

MAT-021B (Koeppel): Midterm #2

Fill in your name and student id:

Name: Solutions

Student Id:

We fill the following when we grade this exam:

Problem: 1 2 3 4 5 6 7 8 Total

Points:

of: 12 12 12 12 13 13 13 13 100
(possible points)

Note: In some of the applications to physics, complicated numbers may appear; since calculators are not allowed in the exams, it is not necessary to do the arithmetic in these cases.

Write your answers in the space provided. Show your work.

Find the volume of the described solid.

- 1) The base of a solid is the region between the curve $y = 3\cos x$ and the x -axis from $x = 0$ to $x = \pi/2$. The cross sections perpendicular to the x -axis are squares with diagonals running from the x -axis to the curve.

The correct cross section:



Area of this square = $\frac{9\cos^2 x}{2}$

$$\begin{aligned} \text{So volume} &= \int_0^{\pi/2} \frac{9}{2} \cos^2 x \, dx = \frac{9}{2} \int_0^{\pi/2} \frac{1}{2} (1 + \cos 2x) \, dx \\ &= \frac{9}{4} \int_0^{\pi/2} 1 + \cos 2x \, dx \\ &= \frac{9}{4} \left[x + \frac{\sin 2x}{2} \Big|_0^{\pi/2} \right] = \frac{9}{4} \cdot \frac{\pi}{2} = \boxed{\frac{9\pi}{8}} \end{aligned}$$

Grading: Since most interpretations of the cross-section resulted in an integral with a $\cos^2 x$, I gave 6 points for correctly integrating $\cos^2 x$, and 6 points for having the correct cross-section formula & integral.