Homework 2, due to February 3, 2006

Problem 1: Solve exercise 5.1

Problem 2: Let $A \in \mathbb{C}^{m \times n}$ $(m \geq n)$ with *reduced* SVD $A = U\Sigma V^*$, i.e. $U \in \mathbb{C}^{m \times n}$ and $V \in \mathbb{C}^{n \times n}$. Let $\mathcal{Q} := \{Q \in \mathbb{C}^{m \times n} : Q^*Q = I_n\}$ (i.e. the columns of Q are orthonormal). Show that the solution to the optimization problem

$$\min_{Q \in \mathcal{Q}} \|A - Q\|_F$$

is given by $Q := UV^*$.

Problem 3: Image compression experiments with SVD.

- Using Matlab load the image called "mandrill.mat" and display it on the screen (using the gray scale colormap). Print that plot.
- Compute the SVD of this mandrill image and plot the distribution of its singular values. Print that plot.
- Let σ_j, u_j, v_j be the j-th singular value, left and right singluar vector respectively of the mandrill image. Define the rank-k approximation of an image $X \in \mathbb{R}^{m \times n}$ as

  $$X_k := \sum_{j=1}^{k} \sigma_j u_j v_j^*, \quad k = 1, \ldots, \min(m, n).$$

  Then, for $k = 1, 6, 11, 31$ compute X_k of the mandrill and display the results. Fit these four images on one plot by using the `subplot` function and print them out.
- For $k = 1, 6, 11, 31$ display the residuals, i.e., $X - X_k$. Again, print them out on one page.
- In light of the distribution of the singular values what results would you predict (without computing the images) for X_k for larger k (say $k = 50$ or $k = 100$)? Would the SVD make a good image compression method? Why? Why not?

Problem 4: This problem is from the chapter **Fourier analysis** of Cleve Moler's book (you can download the pdf-file from my 229A webpage). Do exercise 8.1 (the one about the telephone number).

You can download the relevant Matlab files from www.mathworks.com, click on *Academia*, click on *Cleve Moler's textbook*, and you will get the option to download the Matlab files (including those needed for this exercise) accompanying his book.

Problem 5: Write a program that plots the time it takes Matlab to compute an FFT of a (random) vector of length n, where $n = 2, 3, \ldots, 1023, 1024$ (you can use `tic` and `toc` to measure the elapsed time in Matlab). Comment on the plot, what conclusions can you draw?

Problem 6: Solve exercise 6.1

Problem 7: Solve exercise 6.3