Solutions

Quiz 1
Thursday August 7

1. (5 pts) A function f is uniformly continuous if the following property holds:

$$\forall \epsilon > 0, \exists \delta > 0 \text{ so that if } |x-y| < \delta, \text{ then } |f(x) - f(y)| < \epsilon.$$

Write the negation of the above property. (There should be no $\sim$ in your final answer.)

$\sim(\forall \varepsilon > 0\ \exists \delta > 0$ so that $|x-y| < \delta \longrightarrow |f(x)-f(y)| < \varepsilon)$

$\equiv\ \exists \varepsilon > 0 \sim (\exists \delta > 0$ so that $|x-y| < \delta \longrightarrow |f(x)-f(y)| < \varepsilon)$

$\equiv\ \exists \varepsilon > 0\ \forall \delta > 0\ \sim (|x-y| < \delta \rightarrow |f(x)-f(y)| < \varepsilon)$

$\equiv\ \exists \varepsilon > 0\ \forall \delta > 0\ |x-y| < \delta$ <u>and</u> $|f(x)-f(y)| \geq \varepsilon$.

(<u>Note</u>: the original statement implicitly has $\forall x, y$ in front of the $|x-y| < \delta$ so technically in the negation there should be a $\exists x, y$ before the $|x-y| < \delta$)

2. (5 pts) Prove:

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

<u>Part 1</u>

$A \subseteq B \longrightarrow A \cap B = A$.

<u>Part 1</u>. $A \subseteq B \rightarrow A \cap B \subseteq A$

Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. (Don't need $x \in B$).
$\hookrightarrow$ so $A \cap B \subseteq A$.

<u>Part 2</u>. $A \subseteq B \rightarrow A \subseteq A \cap B$.

Let $x \in A$. Then $x \in B$ b/c $A \subseteq B$. So $x \in A$ and $x \in B$. therefore $x \in A \cap B$. Therefore $A \subseteq A \cap B$.

Since $A \subseteq B \rightarrow A \cap B \subseteq A$ and $A \subseteq B \rightarrow A \subseteq A \cap B$, $A \subseteq B \rightarrow A = A \cap B$.

<u>Part 2</u>.

$A \cap B = A \rightarrow A \subseteq B$.

Let $x \in A$. Since $A = A \cap B$, in particular $A \subseteq A \cap B$. Therefore $x \in A \cap B$. So $x \in B$. So $A \subseteq B$.

3. (a) (5 pts) Suppose you want to prove that $p \to q$. Explain what a proof by contrapositive is, and show via truth table it is equivalent.

The <u>contrapositive</u> of $p \to q$ is $\sim q \to \sim p$

They are equivalent b/c they have the same truth table:

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

same truth table.

(b) (5 pts) Let f be the function $f(x) = 3x - 5$. Use the contrapositive to prove: if $x \neq y$, then $f(x) \neq f(y)$. (This says that f is injective).

The contrapositive of $x \neq y \to f(x) \neq f(y)$

is $f(x) = f(y) \to x = y$.

<u>Proof</u>: $f(x) = f(y)$

$\Rightarrow 3x - 5 = 3y - 5$

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$.