Lecture 0: Introduction to logic, and techniques of proof.

1. Truth tables and quantifiers

- Definition of a *statement*.
  Ex: $2 + 2 = 4$.
  Non-ex: $x^2 - 5x + 6 = 0$.
  Ex: For all x in the set $\{0, 1, 2, \dots\}$, $x^2 - 5x + 6 = 0$.
- Truth tables for $\sim p$, $p \wedge q$, and $p \vee q$.
- A contradiction, $p \wedge \sim p$, and a tautology $p \vee \sim p$.
- Negating $p \wedge q$, and $p \vee q$.
  Ex: H is a normal subgroup of G.
  Ex: K is convex and bounded.
  Ex: I is open or closed.
- $p \Leftrightarrow q$ and logical equivalence.
  Work out truth table to see why p and q are equivalent if $p \Leftrightarrow q$ is true.
- Negating $p \to q$ using $p \wedge \sim q$. Show via truth table.
  Ex: If a sequence is Cauchy, then it's convergent.
- Definitions of universal and existential *quantifiers*.
  Ex: For all x, $x^2 - 5x + 6 = 0$.
  Ex: There exists an x so that $x^2 - 5x + 6 = 0$.
- Negations of statements involving quantifiers.
  Ex: There exists a positive number x such that $x^2 = 5$.
  Ex: For every positive number M there exists a positive number N such that $N < 1/M$.
  Ex: If $n \geq N$, then $|f_n(x) - f(x)| \leq 3$ for all x in A.
  Ex: $\forall \epsilon > 0$ $\exists N$ so that $\forall n \geq N$, then $\forall x \in S$, $|f_n(x) - f(x)| < \epsilon$.

2. Techniques of proof

- Examples of disproof by counterexample.
  Ex: For every real number x, if $x^2 > 4$ then $x > 2$.
- Examples of proof by contrapositive and contradiction.
  Show the contrapositive $\sim q \Rightarrow \sim p$ is equivalent to $p \Rightarrow q$.
  Ex: Use the contrapositive to prove "If $7m$ is an odd number, then m is an odd number."
  Proof by contradiction: Assume p true and q false. Contradict a known theorem (or axiom).
  (Note the fine line between contradiction and contrapositive).
  Ex: Let x be a real number. If $x > 0$ then $1/x > 0$.
- Examples of the logical structure of proofs involving quantifiers.
  Theorem: For every $\epsilon > 0$, $\exists \delta > 0$ such that $1 - \delta < x < 1 + \delta$ implies that $5 - \epsilon < 2x + 3 < 5 + \epsilon$.
  Theorem: Let f be a continuous function. If $\int_0^1 f(x)dx \neq 0$, then $\exists x \in [0, 1]$ so that $f(x) \neq 0$.