

## Homework 3, due to February 17, 2006

All exercise numbers refer to the book ‘Numerical Linear Algebra’ of Trefethen and Bau.

**Problem 1:** Solve exercise 7.1

**Problem 2:** Solve exercise 7.4

**Problem 3:** Solve exercise 9.1

**Problem 4:** Solve exercise 9.2

**Problem 3:** Solve exercise 10.1

**Problem 5:** Solve exercise 10.2

**Problem 6:** Solve exercise 10.3

**Problem 7:** Solve exercise 11.3

**Problem 8:** Write a Matlab function `[Q,R] = clgs(A)` that computes a reduced QR factorization  $A = \hat{Q}\hat{R}$  of a real-valued  $m \times n$  matrix  $A$  with  $m \geq n$  using the classical Gram Schmidt algorithm (whence `clgs`). The output variables are a matrix  $\hat{Q} \in \mathbb{R}^{m \times n}$  with orthonormal columns and an upper triangular matrix  $\hat{R} \in \mathbb{R}^{n \times n}$ . Test your algorithm by applying it to a random matrix (generated via `randn`) and to the Hilbert matrix (generated via `hilb`) of size  $10 \times 10$ ,  $50 \times 50$ , and  $100 \times 100$ . Check the accuracy by computing  $\|A - \hat{Q}\hat{R}\|_F$  and  $\|\hat{Q}^*\hat{Q} - I\|_F$  (the latter tests orthogonality of  $\hat{Q}$ ).