

$$\int_{-3}^{6} h(x)dx = \int_{-3}^{3} h(x)dx + \int_{3}^{6} h(x)dx = 2 - 10 = -8.$$

and

$$\int_{6}^{-3} h(t)dt = -\int_{-3}^{6} h(x)dx = 8.$$

4.

$$\begin{aligned}\int_0^{\pi} \frac{5 - \sin(4x)}{4} dx &= \int_0^{\pi} \frac{5}{4} - \frac{\sin(4x)}{4} dx \\ &= \left. \frac{5}{4}x + \frac{\cos(4x)}{16} \right]_0^{\pi} \\ &= \frac{5}{4}\pi + \frac{1}{16} - \frac{1}{16} = \frac{5}{4}\pi\end{aligned}$$

5.

$$\frac{d}{dt} \int_0^{\sin t} \frac{1}{16 - u^2} du = \frac{\cos t}{16 - \sin^2 t}$$

by the fundamental theorem of calculus.

6.

$$\begin{aligned}\int_4^{16} (x+3)\sqrt{x} dx &= \int_4^{16} x^{3/2} + 3x^{1/2} dx \\ &= \left. \frac{2}{5}x^{5/2} + 2x^{3/2} \right]_4^{16} \\ &= \frac{2}{5}\left(16^{5/2} - 4^{5/2}\right) + 2\left(16^{3/2} - 4^{3/2}\right)\end{aligned}$$

7. Let $u = x^{3/2} - 4$ so that

$$du = \frac{3}{2}x^{1/2} dx.$$

Then,

$$\begin{aligned}\int \sqrt{x}\cos^2(x^{3/2}-4)dx &= \frac{2}{3}\int \cos^2(u)du \\ &= \frac{1}{3}\int (\cos(2u)+1)du \\ &= \frac{1}{6}\sin(2u)+\frac{1}{3}u+c \\ &= \frac{1}{6}\sin(2x^{3/2}-8)+\frac{1}{3}(2x^{3/2}-4)+c\end{aligned}$$

Notice it is important to know the identity $2\cos^2 x = \cos(2x)+1$.

8. In this problem you'll need to know the following fact

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)+c.$$

Then,

$$\int \frac{5dx}{\sqrt{64-25x^2}} = \frac{5}{8}\int \frac{dx}{\sqrt{1-\left(\frac{5}{8}x\right)^2}}$$

Making the u-substitution $u = \frac{5}{8}x$, we obtain

$$\begin{aligned}\frac{5}{8}\int \frac{dx}{\sqrt{1-\left(\frac{5}{8}x\right)^2}} = \int \frac{du}{\sqrt{1-u^2}} &= \sin^{-1}(u)+c \\ &= \sin^{-1}\left(\frac{5}{8}x\right)+c\end{aligned}$$

9. Let $u = \cos\sqrt{t}$. Then

$$du = -\frac{\sin\sqrt{t}}{2\sqrt{t}}dt.$$

Therefore,

$$\begin{aligned}\int \frac{\sin\sqrt{t}}{\sqrt{t\cos^3\sqrt{t}}}dt &= -2\int \frac{1}{\sqrt{u^3}}du \\ &= u^{-1/2}+c \\ &= \frac{1}{\sqrt{\cos\sqrt{t}}}+c\end{aligned}$$

Note it may have been easier to do this in two steps: first set $u = \sqrt{t}$, then set $v = \cos u$.

10. For this problem you have to know that velocity is the anti-derivative of acceleration, and position is the anti-derivative of velocity.

$$v(t) = \int a(t)dt = \int (\sin(4t) + 10)dt = \frac{-\cos(4t)}{4} + 10t + c.$$

Using the initial condition $v(0) = -1/4$, we get $c = 0$.

Then,

$$s(t) = \int v(t)dt = \int \left( \frac{-\cos(4t)}{4} + 10t \right) dt = \frac{-\sin(4t)}{16} + 5t^2 + c$$

. Again, using the initial condition $s(0) = 1$, we obtain $c = 1$, so the final answer is

$$\frac{-\sin(4t)}{16} + 5t^2 + 1$$

11. The area between $f$ and $g$ is:

$$\begin{aligned} \int_{-4}^{0} (f(x) - g(x))dx &= \int_{-4}^{0} (x^3 + x^2 - 12x)dx \\ &= \left. \frac{x^4}{4} + \frac{x^3}{3} - 6x^2 \right]_{-4}^{0} \\ &= -\left( 4^3 - \frac{4^3}{3} - 6(4^2) \right) = 160/3 \end{aligned}$$

12.

$$\begin{aligned} \int_{0}^{\ln 5} (e^{3x} - e^x)dx &= \left. \frac{e^{3x}}{3} - e^x \right]_{0}^{\ln 5} \\ &= \left( \frac{e^{3\ln 5}}{3} - e^{\ln 5} \right) - (1/3 - 1) \\ &= \left( \frac{5^3}{3} - 5 \right) + \frac{2}{3} = 112/3 \end{aligned}$$