Image deblurring

This test problem is a discrete version of an integral equation of the form

$$\int K(s,t)f(t)dt + \delta(s) = g(s),$$

where the right-hand side g represents the blurred image and the kernel $K(s,t)$ represents the blurring function, δ is additive Gaussian noise. We will approximate the integral equation by a matrix system of equations of the form

$$Ax = b + \delta,$$

where A is the blurring operator, a Toeplitz-type band matrix. x is the original image, in our case, of dimension 16×16, and b is the blurred image, also of dimension 16×16. We can make x and b into vectors by stacking their columns, then x and b become 256×1 vectors. Consequently A will be a 256×256 matrix (use Matlabs commands $x(:)$ and $reshape(x, 16, 16)$ to transform x into a vector and back into a 16×16 image, respectively.)

The matrix A, and the images (vectors) x and b can be generated by using the function *blur*, which can be downloaded from my webpage. Use $[A, b, x] = blur(16, 3, 0.7)$ for this example. Note that A is a sparse matrix represented by using the *sparse*-format in Matlab. you can bring it into standard form if you want by using the Matlab command *full*.

We consider three different cases: (1) noisefree case (2) 1% noise (3) 5% noise. Here 1% noise means the right hand side is given by $b^\delta = b + \delta$, where $\delta = 0.01\|b\|_2 \cdot \text{randn}(m, 1)/\sqrt{m}$ and m is the length of b.

For all three cases use QR, SVD, truncated SVD, and Tikhonov regularization to compute a (regularized) approximation $\tilde{x}$ to x. For each of the noisy cases repeat the experiments 5 times with different random noises (but using the same percentage). Compute the relative errors $\|x - \tilde{x}\|_2 / \|x\|\|_2$ for all methods.

For all three cases plot the resulting images (only for one experiment for the noisy cases, not for all five). How did you select the regularization parameter? How do the results change when you vary the regularization parameters? Comments on your results.