MAT21C
Homework 4 (Assigned on April 22, 2009).

Please try to do as many problems as you can. Some of the problems from this homework may appear on Quiz 2.

Problem 1.

Find the taylor polynomials of orders 0, 1, 2, and 3 generated by $f(x)$ at $x = a$.

a) $f(x) = \ln(2 + x)$ at $x = 0$,

b) $f(x) = x^3$ at $x = 2$,

c) $f(x) = \sqrt{2x + 1}$ at $x = 0$,

d) $f(x) = \sin^2 x$ at $x = 0$,

e) $f(x) = \exp(x^2)$ at $x = 0$,

f) $f(x) = 1/x^2$ at $x = 1$.

Problem 2.

Find the MacLaurin series for the following functions:

a) $f(x) = e^{-x-5}$,

b) $f(x) = \sinh x = \frac{e^x - e^{-x}}{2}$,

c) $f(x) = x^{2009} - 2x^{2008} + 3x^{2007} - 4x^{2006} + \ldots - 2008x^2 + 2009x - 2010$,

d) $f(x) = \frac{2}{(1-x)^2}$.

Problem 3.

Problem 31 from page 795.

Problem 4.

Find the Taylor series generated by f at $x = a$

a) $f(x) = e^{2x}, \quad a = 1$,

b) $f(x) = 1/x^4, \quad a = 1$,

c) $f(x) = \frac{x}{2-x}, \quad a = 0$,

d) $f(x) = \sin(x), \quad a = \pi$,

e) $f(x) = x^{2009}e^x, \quad a = 0$,

f) $f(x) = \cos(x^{5/2}), \quad a = 0$,

g) $f(x) = \sin^2 x, \quad a = \pi,$

h) $f(x) = (x^2 + 1)\ln(1 + x), \quad a = 0.$

Problem 5.

Problem 34 from page 804.

Problem 6.

Problem 42 from page 805.

Problem 7.

If $\sin x$ is replaced by $x - (x^3/6)$ and $|x| < 0.3$, what estimate can be made of the error? Does $x - (x^3/6)$ tend to be too large or too small? Give reasons for your answer.

Problem 8.

For what positive values of x you can replace $\sin 2x$ by $2x - \frac{4}{3} * x^3$ with an error no greater than one percent of the value of x?

Problem 9.

Problem 44 from page 805.

Problem 10.

Use the Taylor series for $\cos x$ and the Alternating Series Estimate to show that

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}, \quad x \neq 0.$$