MAT235b/STA235b
Homework 4 (Assigned on February 18, 2016).

Problem 1.

Prove that ther number of simple random walk trajectories of length $2n$ that start at zero, end at zero, and never go below the horizontal axis (i.e. $X(0) = X(2n) = 0$, $|X(t+1) - X(t)| = 1$, and $X(t) \geq 0$ for all $0 \leq t \leq 2n$) is equal the Catalan number $\frac{(2n)!}{n!(n+1)!}$.

Hint: Note that $\frac{(2n)!}{n!(n+1)!} = C_{2n}^{n} - C_{2n}^{n+1}$, where $C_m^k = \frac{m!}{k!(m-k)!}$ is a binomial coefficient. Since C_{2n}^{n} is the total number of simple random walk trajectories of length $2n$ that start at zero and end at zero, it is enough to show that the number of those trajectories that cross the horizontal axis is equal to C_{2n}^{n+1}. To do this you may use an argument based on the reflection principle.

Problem 2.

Exercise 6.2.3 from page 281.

Problem 3.

Exercise 6.2.4 from page 281.

Problem 4.

Exercise 6.2.6 from page 282.

Problem 5.

Exercise 6.2.7 from page 282.

Problem 6.

Exercise 6.2.8 from page 282.

Problem 7.

Exercise 6.3.3 from page 287.

Problem 8.

Exercise 6.3.4 from page 287.