Quantized Maximum Ratio Transmission for Multiple-Input Multiple-Output Wireless Systems

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***Abstract*— Multiple-input multiple-output (MIMO) wireless systems can provide substantial gains in capacity and quality compared to single-input single-output (SISO) wireless systems. Maximum ratio transmission has been shown to be a low complexity solution to improving average receive signal-to-noise ratio (SNR), however, it requires feedback. Since in practice full channel knowledge at the transmitter is difficult to realize, we propose a technique where the receiver sends channel state information in the form of a codebook label. A codebook design method is based on a quantized version of traditional maximum ratio transmission with maximum ratio combining at the receiver. The codebook design criterion exploits the quantization problem's relationship with Grassmannian line packing. Systems using the transmit diversity codebooks are shown to have a diversity order of the product of the number of transmit and the number of receive antennas. Monte Carlo simulation compares the performance of systems using this new codebook method with previously proposed systems.**

I. Introduction

Beamforming, or optimal combining, is a low complexity technique for exploiting the spatial diversity available from multiple antennas and can be employed at both the transmitter and the receiver [1]. Optimal combining has long been used in receive diversity combiners for single-input multiple-output (SIMO) wireless systems [2]. When the transmitter has channel knowledge, the extension of the SIMO combining method to multiple-input single-output (MISO) beamforming is straightforward due to the fact that both systems encounter vector channels. The design methods used for vector channels, however, can not be directly applied to the matrix channels encountered in multiple-input multiple-output (MIMO) wireless systems [3]. Since an optimal combiner and beamformer must be designed transmission techniques such as maximum ratio transmission (MRT) [4], equal gain transmission (EGT) [3], and selection diversity transmission (SDT) [5] have all been extended to the case of MIMO wireless systems.

Each of these transmit diversity techniques requires channel information at the transmitter. SDT is the only method of the three that lends itself to straightforward digital implementation. MRT and EGT both require, in theory, infinite precision feedback. Since infinite resolution is impossible, it is preferable to quantize the beamforming vectors that the receiver sends to the transmitter. Limited feedback equal gain transmit diversity was explored in ([3],[6]) and called quantized equal gain transmission (QEGT). Quantized maximum ratio transmission systems were first addressed in [7] for the MISO case. Codebooks were designed using vector quantization techniques in [7], but a specific design criteria was not developed.

The objective of this paper is to develop design criterion for quantized maximum ratio transmission (QMRT) codebooks for independent identically distributed (i.i.d.) MIMO Rayleigh fading channels. We show that this new quantization technique should be based on a phase invariant mean squared error distortion function. Codebooks are designed for systems using quantized maximum ratio transmission at the transmitter combined with maximum ratio combining (MRC) at the receiver. A codebook design criterion is developed using a probabilistic analysis of the the optimal maximum ratio transmission vector and then tying the quantization of this vector to Grassmannian line packing.

This paper is organized as follows. Section II overviews MIMO beamforming systems. Grassmannian line packing and spherical codes are discussed in Section III. Section IV looks at the distribution of an optimal beamforming vector and ties the probabilistic results to Grassmannian line packing. The performance of quantized transmit diversity systems is analyzed in Section V. Section VI presents Monte Carlo simulation results. The paper concludes in Section VII.

II. System Overview

A MIMO system with transmit beamforming and receive combining is described by Fig. 1 with M_t transmit antennas and M_r receive antennas. A symbol s ($s \in \mathbb{C}$, the field of complex numbers) to be transmitted is multiplied by weight w_l ($w_l \in \mathbb{C}$) at the l^{th} ($0 < l \leq M_t$) transmit antenna. The weighted data received by the k^{th} ($0 < k \leq M_r$) receive antenna, sent by the l^{th} transmit antenna, is multiplied by a gain $h_{k,l}$ where $h_{k,l}$ is distributed according to $\mathcal{CN}(0, \sigma^2)$. This channel model is valid when there is a significant amount of scattering. This paper assumes $h_{k,l}$ is independent of $h_{m,n}$ if

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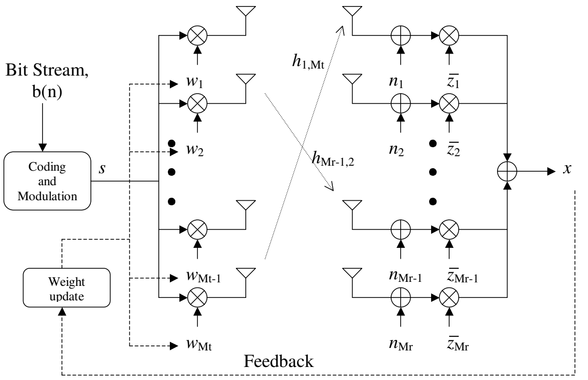

Fig. 1. Block diagram of a MIMO system.

$k \neq m$ or $l \neq n$. The data received by the k^{th} receive antenna is then added with white Gaussian noise n_k where n_k is distributed according to $\mathcal{CN}(0, N_0)$ and weighted by $\overline{z_k}$ ($z_k \in \mathbb{C}$ with $\bar{\cdot}$ denoting conjugation). The weighted output of each of the M_r receive antennas is summed yielding the combiner output x.

This formulation allows the input/output relationship to be written in matrix form. Let $\mathbf{w} = [w_1\ w_2\ ...\ w_{M_t}]^T$ (with ^{T} denoting matrix transposition), $\mathbf{z} = [z_1\ z_2\ ...\ z_{M_r}]^T$, $\mathbf{n} = [n_1\ n_2\ \ldots\ n_{M_r}]^T$, and $\mathbf{H}$ be the $M_r \times M_t$ matrix with coordinate (k, l) equal to $h_{k,l}$. Then the combiner output is $x = (\mathbf{z}^H\mathbf{H}\mathbf{w})s + \mathbf{z}^H\mathbf{n}$ where ^{H} denotes the matrix conjugate transpose.

Given the system in Fig. 1, we would like to design $\mathbf{w}$ and $\mathbf{z}$ to maximize error rate performance. It was shown in [3] that minimizing the average probability of symbol error (APSE) requires that we maximize $\mathcal{E}_r$, the receive energy. The formulation above allows us to express $\mathcal{E}_r$ in our system as

$$\begin{aligned}\mathcal{E}_r &= \mathcal{E}_t|\mathbf{z}^H\mathbf{H}\mathbf{w}|^2/||\mathbf{w}||_2^2 \\ &= \mathcal{E}_t\mathbf{\Gamma}_r\end{aligned} \tag{1}$$

where $\mathbf{\Gamma}_r$ is the weighted channel gain given by $\mathbf{\Gamma}_r = |\mathbf{z}^H\mathbf{H}\mathbf{w}|^2/||\mathbf{w}||_2^2$, $|\cdot|$ is the complex magnitude, $||\cdot||_2$ is the matrix 2-norm, and $\mathcal{E}_t$ is the transmit energy. Thus to maximize $\mathcal{E}_r$ we must maximize $\mathbf{\Gamma}_r$. To enforce the transmit power capacity, we will constrain $||\mathbf{w}||_2 = 1$ and assume $\mathcal{E}_t$ held constant. The SNR for this system is given by $\mathcal{E}_t\mathbf{\Gamma}_r/(||\mathbf{z}||_2\sigma^2)$. Therefore without loss of generality we also constrain $||\mathbf{z}||_2 = 1$.

A transmitter where $\mathbf{w}$ maximizes $|\mathbf{z}^H\mathbf{H}\mathbf{w}|$ for an arbitrary $\mathbf{z}$ is called maximum ratio transmission (MRT). In MIMO systems, the receive vector also needs to be chosen. A receiver where $\mathbf{z}$ maximizes $|\mathbf{z}^H\mathbf{H}\mathbf{w}|$ given $\mathbf{w}$ is called a maximum ratio combiner (MRC). This paper assumes the receiver uses MRC.

Given no design constraints on the form of $\mathbf{w}$ or $\mathbf{z}$, the optimal solutions in an APSE sense are the transmit and receive weights that maximize $\mathcal{E}_r$. For a combining scheme that solves for transmit weight vector $\mathbf{w}$ using the solution set $\mathcal{W}$ ($\mathcal{W} \subseteq \Omega_{M_t}$ with Ω_{M_t} denoting the set of unit vectors in $\mathbb{C}^{M_t}$) with an MRC receiver, $\mathbf{w}$ is given by [8]

$$\mathbf{w} \in \arg\max_{\mathbf{x}\in\mathcal{W}} ||\mathbf{H}\mathbf{x}||_2. \tag{2}$$

This result follows since the optimal MRC weight vector $\mathbf{z}$ for any weight vector $\mathbf{w}$ is known to be $\mathbf{z} = \mathbf{H}\mathbf{w}/||\mathbf{H}\mathbf{w}||_2$.

The transmitter that this paper considers has no knowledge of the channel so the receiver must send $\mathbf{w}$ to the transmitter. Since $\mathbf{w}$ can be any unit vector in a possibly large solution set (Ω_{M_t} for MRT) it is necessary to introduce some method of quantization. A reasonable solution is let the receiver and transmitter both use a codebook of N vectors. This allows the receiver to replace the costly singular value decomposition (SVD) of $\mathbf{H}$ needed for an MRT/MRC system (a system using MRT at the transmitter and MRC at the receiver) [3] with a simple brute force search of the N vectors. The main benefit of using a reduced solution set for $\mathbf{w}$ is that the number of bits sent back can be kept to a manageable number given by $\lceil \log_2 N \rceil$. Unfortunately, it is not obvious how to design an N vector codebook.

III. Grassmannian Line Packing & Spherical Codes

The Grassmannian line packing problem and spherical codes play key role in the design of our QMRT codebooks. In this section, we give a brief overview of both topics.

The Grassmannian space $\mathcal{G}(m, n)$ is the set of all n-dimensional subspaces of the space $\mathcal{M}^m$, where $\mathcal{M} = \mathbb{R}$ or $\mathcal{M} = \mathbb{C}$. Since the matrix channels that our MIMO system deals with are complex, we will only consider $\mathcal{M} = \mathbb{C}$. We are also interested in the application of Grassmannian subspace packing to the quantization of transmit diversity weight vectors therefore we will deal exclusively with $\mathcal{G}(m, 1)$.

The *Grassmannian packing problem* is the problem of finding the best packing of N n-dimensional subspaces in $\mathcal{M}^m$. In other words, we want to find N points in $\mathcal{G}(m, n)$ so that the minimal distance between any two of them is as large as possible. Since we are dealing with $\mathcal{G}(m, 1)$ for $\mathcal{M} = \mathbb{C}$, the subspaces are lines through the origin in $\mathbb{C}^m$ and the goal is to arrange the lines such that the angle between any of two of the lines becomes as large as possible [9]. Note that if $\mathbf{w}$ lies on a line then so does $e^{j\phi}\mathbf{w}$ for all $\phi \in [0, 2\pi)$. Thus the problem simplifies down to arranging N vectors so that the magnitude correlation between any two lines is as small as possible.

Let ρ be a function that maps the set of $m \times N$ matrices with complex unit vector columns, $\mathcal{U}_m^N$, to the set of nonnegative reals, defined by

$$\rho(\mathbf{W}) = \max_{0<k<l\leq N} |\mathbf{w}_k^H\mathbf{w}_l| \tag{3}$$

where $\mathbf{w}_k$ is the k^{th} column of $\mathbf{W}$. A set of N lines (vectors) is defined to be equiangular if $\left|\mathbf{w}_k^H\mathbf{w}_l\right| = \cos(\theta)$ for all k, l with $k \neq l$ with $\theta \in [0, \pi/2]$.

A set of N vectors representing N lines is a type of spherical code. A *spherical code* of dimension m is defined to be a set of points on Ω_m [10]. Each spherical code has a corresponding matrix in $\mathcal{U}_m^N$. We will define a *complex antipodal spherical code* to be a code with the property that if $\mathbf{w}$ is a member of the spherical code then $e^{j\phi}\mathbf{w}$ is a member of the spherical code for all $\phi \in [0, 2\pi)$. We will restrict the complex antipodal spherical codes to be uniquely represented by a set of N non-collinear vectors. A useful measure of a spherical code's quality is its *density.* Let $\mathbf{W}$ be a $m \times N$ matrix whose columns form an N vector complex antipodal spherical code (i.e. $\mathbf{W} \in \mathcal{U}_m^N$). For a $\gamma \in [0, 1)$ and $\mathbf{w} \in \Omega_m$, we will define the spherical cap of radius γ around $\mathbf{w}$ as the set

$$\mathcal{C}_{\mathbf{w}}(\gamma) := \left\{\mathbf{x} \in \Omega_m \ \mid \ |\mathbf{x}^H\mathbf{w}| \geq \gamma\right\}. \tag{4}$$

For a complex antipodal spherical code, the maximum angular radius is given by $\theta_r := \arccos(\rho(\mathbf{W}))/2$.

We will define the density of an N vector spherical code as

$$\begin{aligned}\Delta(\mathbf{W}) &:= \frac{\sum_{k=1}^{N} S\left(\mathcal{C}_{\mathbf{w}_k}\left(\cos\left(\theta_r\right)\right)\right)}{S\left(\Omega_m\right)} \\ &= \frac{NS\left(\mathcal{C}_{\mathbf{w}_1}\left(\cos\left(\theta_r\right)\right)\right)}{S\left(\Omega_m\right)} \end{aligned} \tag{5}$$

where S is a function that gives the surface area of a set of points on the unit sphere and $\Delta : \mathcal{U}_m^N \rightarrow [0, 1]$. The following theorem points out an important fact about the density function.

Theorem 1: Let $\mathbf{W}, \widetilde{\mathbf{W}} \in \mathcal{U}_m^N$. $\Delta(\mathbf{W}) > \Delta(\widetilde{\mathbf{W}})$ if and only if $\rho(\mathbf{W}) < \rho(\widetilde{\mathbf{W}})$ [8].
Thus, to maximize the density we must minimize the maximum absolute correlation.

IV. Codebook Analysis and Design

In [3] it is shown that an optimal beamforming vector for MRT systems is the dominant right singular vector of $\mathbf{H}$ with $\mathbf{H}$ defined as in Section II. Therefore, $\mathbf{w}_{MRT}$ that satisfies (2) ($\mathcal{W} = \Omega_{M_t}$) is an optimal MRT solution. Another restatement of this is

$$\mathbf{w}_{MRT} \in \arg\max_{\mathbf{x}\in\Omega_{M_t}} |\mathbf{x}^H\mathbf{H}^H\mathbf{H}\mathbf{x}|. \tag{6}$$

It is also important to note that if $\mathbf{w}_{MRT}$ satisfies (6), then $e^{j\phi}\mathbf{w}_{MRT}$ also satisfies (6) since $e^{j\phi}$ has unit magnitude. Therefore the beamforming vector is not unique.

Let us define this property in terms of points on a complex line instead of vectors. Let $\mathcal{P}_{\mathbf{w}}$ denote the column space of the vector $\mathbf{w}$. This subspace $\mathcal{P}_{\mathbf{w}}$ can easily be understood as a line in $\mathbb{C}^{M_t}$ passing through the origin. We will define a *representative vector* of $\mathcal{P}_{\mathbf{w}}$ as any vector in Ω_{M_t} whose column space is equal to $\mathcal{P}_{\mathbf{w}}$. We can now state an important property of transmit diversity systems. *For any line $\mathcal{P}$ in $\mathbb{C}^{M_t}$ that passes through the origin, all vectors $\mathbf{w} \in (\mathcal{P} \bigcap \Omega_{M_t})$ provide the same receive SNR and the same probability of symbol error.*

The distribution of the optimal weight vector must now be addressed. Let $\mathbf{H}$ be defined as in Section II with all entries independent. The distribution of $\mathbf{X} = \mathbf{H}^H\mathbf{H}$ is the complex Wishart distribution [11]. An important property of $\mathbf{X}$ is summarized in Theorem 2 [11].

Theorem 2: If $\mathbf{X}$ is complex Wishart distributed, then $\mathbf{X}$ is equivalent in distribution to $\mathbf{UDU}^H$ where $\mathbf{U}$ is Haar distributed on the group of $M_t \times M_t$ unitary matrices, and $\mathbf{D} = diag_{M_t}\{\lambda_1, \ldots, \lambda_{M_x}\}$ $(\lambda_i \geq \lambda_k \ \forall i \geq k)$, the diagonal matrix with $\lambda_1, \ \ldots, \lambda_{M_x}$ on the first M_x diagonal entries and zeros on the remaining diagonal terms, with $M_x = \min(M_r, M_t)$ has a distribution found in [11].

A very important property of Haar distributed matrices that will be exploited is given in the following theorem from [8].

Theorem 3: Let $\mathbf{U}$ be a Haar distributed $\mathrm{M}_t \times M_t$ random matrix. If $\mathbf{v} \in \Omega_{M_t}$ then $\mathbf{Uv}$ is uniformly distributed on Ω_{M_t}.

One solution to (6) is equivalent in distribution to $\mathbf{w}_{MRT} = \mathbf{U}[1 \ 0 \ 0 \ \cdots \ 0]^T$ with $\mathbf{U}$ given in Theorem 2. Since $\mathbf{U}$ is Haar distributed on $\mathcal{U}_{M_t}$ and $[1 \ 0 \ 0 \ \cdots \ 0]^T$ is a unit vector, Theorem 3 states that $\mathbf{w}_{MRT} = \mathbf{U}[1 \ 0 \ 0 \ \cdots \ 0]^T$ is distributed uniformly on Ω_{M_t}. It similarly follows that each of the columns of $\mathbf{U}$ and any unit norm linear combination of columns of $\mathbf{U}$ is uniformly distributed on Ω_{M_t}.

This result taken along with Theorem 2 reveals that the distribution of the optimal transmit weight vector is independent of the number of receive antennas. Thus *the problem of finding quantized transmit weights for MISO systems is the same problem as that of finding quantized transmit weights for MIMO systems.* Therefore MISO quantized transmit diversity analysis is directly applicable to MIMO systems.

We wish to obtain a codebook matrix $\mathbf{W}$ with unit code vector columns. We will denote the i^{th} column of $\mathbf{W}$ by $\mathbf{w}_i$. Because we wish to maximize the receive SNR, we use an encoding function $\mathcal{Q}_{\mathbf{w}} : \mathbb{C}^{M_r \times \mathrm{M}_t} \rightarrow \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N\}$ given by

$$\mathcal{Q}_{\mathbf{w}}(\mathbf{H}) = \arg\max_{\mathbf{x}\in\{\mathbf{w}_1,\mathbf{w}_2,\ldots,\mathbf{w}_N\}} \|\mathbf{H}\mathbf{x}\|_2^2. \tag{7}$$

Notice that this encoding function is not solely a function of the maximum singular value direction in the matrix channel case. The reason for this is that situations arise where it is better to use the codebook vector that is close to some linear combination of singular vectors. For example, channels where several of the singular values are equal would fall into this case.

We want to solve for $\mathbf{W}$ that maximizes the performance metric $G(\mathbf{W}) = E_{\mathbf{H}}\|\mathbf{H}\mathcal{Q}_{\mathbf{w}}(\mathbf{H})\|_2^2$ with $E_{\mathbf{H}}$ denoting expectation over $\mathbf{H}$. This can be rewritten as [8]

$$\begin{aligned} G(\mathbf{W}) &= E_{\mathbf{H}}\left[\max_{0<i\leq N} \sum_{k=1}^{M_t} \lambda_k |\mathbf{v}_k^H\mathbf{w}_i|^2\right] \\ &\geq E_{\mathbf{H}}\left[\lambda_1\right] E_{\mathbf{H}}\left[\max_{0<i\leq N} |\mathbf{v}_1^H\mathbf{w}_i|^2\right] \end{aligned} \tag{8}$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{M_t}$ are the M_t eigenvalues and $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{M_t}$ are the corresponding eigenvectors of the matrix $\mathbf{H}^H\mathbf{H}$. The set of vectors that maximizes the lower bound in (8) is the set of vectors that minimizes the phase-invariant minimum mean-squared error (MMSE) distortion function for an isotropically distributed unit vector source.

For spherical codes we believe that the following conjecture provides a necessary and sufficient characterization of optimal quantizers on the sphere Ω_m.

Conjecture 1: The optimal codebook for a phase-invariant MMSE quantizer for a source uniformly distributed on the complex unit sphere is a codebook whose vectors are maximally spaced and equiangular.

An explanation of the conjecture is given in [8]. Assuming the conjecture is true, there are two main difficulties with designing optimal codebooks. Equiangular codebooks are difficult if not impossible to find for most pairs (M_t, N). As well it is only possible to find equiangular codebooks if $N < M_t^2$ [9].

In order to address this problem we will further lower bound the result in (8). We can say

$$G(\mathbf{W}) \geq E_{\mathbf{H}}\left[\lambda_1\right]\cos^2\left(\frac{\theta_{min}}{2}\right)\Delta(\mathbf{W}). \tag{9}$$

The result in (9) follows from the fact that $\mathbf{v}_1$ is distributed uniformly on Ω_{M_t}.

Therefore we can attempt to maximize G by maximizing the lower bound and thus the spherical code density. Using the results in Theorem 1 we must therefore minimize the maximum absolute inner product between any pair of codebook vectors. Thus we propose the following criterion for designing Grassmannian beamforming codebooks.

Grassmannian Beamforming Criterion: Design the set of codebook vectors $\{\mathbf{w}_i\}_{i=1}^N$ such that the corresponding codebook matrix $\mathbf{W}$ minimizes $\rho(\mathbf{W}) = \max_{0<i<j\leq N} |\mathbf{w}_i^H \mathbf{w}_j|$.

As discussed in Section III, this criterion is exactly the same as spacing representative vectors so that the closest pair of corresponding complex lines has maximum angular separation. This problem is mathematically challenging from an analytical point of view [10], but can be approached from a brute force perspective where numerical simulations can yield good results.

A quantized version of maximum ratio transmission, which poses no restrictions except unit energy on the transmit weight vector, can thus be designed using the criterion. Thus the optimal codebook matrix $\mathbf{W}$ would be given by

$$\mathbf{W} \in \arg\min_{\mathbf{X}\in\mathcal{U}_{M_T}^N} \rho(\mathbf{X}). \tag{10}$$

Codebook design becomes, for a given M_t and N, finding the matrix in $\mathcal{U}_{M_t}^N$ with the smallest maximum absolute correlation between any two columns. Codebooks of this type can be design analytically when applicable [9], but in most cases computer searches must be employed. Notice that when $N \leq M_t$ maximally spaced packings are easy to find: simply take N columns of any $M_t \times M_t$ unitary matrix.

V. Codebook Performance

Closed form results on the average probability of symbol error for quantized transmit diversity systems are difficult if not impossible to determine. We will therefore bound the diversity, the asymptotic slope of the average probability of symbol error curve, for quantized transmit diversity systems using maximum ratio combining. The following analysis will hold for any transmit diversity technique that uses a codebook with at least M_t vectors. The only requirement is that the column vectors contained in the codebook span $\mathbb{C}^{M_t}$.

First consider the receive SNR, $\mathcal{E}_t\Gamma_r/\sigma^2$. Since $\mathcal{E}_t$ and σ^2 are assumed fixed, we only need to consider $\Gamma_r = |\mathbf{z}^H\mathbf{H}\mathbf{w}|^2$. The receive SNR of a quantized beamforming system will always be less than that of an unquantized MRT system. It was shown in [4] that an MRT/MRC system has a diversity advantage on the order of $M_r M_t$.

We will compute the SNR lower bound for our quantized transmit diversity and MRC system assuming that the codebook is overcomplete. For an N vector transmit diversity codebook system with MRC at the receiver, the effective channel gain is $\Gamma_r = \max_{0<i\leq N} ||\mathbf{H}\mathbf{w}_i||_2^2$ where $\mathbf{W}$ is the $M_t \times N$ codebook matrix. Because the columns of the codebook matrix $\mathbf{W}$ span $\mathbb{C}^{M_t}$, $\mathbf{W}$ can be factored via a singular value decomposition (SVD) into the form $\mathbf{W} = \mathbf{U}_1[\mathbf{D}\ \mathbf{O}]\mathbf{U}_2$ where $\mathbf{U}_1$ is an $M_t \times M_t$ unitary matrix, $\mathbf{O}$ is an $M_t \times (N - M_t)$ matrix of zeros, $\mathbf{U}_2$ is an $N \times N$ unitary matrix, and $\mathbf{D}$ is a real diagonal matrix with $d_{1,1} \geq d_{2,2} \geq \ldots \geq d_{M_t,M_t} > 0$. By the invariance of complex Gaussian random matrices [11], $\mathbf{H}\mathbf{U}_1^H$ is equivalent in distribution to $\mathbf{H}$. Therefore,

$$\Gamma_r \overset{d}{=} \widetilde{\Gamma}_r = \max_{0<i\leq N} ||\mathbf{H}\mathbf{U}_1^H\mathbf{w}_i||_2^2. \tag{11}$$

Now using matrix norm inequalities taken from [12], stated for the real case but easily seen to extend to the complex case, we find that

$$\widetilde{\Gamma}_r \geq \frac{1}{M_r}||\mathbf{H}\mathbf{U}_1^H\mathbf{W}||_1^2 \tag{12}$$

with $||\cdot||_1$ denoting the matrix 1-norm. Using the SVD, (12) can be rewritten as

$$\widetilde{\Gamma}_r \geq \frac{1}{M_r}||[\mathbf{H}\mathbf{D}\ \widetilde{\mathbf{O}}]\mathbf{U}_2||_1^2 \tag{13}$$

where $\widetilde{\mathbf{O}}$ is an $M_r \times (N - M_t)$ matrix of zeros.

By the matrix submultiplicative property [12],

$$||[\mathbf{H}\mathbf{D}\ \widetilde{\mathbf{O}}]\mathbf{U}_2||_1^2 \geq \frac{1}{N}||\mathbf{H}\mathbf{D}||_1^2. \tag{14}$$

Applying this bound we find that

$$\begin{aligned}\frac{1}{M_r}||[\mathbf{H}\mathbf{D}\ \widetilde{\mathbf{O}}]\mathbf{U}_2||_1^2 &\geq \frac{1}{NM_r}||\mathbf{H}\mathbf{D}||_1^2 \\ &\geq \frac{d_{M_t,M_t}^2}{NM_r}\max_{k,l}|h_{k,l}|^2. \end{aligned}\tag{15}$$

The result in (15) is the receive SNR of a MIMO selection diversity system with a constant gain factor. It was shown in [2] that selection diversity systems have a diversity gain of $M_r M_t$. Therefore, systems using quantized transmit diversity with MRC provide a diversity advantage on the order of $M_r M_t$.

VI. SIMULATIONS

All simulations used binary phase shift keying (BPSK) modulation and i.i.d. Rayleigh fading (where $h_{k,l}$ is distributed according to $\mathcal{CN}(0,1)$). The APSE is estimated using at least 1.5 million iterations per SNR point. Codebooks were designed using closed-form expressions when applicable [9] and random searches. Other simulation results are shown in [8].

Experiment # 1 The first experiment used $M_r = M_t = 2$. A QMRT/MRC system performance is shown for several different quantizations. The simulated error rate curve of an MRT/MRC system and the actual error rate curve for an SDT/MRC system are shown for comparison. An array gain of around 0.7dB is attained when going from 1 bit to 2 bits of quantization. However, the array gain going from 2 bits to 3 bits of quantization is only around 0.25dB. The simulation results are shown in Fig. 2.

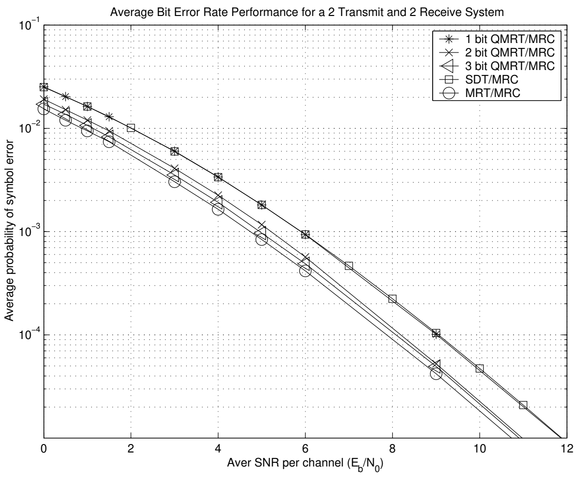

Fig. 2. Bounds and symbol probability of error for 2×2 systems using QMRT/MRC.

Experiment # 2 In [7], vector quantization techniques were used to design QMRT codebooks. In the third experiment, we compare a system using Grassmannian beamforming codebooks with a system using a codebook designed with Lloyd algorithm. Codebooks containing 8 vectors were designed for a $M_r = M_t = 2$ system. The results are shown in Fig. 3. This simulation verifies the validity of the proposed design criterion.

VII. CONCLUSION

This paper examined quantized transmit diversity schemes for MIMO wireless systems. The distribution of an optimal transmit weight vector for a system using MRT/MRC was

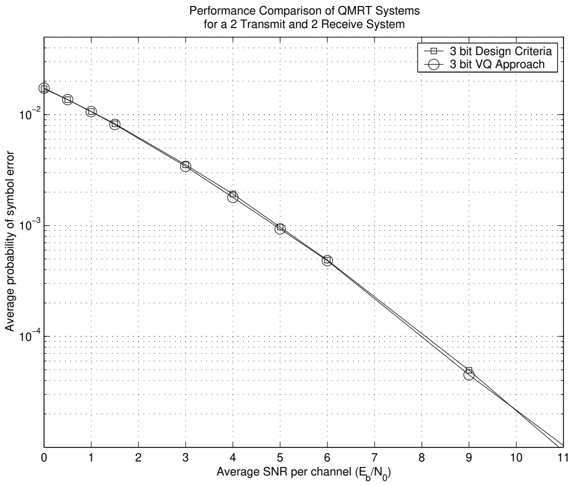

Fig. 3. Symbol probability of error for 2×2 systems using QMRT codebooks designed with the proposed criterion and with vector quantization.

found to be uniform on the complex unit sphere. It is conjectured that the optimal quantizer for this weight vector is a set of equiangular vectors. This conjecture leads to codebook design criterion using Grassmannian line packing.

Future work abounds in this area. Probabilistic performance analysis is needed to determine how close the performance of a quantized transmit diversity scheme can come to optimal performance. Also, when the channel is fading slowly it may be possible to derive codebooks adaptive to channel statistics.

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