

MAT21C
Homework 1 (Assigned on April 1, 2009).

Please try to do as many problems as you can. Some of the problems from this homework may appear on the Quiz 1.

Problem 1.

Find a formula for the n -th term of the following sequences:

- a) The sequence 1, 4, 7, 10, 13, 16, 19, ...
- b) The sequence 1, -8, 27, -64, 125, -216, ... (Cubes of the positive integers; with alternating signs.)
- c) The sequence 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, ... (Each positive integer repeated three times.)
- d) The sequence -2009, 2009, -2009, 2009, -2009, 2009, -2009, 2009, ... (2009s with alternating signs.)

Problem 2.

Which of the sequences $\{a_n\}$ converge and which diverge? Find the limit of each converging sequence.

- a) $a_n = \frac{2n^2-1}{n^2}$
- b) $a_n = \tan(3 + (0.2)^n)$
- c) $a_n = \frac{10^n+5}{n^2}$
- d) $a_n = (-1)^n n$
- e) $a_n = \exp((1 + 1/n)^n)$
- f) $a_n = n^2 - \sqrt{n^4 - n^2}$
- g) $a_n = n \sin(1/n)$
- j) $a_n = \frac{n!}{(n/2)^{n/2}}$
- i) $a_n = \frac{0.8^n+0.7^n}{0.6^n+0.9^n}$
- k) $a_n = \cosh(\ln n + 10)$

Problem 3.

Determine if the sequence is nondecreasing and if it is bounded from above.

- a) $a_n = \frac{5^n}{(n-1)!}$

b) $a_n = \frac{5n+2}{n+1}$

c) $a_n = 10n - e^{-n}$

d) $a_n = n + 1/n$

Problem 4.

Prove Theorem 2.

Problem 5.

Prove that if $\{b_n\}$ and $\{c_n\}$ both converge to the same limit L , then the sequence

$$b_1, c_1, b_2, c_2, b_3, c_3, b_4, c_4, \dots$$

also converges to L .

Problem 6.

Find the limits of the following sequences.

a) $a_n = 2009^{2/n^2}$

b) $a_n = \frac{\ln(n+1)}{n^3}$

c) $a_n = nf(1/n)$, where $f(x)$ is differentiable on $[0, 1]$ and $f(0) = 0$.

d) $a_n = \int_2^n 1/x^p dx$, where $p > 1$.

Problem 7.

Exercise 107 (page 743).

Problem 8.

Exercise 110 (page 743).

Problem 9.

Which of the following series converge and which diverge? Give reasons for your answers. If a series converges, find its sum.

a) $\sum_{n=0}^{\infty} \sin(\pi n/4)$

b) $\sum_{n=0}^{\infty} (-1)^n \sqrt{n}$

c) $\sum_{n=0}^{\infty} \cos((n+1)/(n+2))$

d) $\sum_{n=0}^{\infty} (1 - 10/n)^n$

Problem 10.

Find the sum of the series.

a) $\sum_{n=0}^{\infty} \frac{10(-1)^n}{7^n}$

b) $\sum_{n=0}^{\infty} \cos(x)^n$, where $x \in (0, \pi)$.

c) $\sum_{n=0}^{\infty} \frac{2}{(5n+2)(5n-3)}$

d) $\sum_{n=0}^{\infty} (e^{-n} - e^{-n-1})$