Lecture 9

- Definition of a bounded sequence.
- Theorem: If (s_n) converges, it is bounded.
- Give a counterexample to the statement: If (s_n) is bounded, then it converges.
- Theorem: If $s_n \to s$ and $t_n \to t$, then $s_n + t_n \to s + t$.
- Theorem: If $s_n \to s$ and $t_n \to t$, then $s_n t_n \to st$.
- Theorem: If $s_n \to s$ and $s_n \neq 0\ \forall n$ and $s \neq 0$, then $\frac{1}{s_n} \to \frac{1}{s}$.
- Corollary: If $s_n \to s$ and $t_n \to t$, $s_n \neq 0\ \forall n$, and $s \neq 0$, then $\frac{t_n}{s_n} \to \frac{t}{s}$.
- Example: Use the above theorems to find

$$\lim_{n\to\infty} \frac{n^3 + 6n^2 + 7}{4n^3 + 3n - 4}.$$

- Make sure to read 9.7(a)-(d), and be able to reproduce the proofs.
- Definition of a sequence diverging to $\pm\infty$. What is the difference between this and merely being unbounded?
- Prove $n^2 - 3n$ diverges to ∞.
- Prove $\frac{n^2+3}{n+1}$ diverges to ∞.
- Theorem: If (s_n) diverges to ∞ and $\lim t_n$ exists and is positive, then $(s_n t_n)$ diverges to ∞.
- Theorem: (s_n) diverges to ∞ iff $\frac{1}{s_n} \to 0$.
- Theorem: If $\exists M$ so that $\forall n \geq M\ s_n \leq t_n$, and if $s_n \to s$ and $t_n \to t$, then $s \leq t$.