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HOW TO RECOVER SMOOTH OBJECT BOUNDARIES IN NOISY MEDICAL IMAGES

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ABSTRACT

Diagnostics in medicine is often based on analysis of medical images. Extraction of the shape of objects is one basic step for further image analysis. It consists of two steps: (i) edge detection and (ii) contour recovery based on the detected edge points. In this paper we focus on the second step. We describe the boundary of an object by trigonometric polynomials and present a fast and robust method for the approximation of the boundary from a given set of nonuniformly distributed noisy edge points. We show how the proposed technique can be efficiently generalized to 3-D surface recovery. Examples from 2-D and 3-D echocardiography are given.

1. INTRODUCTION

Medical imaging has become an important support for diagnostics in medicine. The automatic extraction of relevant parameters from medical images is of basic interest. Such parameters often depend on the shape of the object of interest. The recovery of an object boundary comprises two steps: (i) edge detection (ii) contour recovery based on detected edge points. The first step can be performed through well-established algorithms (e.g. wavelet-based or neural network based edge detectors). However these edge detection algorithms are heavily hampered by the presence of interfering biological structures (such as papillar muscles in echocardiography), the uneveness of boundary contrast and various kinds of noise. Therefore recovery of a smooth object boundary from detected edge points is a non-trivial task.

In the proposed approach we model the boundary of a smooth object by a trigonometric polynomial p and present and efficient method for the computation of p, based on an algorithm originally designed for the reconstruction of band-limited signals from nonuniformly distributed samples. Due to the noise we are interested in an approximation of the boundary rather than an interpolation of the edge points.

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2. SMOOTH CONTOUR APPROXIMATION

In the sequel we present an efficient method for the recovery of smooth contours from nonuniformly spread edge points. In a nutshell the contour points are interpreted as samples of a complex function and the Euclidean distance between the contour points gives the sampling interval.

In two dimensions the boundary of an object is a closed curve f in the plane. We parametrize f by $f(t) = (x_t, y_t)$, where x_t and y_t are the coordinates of f at time t in the x- and y-direction. Obviously we can interpret f as a one-dimensional continuous complex and periodic function, where x represents the real part and y represents the imaginary part respectively. By a slight abuse of notation we may also write $f(t_j) = x_j + iy_j$. It follows from the Theorem of Weierstrass (and from the Theorem of Stone-Weierstrass [1] for higher dimensions) that a continuous periodic function can be approximated uniformly by trigonometric polynomials. For smooth functions we can fairly assume that trigonometric polynomials of low degree provide an approximation of sufficient precision.

To give a more detailed description of the situation, let us assume that $f(t) = (x_t, y_t)$ represents the object boundary we want to reconstruct. Assume that an edge detection provides the points

$$c_j = (u_j, v_j) = (x_j, y_j) + \varepsilon_j, \qquad j = 1, \dots r. \tag{1}$$

These points are in general contaminated by noise (ε

represents the noise) and nonuniformly spread due to interfering structures or image regions of low contrast. We relate the curve parameter t to the contour points $c_j = (u_j, v_j)$ by computing the distance between two successive edge points, i.e. by setting $t_1 = 0$ and

$$t_j = t_{j-1} + \sqrt{(u_j - u_{j-1})^2 + (v_j - v_{j-1})^2} \qquad (2)$$

for $j = 2, \ldots, r$.

From this point of view, we are given the sampling points $\{t_j\}_{j=1}^r$, the sampling values $s_j = u_j + iv_j$ and we are looking for a trigonometric polynomial $p(t) = \sum_{k=-M}^{M} a_k e^{2\pi ikt}$ of degree $M \leq r/2$ that approximates f. In other words we have to solve the least squares problem

$$\sum_{j=1}^{r} |p(t_j) - s_j|^2 = minimum\,, \qquad (3)$$

where the minimum is taken over all trigonometric polynomials p of degree M.

2.1. An efficient algorithm for contour recovery

There exist several methods to solve this least squares problem (3), we choose the ACT method developed in [2], since it seems to be the most efficient and flexible one. Moreover it can be easily generalized to higher dimensions. This method is based on the fact, that the least square problem (3) can be formulated as Toeplitz system of equations, where the Toeplitz matrix is of size $(2M+1) \times (2M+1)$. The coefficients $\tilde{a}$ of the trigonometric polynomial $\tilde{p}(t) = \sum_{k=-M}^{M} \tilde{a}_k e^{-2\pi ikt}$ which minimizes (3) are computed by solving the Toeplitz system $T\tilde{a} = b$ iteratively by the conjugate gradient method.

Algorithm 1
Input: Sampling points t_j, sampling values s_j, and the degree M.
Step 1: Compute the entries of T and b by

$$T_{kl} = \sum_{j=1}^{r} e^{2\pi i(k-l)t_j}, \qquad (4)$$

$$b_k = \sum_{j=1}^{r} s_j e^{2\pi ikt_j} \qquad (5)$$

for $|k|, |l| \leq M$.
Step 2: Initialize $r_0 = b, a_0 = 0, n = 0$ and compute $\tilde{a}$ iteratively:

while $r_n \neq 0$
  $n = n + 1$
  if $n = 1$
    $q_1 = r_0$
  else
    $\beta_n = r_{n-1}^* r_{n-1} / r_{n-2}^* r_{n-2}$
    $q_n = r_{n-1} + \beta_n q_{n-1}$
  end
  $\alpha_n = r_{n-1}^* r_{n-1} / q_n^* T q_n$
  $a_n = a_{n-1} + \alpha_n q_n$
  $r_n = r_{n-1} - \alpha_n T q_n$
end
$\tilde{a} = a_n$

Step 3: The solution $\tilde{p}$ is then given by

$$\tilde{p}(t) = \sum_{k=-M}^{M} \tilde{a}_k e^{2\pi ikt} \qquad (6)$$

It follows from the Fundamental Theorem of Algebra that a trigonometric polynomial of degree M is uniquely defined by $2M + 1$ points. As an immediate consequence we obtain that T is invertible if $r \geq 2M + 1$, see [2]. For a more detailed discussion of the algorithm (e.g. an estimation of the rate of convergence) we refer to [2]. We only mention that the matrix-vector multiplication in Step 2 and also Step 3 can be carried out by FFT.

Algorithm 1 provides an efficient method to compute the optimal approximation p to f for given M. The question arises how to determine the optimal degree M. Since the degree of the polynomial defines the smoothness of the solution, the choice of M depends on the noise level, but also on the distribution of the edge points c_j. One way to compute a reasonable M is based on the so-called L-curve, recently propagated by Hansen [3].

The idea is as follows: Our approximation $\tilde{p}$ has to satisfy two main conditions: (i) $\tilde{p}$ has to fit the samples s_j at the points t_j; (ii) $\tilde{p}$ should be a smooth function without high oscillations.

For noisy data these are two contrary conditions: If we want to satisfy condition (i), we have to choose a high degree for $\tilde{p}$, since the samples s_j are contaminated by noise and thus is smooth curve would not pass trough all given samples. On the one hand fitting exactly all samples would result in a highly oscillating polynomial of large degree. On the other hand choosing a polynomial of low degree on the other hand may result in a large deviation at the samples. The goal is to find a balance between these two conditions. The procedure is as follows. We run Algorithm 1 for different values of M and calculate for each M the least squares error $\sum_{j=1}^{r} |\tilde{p}_M(t_j) - s_j|^2$. If we plot this error versus the degree M, the resulting graph has a typical

L-curve-like shape, which is observed in many regularization problems. The point where the "L" has its corner determines the value of M which gives the best compromise between smoothness of the solution and approximation error at the samples.

Clearly this procedure increases the computational effort, because we have to run the algorithm several times for different choices of M. But fortunately the additional amount can be kept small, which is due to following observations. The entries of T have to be calculated only once for the largest M, since T contains all block Toeplitz matrices for smaller choices of M as sub-matrices. Furthermore for increasing M we use each solution $\tilde{a}$ as start value a_0 in Algorithm 1 for the next-higher choice of M, which reduces the number of iterations considerably.

2.2. Contour recovery in 2-D echocardiography

The problem of extracting the boundary of an object arises for instance in clinical cardiac studies, where the evaluation of cardiac function using parameters of left ventricular contractility is an important constituent of an echocardiographic examination [4]. These parameters are derived using contour tracing of endocardial borders. Therefore numerous attempts have been made to develop automated algorithms for contour detection, see e.g. [5, 6]. In [7] Leger et al. also propose trigonometric polynomials to describe the contour of the Left Ventricle (LV) LV in two dimensions. However they restrict themselves to polynomials of degree 3, our experiments showed that this restriction can often give inadequate results. Usually polynomials of higher degree are required to get a good approximation of the endocardial border. A physical interpretation of the Fourier coefficients of the interpolating polynomial can also be found in [7].

We have applied Algorithm 1 to two-dimensional short-axis echocardiographic images. The edge detection has been carried out by a neural network, cf. [8] for details. The upper two images in Figure 1 show the Left Ventricle and a 2-D ultrasonic image. The lower left image displays the detected contour points of the LV. The reconstructed contour is shown in the lower right image.

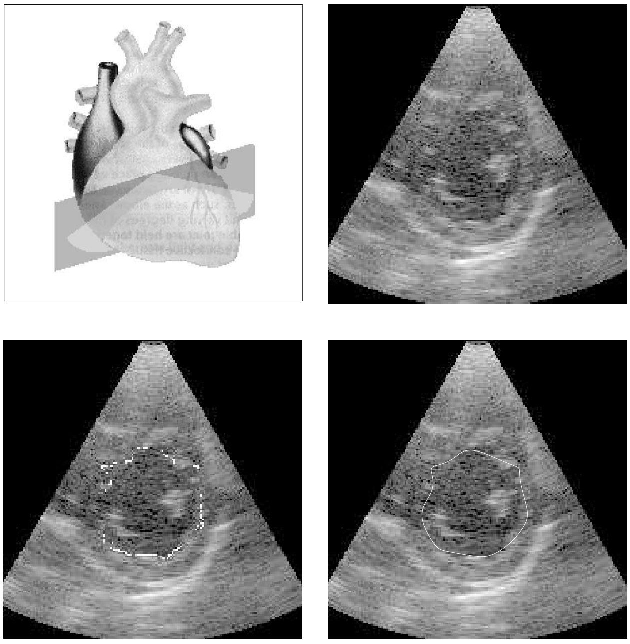

Figure 1:

3. APPROXIMATION OF SMOOTH SURFACES

Different systems such as computerized tomography, magnetic resonance imaging or ultrasound scanners provide digital images that are naturally two-dimensional (2-D cross sections or 2-D projections). Hence the challenge arises to develop reliable algorithms for creating 3-D images enabling the combination of data obtained from subsequent examinations in time. In the literature several methods are described for constructing a surface over a set of cross-sectional contours, see e.g. [9, 6]. We consider the reconstruction of a smooth closed surface σ from a given sequence of 2-D images. We assume that the given planar sections are perpendicular to the z-axis and that each of the intersections with the surface σ is a closed curve.

The representation of σ in cylindric coordinates is given by

$$\begin{aligned} x &= x_0 + \sigma(w, \Theta)\cos(\Theta) \\ y &= y_0 + \sigma(w, \Theta)\sin(\Theta) \\ z &= w \end{aligned}$$

for $a \le w \le b$ and $0 \le \Theta \le \pi$.

Assume we are given K images containing intersections of σ. We apply an edge detector to each image and obtain the data values

$$c_{jk} = (u_{jk}, v_{jk}, k) = (x_{jk} + \varepsilon_{x_{jk}}, y_{jk} + \varepsilon_{y_{jk}}, k) \qquad (7)$$

for $j = 1, \ldots, n_k, k = 1, \ldots, K$.

Since σ is a closed smooth surface we suggest approximating $\sigma = \sigma(x, y, z)$ by a 2-D trigonometric polynomial

$$p(s,t) = \sum_{l_1=-M_1}^{M_1} \sum_{l_2=-M_2}^{M_2} a_{l_1,l_2} e^{2\pi i(l_1 s + l_2 t)} . \qquad (8)$$

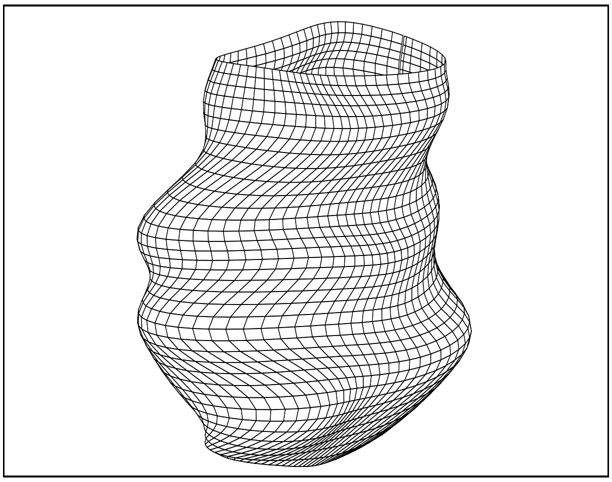

Figure 2: Approximation of the Left Ventricle by a 2-D trigonometric polynomial from 22 subsequent images

such that $(x, y, z) \simeq (Re(p(s,t)), Im(p(s,t)), t)$ and x_{jk}, y_{jk} and z_{jk} are related to s, t as described in equation (10).

To be more precise, given the data points $\{c_{jk}\}$ we are looking for the trigonometric polynomial p of degree $M_1 M_2$ which solves following least squares problem:

$$\min \sum_{j,k} |p(s_{jk}, t_{jk}) - (u_{jk} + iv_{jk})|^2 \qquad (9)$$

where the sampling coordinates are given by $t_{jk} = k \, \forall j$, $s_{1k} = 0$ and for $j = 2, \ldots, n_k$:

$$s_{jk} = s_{j-1k} + \sqrt{(u_{jk} - u_{j-1k})^2 + (v_{jk} - v_{j-1k})^2}\,. \quad (10)$$

We have transformed the 3-D surface reconstruction problem to a 2-D nonuniform sampling problem. In [10] it is shown that the least squares problem (3) can be solved efficiently by generalizing Algorithm 1 to two dimensions. The system matrix is now of block Toeplitz type, hence we can use 2-D FFT's to carry out the matrix-vector multiplications in the CG method. For a detailed presentation of the algorithm we refer the interested reader to [10]. We point out once more that the approximation of σ is computed using all data points simultaneously and not by recovering each 2-D contour independently followed by interpolating across these contours.

Typical applications for the proposed method arise e.g. in the 3-D visualization of tumors from CT or MRI images. Another important topic is the recovery of the 3-D shape of the Left Ventricle. Figure 2 shows a reconstruction of the surface from the LV, based on 22 subsequent 2-D transesophageal echoscans.

4. DISCUSSION

Since the presented algorithms are very fast, real-time implementations are possible. A clinical study where the 2-D contours computed by Algorithm 1 in combination with the edge detection proposed in [8] are compared to hand drawn contours by an expert is currently carried out at the Department of Cardioloy of the University of Vienna.

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