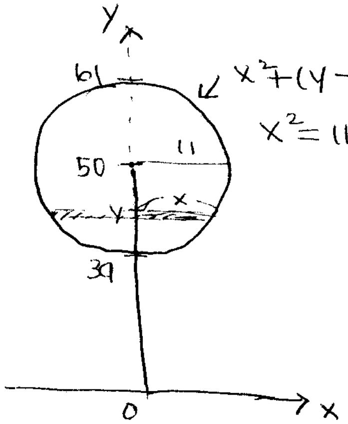
#6.



$x^2+(y-50)^2=11^2$

$x^2=11^2-(y-50)^2$

$\Delta V = \pi r^2 \Delta y = \pi x^2 \Delta y$

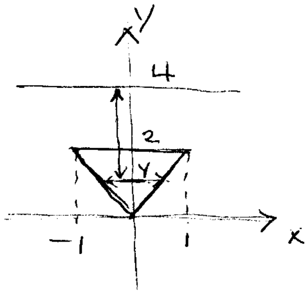
$= \pi (11^2-(y-50)^2)\Delta y$

$F = w \cdot \Delta V = 62.4\pi(11^2-(y-50)^2)\Delta y$

$\Delta W = 62.4\pi(11^2-(y-50)^2)\, y\, \Delta y$

$\Rightarrow W = \int_{39}^{60} 62.4\pi(11^2-(y-50)^2)\, y\, dy$

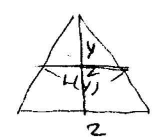
#7



$F = \int w \begin{pmatrix}\text{strip}\\ \text{depth}\end{pmatrix} L(y)\, dy$

strip depth $= 4-y$

$L(y) = y$ (similar triangle,



$\frac{2}{2} = \frac{y}{L(y)} \Rightarrow L(y)=y$)

or center of mass : $(0, \frac{4}{3})$

$\bar{h}$ = distance between center of mass and surface

$F = w\bar{h}A$

$= w\cdot(4-\frac{4}{3})\cdot(\frac{1}{2}\cdot 2\cdot 2)$

$= w\cdot\frac{8}{3}\cdot 2 = \frac{16w}{3}$.

$\Rightarrow F = \int_0^2 w(4-y)y\, dy = \int_0^2 w(4y-y^2)\, dy$

$= w(2y^2 - \frac{1}{3}y^3)\Big|_0^2$

$= w(8-\frac{8}{3}) = 8w(1-\frac{1}{3}) = \frac{16}{3}w$

#8 $\int \frac{(2r-1)\cos\sqrt{3(2r-1)^2+5}}{\sqrt{3(2r-1)^2+5}}\, dr = \int \cos u \frac{du}{6} = \frac{1}{6}\int \cos u\, du$

$= \frac{1}{6}\sin u + c$

$= \frac{1}{6}\sin\sqrt{3(2r-1)^2+5} + C$

Let $u = \sqrt{3(2r-1)^2+5}$

$\Rightarrow du = \frac{1}{2}(3(2r-1)^2+5)^{-1/2}\cdot 6(2r-1)\cdot \cancel{2}\, dr$

$= \frac{6(2r-1)}{\sqrt{3(2r-1)^2+5}}\, dr$

$\Rightarrow \frac{du}{6} = \frac{(2r-1)}{\sqrt{3(2r-1)^2+5}}\, dr$