

Adaptive and Robust Channel Estimation for Pilot-aided OFDM Systems

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Abstract—A time domain interpolation method is presented and applied to channel estimation for pilot-aided Orthogonal Frequency Division Multiplexing (OFDM) system. One key advantage of this method is that it dynamically selects suitable basis functions to approximate the channel transfer function by studying the shape of its Inverse Fourier transform. Which results in a very good approximation by using as few as possible basis functions, and provides significant reduction of additive white Gaussian noise (AWGN) and Inter Carrier Interference (ICI) at the same time. Theoretical analysis and simulations show that it is an efficient method for approximating various kinds of OFDM channels.

I. INTRODUCTION

OFDM has attracted considerable attention in the last decade due to its desirable properties such as its high data rate transmission capability with high bandwidth efficiency as well as its robustness to multipath delay spread. It has been adopted as a standard for wire and wireless communication such as Digital Subscriber Line (DSL), European digital Audio and Video Broadcasting (DAB/DVB), American IEEE 802.11(a) and European HiperLAN/2 [1], [2].

Coherent OFDM transmission requires an estimation of the channel frequency response. Usually, channel estimation in OFDM is carried out by transmitting known pilot symbols in given positions of the frequency-time grid. There are essentially two kinds of pilot insertions: block-type pilot arrangements and comb-type pilot arrangements [1]. Block type channel estimation methods, such as Least Square (LS) estimation and Minimum Mean-Square Error (MMSE) estimation, have been developed under the assumption of slow fading channels. In fast fading channel, the channel transfer function changes significantly even for adjacent OFDM data blocks, thus the comb-type pilots channel estimation performs much better than block type pilot estimation [1].

The traditional approach consists of two steps for comb-type pilot based estimation. First the channel at pilots frequencies is obtained by LS or MMSE estimation. In the second step the transfer function is estimated at the unknown tones by using some kind of interpolation or approximation based on the pilot information [3]. Among them there are two major types: Time domain estimation and frequency domain interpolation. In frequency-domain methods, first or second order interpolation, low-pass interpolation and cubic spline interpolation are

applied directly to pilot frequencies to approximate the channel frequency response. In time-domain estimation, one focuses on the Fourier transform of the frequency-domain channel response at pilot sub-carriers. Under the assumption that the number of pilot sub-carriers M is greater than the normalized maximum time delay aliasing is avoided [4].

While the more advanced of the aforementioned channel estimation methods work fairly well under certain assumptions, they do not fully utilize the available information provided by the pilots and the characteristics of frequency selective channels. For instance, while the maximum delay spread of the channel may be quite large, the impulse response itself exhibits a sparse structure, i.e., it is composed of only few (say, a dozen) relevant multipath delays.

A natural way to exploit the sparsity of the impulse response would be to use L^1 -minimization techniques [5]. While the recent mathematics and engineering literature has seen a plethora of publications on sparse representations and L^1 -minimization, the algorithms are computationally too expensive to be of practical interest for OFDM channel estimation. Furthermore these algorithms would have difficulties in dealing with non-integer time delays.

In this paper, we propose a robust and numerically efficient pilot-based channel estimation method for OFDM that utilizes the sparsity of the channel, is applicable to non-integer time delays, and performs well both in the low SNR as well as in the high-SNR regime. It outperforms the aforementioned channel estimation methods in a variety of scenarios.

By studying the Inverse Fourier transform of the frequency-domain channel response at pilot sub-carriers, we dynamically select a set of nonorthogonal basis functions to approximate the channel frequency response. The two-step procedure adaptively estimates first the number and positions of the unknown time delays. Then based on this estimate we dynamically select a set of non-orthogonal basis functions to approximate the channel frequency response. At the same time, this set of basis functions works as an adaptive “lowpass” filter which can reduce AWGN and ICI significantly. In addition, our method can also deal with non-uniformly distributed pilots.

The rest of the paper is organized as follows. In section II, the pilot based OFDM system is described. In section III, previous channel estimation methods are reviewed. Our interpolation method is proposed in section IV. An algorithm for estimating the time delays is introduced in section V. In section VI, we present an error analysis for our and other

channel estimation methods. Section VII presents simulation results and comparisons which indicate the bit error rate (BER) improvements. Section VIII concludes this paper. Proofs of the theoretical results presented in this paper are omitted due to space constraints and will be included in the journal version of this paper.

II. SYSTEM DESCRIPTION

We consider a standard cyclic-prefix OFDM system, and assume that the reader is familiar with the basic setup, cf. e.g. [6]. Binary data are grouped and mapped to a signal constellation, such as QPSK or 16-QAM. We denote the data sequence of an OFDM symbol consisting of N subcarriers by $\{X(k)\}_{k=0}^{N-1}$. Since we consider a pilot-aided OFDM system, we assume that the sequence $\{X(k)\}_{k=0}^{N-1}$ also contains the pilots tones, i.e., certain tones that are assigned specific values, the position and value of which are known to the receiver. In a comb-type pilot arrangement, N_p pilot symbols are uniformly inserted¹ into the sequence $\{X(k)\}_{k=0}^{N-1}$ with $X_p(m) := X(mL)$ where $L = \frac{N}{N_p}$ is an integer, $X_p(m)$ is the m -th pilot symbol value and $m = 0, 1, \dots, N_p - 1$.

The data sequence $\{X(k)\}_{k=0}^{N-1}$ is transferred into a time domain signal $\{x(n)\}_{n=0}^{N-1}$ by an Inverse Discrete Fourier Transform (IDFT), where

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}}, \quad n = 0, \dots, N-1. \quad (1)$$

After inserting a cyclic prefix at the beginning of each symbol to prevent inter-symbol interference (ISI), the signal symbols are modulated on a set of sub-carriers and transmitted through the frequency-selective channel.

After removing the cyclic prefix, the received discretized samples $\{y(n)\}_{n=0}^{N-1}$ associated with one OFDM block are sent to the DFT block with $Y = F\{y\}_{n=0}^{N-1}$, where F is the $N \times N$ DFT-matrix. Assuming there is no ISI, then by [7]

$$Y(k) = X(k)H(k) + I(k) + W(k), \quad k = 0, \dots, N-1. \quad (2)$$

Here $W(k)$ is AWGN, $H = F\{h(n)\}_{n=0}^{N-1}$ and I is the inter-carrier interference (ICI) caused by Doppler frequency with expressions

$$H(k) = \sum_{i=1}^r h_i e^{j\pi f_{D_i} T} \frac{\sin(\pi f_{D_i} T)}{\pi f_{D_i} T} e^{-\frac{2j\pi k \tau_i}{N}}, \quad (3)$$

$$I(k) = \sum_{i=1}^r \sum_{n=0, n \neq k}^{N-1} \frac{h_i X(n)}{N} \frac{1 - e^{j2\pi(f_{D_i} T - k + n)}}{1 - e^{j2\pi/N(f_{D_i} T - k + n)}} e^{-\frac{j2\pi \tau_i n}{N}}. \quad (4)$$

Where r denotes the number of propagation paths, h_i is the gain for the i -th path, f_{D_i} is the i -th path's Doppler frequency shift, T is the sampling time and τ_i is the time delay normalized by the sampling time.

¹While the pilots are usually uniformly spaced across the OFDM symbol, we note that for instance 802.16e contains some transmission modes that use (slightly) nonuniformly spaced pilot tones.

III. FREQUENCY AND TIME DOMAIN CHANNEL ESTIMATION METHODS

In comb-type pilot arrangements, the estimation of channel frequency response at pilot sub-carrier H_p is obtained by LS or MMSE estimation. For instance the estimation of channel frequency response at pilot sub-carriers based on LS estimation is given by:

$$H_p(m) = H_e(mL) = \frac{Y(mL)}{X(mL)} = \frac{Y(mL)}{X_p(m)}, \quad (5)$$

for $m = 0, \dots, N_p - 1$. The estimation of the whole channel frequency response H_e is then calculated from the information of H_p . This is where the various channel estimation methods differ. Having computed H_e the transmitted signal symbols are estimated by

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad k = 0, 1, \dots, N-1. \quad (6)$$

Then the source binary information is recovered at the receiver output after signal demapping.

From (3), it is natural to model the real channel frequency response function as

$$\phi(\omega) = \sum_{i=1}^{\tilde{r}} c_i e^{-2\pi j \omega \tau_i}, \quad (7)$$

where $\omega \in [0, 1]$, $c_i \in \mathbb{C}$, $\tilde{r} \in \mathbb{Z}$ and $\tau_i \in \mathbb{R}$. Thus $H(k) = \phi(\frac{k}{N})$ and the channel estimation problem can be formulated as follows: Given noisy samples $\{H_p(m) = \phi(\frac{m}{N_p}) + W(m)\}_{m=0}^{N_p-1}$, where $W(m)$ is AWGN, we try to approximate $\phi(\frac{k}{N})$ for $k = 0, 1, \dots, N-1$.

A variety of methods has been proposed in the literature to solve this channel estimation problem. This includes frequency domain methods such as linear interpolation, second order interpolation [8], low-pass approximation and cubic spline interpolation. Based on zero-padding and DFT/IDFT, a time-domain method was introduced in paper [7], this method is essentially trigonometric approximation. It works well in the low SNR case, however it discards some channel information since the time delays τ_i may be non integer, which degrades its overall performance especially when SNR is high [9]. Furthermore, like the other aforementioned methods, it does not fully utilize the fact that the impulse response has a sparse representation in the time domain (beyond the obvious fact that it is time-limited).

If we know r and the exact time delay τ_i in equation (3) we could compute the best least squares approximation for the channel frequency response H via

$$H_e = A(A_p^H A_p)^{-1} A_p^H H_p, \quad (8)$$

where the $N \times r$ matrix A has entries $A(k, i) = e^{-\frac{j2\pi k \tau_i}{N}}$ and the N_p by r matrix A_p has entries $A_p(m, i) = e^{-\frac{j2\pi m \tau_i}{N_p}}$ for $i = 1, 2, \dots, r$. However for realistic OFDM system, r is unknown and the time delays τ_i are also unknown. This is what makes pilot-based channel estimation challenging (and mathematically interesting)!

In the next two sections, we first show how our interpolation method can preserve the channel information by using non-orthogonal trigonometric polynomials provided the approximate time delay locations are known. Secondly, we propose an algorithm to estimate the unknown time delay locations.

IV. ADAPTIVE INTERPOLATION METHOD

Assume that the channel frequency response function ϕ has the expression (7), whose Fourier expansion is

$$\phi(\omega) = \sum_{n=-\infty}^{\infty} \alpha_n e^{-2\pi j \omega n}. \quad (9)$$

If one or more of the τ_i are not integers, then it follows from basic Fourier analysis that $\alpha_n \rightarrow \mathcal{O}(\frac{1}{|n|})$ as $|n| \rightarrow \infty$. This implies that if we use the $2n + 1$ -term Fourier series $\phi_n := \sum_{j=-n}^n \alpha_j e^{-2\pi i \omega j}$ to approximate ϕ , the truncation error

$$\|\phi - \phi_n\|_2 \rightarrow \mathcal{O}\left(\frac{1}{\sqrt{n}}\right), \quad \text{for } n \rightarrow \infty.$$

In practice this approximation rate is not good enough, especially when n is small! That is also the reason why the method introduced in [7] can not do well in preserving the channel information in such a case.

In the following, we show that by using non-orthogonal exponential polynomials as basis functions, we can achieve a much better approximation. But note that the choice of these nonorthogonal basis functions must be simple, otherwise the approach may not be practical.

Let $\psi_k(\omega) = e^{-2\pi j \omega (k + \frac{1}{2})}$ and $\varphi_k(\omega) := e^{-2\pi j \omega k}$ for $k = 0, 1, \dots$ with $\omega \in [0, 1]$. Furthermore, $P_{\mathbf{A}}(f)$ denotes the orthogonal projection of a function f onto a function space \mathbf{A} . Let $\phi(\omega) = e^{-2\pi j \omega t}$ with $\omega \in [0, 1]$ for $t \in [0, 2]$. We want to approximate this function (our toy channel frequency response) by a linear combination of a few (say 5) fixed trigonometric functions. First we use the standard space of orthogonal trigonometric functions $\mathbf{A} := \{\varphi_{-1}, \dots, \varphi_4\}$. Then we choose a space spanned by the non-orthogonal functions $\mathbf{B} := \{\varphi_0, \varphi_1, \varphi_2, \psi_0, \psi_1\}$. We define the error functions $E_{\mathbf{B}}(t) = \|\phi - P_{\mathbf{B}}(\phi)\|_2$ and $E_{\mathbf{A}}(t) = \|\phi - P_{\mathbf{A}}(\phi)\|_2$. From the numerical results we find that $\|E_{\mathbf{B}}(t)\|_{\infty} = 0.0109$ and $\frac{1}{\sqrt{2}}\|E_{\mathbf{B}}(t)\|_2 = 0.0060$, while $\|E_{\mathbf{A}}(t)\|_{\infty} = 0.2883$ and $\|E_{\mathbf{A}}(t)\|_2 = 0.2045$. Thus it seems that modifying the orthogonal trigonometric basis by adding a few non-orthogonal trigonometric functions at the ‘‘boundaries’’ can improve the channel estimation accuracy by counteracting the boundary effects caused by non-integer time delays.

Indeed, by extending \mathbf{A} by just two basis functions at the interval boundaries, we arrive at the following result:

Theorem 4.1: Let $\mathbf{A} = \text{span}\{\varphi_{-1}, \varphi_0, \dots, \varphi_{T+1}\}$ and $\mathbf{B} = \text{span}\{\psi_1, \psi_{T-1}, \varphi_0, \varphi_1, \dots, \varphi_T\}$. Suppose $\phi(\omega) = e^{-2\pi j \omega t}$ where $\omega \in [0, 1]$ and $t \in [0, T]$, then

$$\|\phi - P_{\mathbf{B}}(\phi)\|_2 < C(t)\|\phi - P_{\mathbf{A}}(\phi)\|_2, \quad (10)$$

where $T \geq 2$ and $0 < C(t) < 1$.

In fact, numerical results show

$$\|\phi - P_{\mathbf{B}}(\phi)\|_2 < 0.65\|\phi - P_{\mathbf{A}}(\phi)\|_2, \quad (11)$$

when $T \leq 50$.

Example 4.2: We consider the function $\phi(\omega) = e^{-2\pi j \omega t}$ with $\omega \in [0, 1]$ for $t \in [0, T]$, where integer $T \geq 2$. Let space $\mathbf{A} = \{\varphi_{-1}, \varphi_0, \dots, \varphi_{T+1}\}$ and space $\mathbf{B} = \{\psi_1, \psi_{T-1}, \varphi_0, \varphi_1, \dots, \varphi_T\}$. Define $E_1(T) = \frac{1}{\sqrt{T}}\|E_{\mathbf{B}}(t)\|_2$ and $E_2(T) = \frac{1}{\sqrt{T}}\|E_{\mathbf{A}}(t)\|_2$, Fig 1 shows the numerical results for T from 2 to 50.

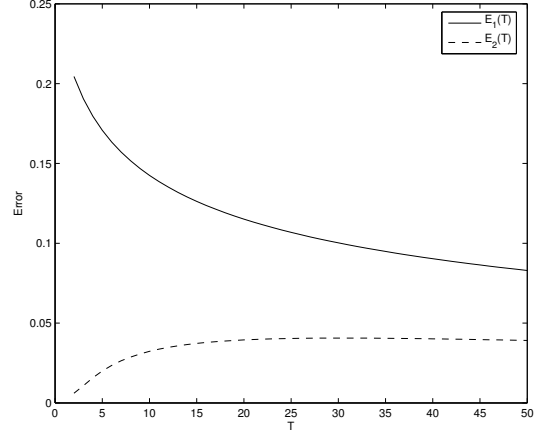


Fig. 1. Numerical results of approximation error

Clearly, we achieve a much better approximation by adding two nonorthogonal basis functions. But in practice the result shown in Fig 1 may be not good enough. For instance, in OFDM channel estimation, we would like the L^2 norm error less than 40 dB in channel estimation, which requires us to add more basis functions. But adding too many basis functions leads to ill conditioned problem. So we will find an equipoint between those problems. Fortunately, we find adding four basis functions is enough for many applications, especially when $T \leq 100$, we can always make $E_2(T) \leq 0.01$, while the condition number of matrix B defined as last example is still small.

For given T , the basis functions are chosen by solving the next problem

$$(n_1, n_2) = \underset{(n_1, n_2)}{\text{argmin}} E_2(T) \quad (12)$$

s.t. $\kappa(\mathbf{B}) \leq M_T$, where space $\mathbf{B} = \{\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3, \tilde{\psi}_4, \varphi_0, \dots, \varphi_T\}$ with $\tilde{\psi}_1 = \psi_{n_1}$, $\tilde{\psi}_2 = \psi_{n_2}$, $\tilde{\psi}_3 = \psi_{T-n_1-1}$ and $\tilde{\psi}_4 = \psi_{T-n_2-1}$. After solving (12), we choose \mathbf{B} as our final basis space. For $T \leq 100$, numerical results show we can always make $E_2(T) \leq 0.01$ when we set $M_T = 60$.

Theorem 4.3: Let space $\mathbf{A} = \{\varphi_1, \varphi_2, \dots, \varphi_M\}$, space $\tilde{\mathbf{A}} = \{\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_M\}$ and $\tilde{\varphi}_k(\omega) = \varphi_k(\omega)e^{-2\pi j r_0 \omega}$ for arbitrary $r_0 \in \mathbb{R}$ and $k = 1, 2, \dots, M$. Assume that g is a bounded function and $\tilde{\psi}(\omega) = g e^{-2\pi j r_0 \omega}$, then we have

$$\|g - P_{\mathbf{A}}(g)\|_2 = \|\tilde{\psi} - P_{\tilde{\mathbf{A}}}(\tilde{\psi})\|_2. \quad (13)$$

The above theorem tells us that for function $\phi(\omega) = e^{-2\pi j \omega t}$ with $t \in [a, T+a]$, we can first consider $t \in [0, T]$ and choose the basis functions, then simply multiply $e^{-2\pi j a \omega}$ onto each basis function to form the new basis space.

Thus, concerning problem (7), if we have a priori knowledge of the approximate time delay locations, i.e., we know $\tau_i \in [0, T]$ or $\tau_i \in \bigcup_{k=1}^{\mu} [a_k, b_k]$, we first choose a set of non-orthogonal exponential polynomials as basis functions as in (12) for each interval $[a_k, b_k]$, then combine all these basis functions. Let space $\mathbf{B} = \{\chi_1, \chi_2, \dots, \chi_{\tilde{m}}\}$ contain all the basis functions we choose. Furthermore, let us introduce the $N \times \tilde{m}$ matrix B with entries $B(k, i) = \chi_i(\frac{k}{N})$ and the $N_p \times \tilde{m}$ matrix B_p with entries $B_p(m, i) = \chi_i(\frac{m}{N_p})$ for $k = 0, 1, \dots, N-1$, $m = 0, 1, \dots, N_p-1$ and $i = 1, 2, \dots, \tilde{m}$. Then the approximation of the channel frequency response is given by

$$H_e = B(B_p^H B_p)^{-1} B_p^H H_p. \quad (14)$$

In real application we do not have any a priori information about τ_i ! Thus finding the approximate time delay locations is a critical problem. In the next section, we build an algorithm to accomplish this task.

V. ESTIMATING THE TIME DELAY LOCATIONS

In order to find the approximate time delay locations, we consider the exact function

$$\phi(\omega) = \begin{cases} \sum_{i=1}^{\tilde{r}} c_i e^{-2\pi j \omega \tau_i} & \omega \in [0, 1] \\ 0 & \omega \notin [0, 1] \end{cases} \quad (15)$$

and its inverse Fourier Transform $\Phi(t) = \sum_{i=1}^{\tilde{r}} c_i \text{sinc}(t - \tau_i)$. Since the peak of the function $\text{sinc}(t - \tau_i)$ is located at τ_i , it is not surprising that all the time delays τ_i are close to some peaks of the function $|\Phi(t)|$.

Theorem 5.1: Let $\phi(\omega)$ have expression (15), where $\tau_i \in [a, b]$. $\Phi(t)$ is its Inverse Fourier Transform. $|\Phi(t)|$ has its maximum at t_0 , then

$$t_0 \in (a - 2, b + 2). \quad (16)$$

From this theorem we see, by studying the peaks of the function $|\Phi(t)|$, we can estimate the time delay locations.

In this iterative algorithm, we consider the biggest peak of the function $|\Phi(t)|$ in each iteration and find a proper interval $[a_k, b_k]$ which contains this peak. Two or more peaks which are close to each other will be considered in the same iteration, which means we find one interval $[a_k, b_k]$ to contain these peaks. Let $\hat{f} = H_p$, the procedure of this algorithm in its i -th iteration is given by:

Step 1: Define function $|\Phi(t)|$, where

$$\Phi(t) = \begin{cases} \sum_{k=0}^{N_p-1} e^{j2\pi t \frac{k}{N_p}} \hat{f}(k) & t \in [0, N_p - 1] \\ 0 & t \notin [0, N_p - 1] \end{cases} \quad (17)$$

Step 2: Find region

1. Find the biggest peak of function $|\Phi(t)|$ at t_0 and let $E_i = \Phi(t_0)$.

2. Find all the peaks in $\Gamma = \{t_n\}$, which satisfy $|\Phi(t_n)| > \gamma |\Phi(t_0)|$, and for any t_n , there is a $t_m \in \Gamma$ such that $0 \leq (t_n - t_m) \text{sign}(t_m - t_0) < \theta$.

3. Find integers a_i, b_i such that $t_n \in [a_i, b_i]$ for all t_n , with $\Phi(a_i) > \gamma \Phi(t^a)$ and $\Phi(b_i) > \gamma \Phi(t^b)$, where t^a is the nearest peak to a_i and t^b is the nearest peak to b_i for $0 < \gamma < 1$.

Step 3: Select basis space $\mathbf{B}^{(i)} = [\chi_1, \chi_2, \dots, \chi_{m_i}]$ based on interval $[a_i, b_i]$ as in problem (12). let $\bar{g} =$

$B^{(i)}(B_p^{(i)H} B_p^{(i)})^{-1} B_p^{(i)H}(\hat{f})$, where $B^{(i)}(n, i) = \chi_i(\frac{n}{N})$ and $B_p^{(i)}(k, i) = \chi_i(\frac{k}{N_p})$ for $i = 1, 2, \dots, m_i$.

Step 4: $\hat{f} = \hat{f} - \bar{g}$, stop when one of the following is true, else go to **step 1** of the next iteration.

1. $\|\bar{g}\|_2 \leq \alpha \sqrt{\frac{m_i}{N_p}} \|\hat{f}\|_2$, where m_i is the number of basis functions used in this iteration.

2. $\frac{E_i}{E_1} \leq \eta$, where η is a small number.

In our algorithm, we set $\theta = 3, \gamma = 0.5, \alpha = 1.5$ and $\eta = \frac{1}{30}$. In the first stopping criterion, $\sqrt{\frac{m_i}{N_p}} \|\hat{f}\|_2$ is the average power in frequency domain, when the energy in the biggest peak is close to the average energy, we treat it as noise and stop. In the second stop criterion, when the current i -th peak is very small, we may treat it as approximation error and stop.

After we find all the $[a_j, b_j]$, we solve problem:

$$\min_{([\bar{a}_1, \bar{b}_1], \dots, [\bar{a}_k, \bar{b}_k])} \sum_{i=1}^k \bar{b}_i - \bar{a}_i \quad (18)$$

s.t. for integer \bar{a}_i, \bar{b}_i , $\cup_{i=1}^k [\bar{a}_i, \bar{b}_i]$ contains domain $\cup_i [a_i, b_i]$ and $\bar{b}_{i+1} - \bar{a}_i \geq \theta$. Then $\cup_{i=1}^k [\bar{a}_i, \bar{b}_i]$ is the final solution for the approximate time delay locations. After we have this, we can go back to problem (14) to find the approximate channel frequency response H_e .

VI. ERROR ANALYSIS

In this section we give a brief error analysis for the interpolation method proposed in Section IV. Formally, an interpolation method involves an operator mapping discrete data to a continuous-variable function. Here we consider functions with finite entry. Let the interpolation operator $T : \mathbb{C}^N \rightarrow L^2(\mathbb{R})$, then the function $\phi_e = T(H_p + W)$ is the approximation we get after interpolation. We call $\phi - T(H_p)$ the interpolation error and $T(H_p + W) - T(H_p)$ the interpolated noise. The aforementioned interpolation methods are linear operators, we thus have in this case that $T(H_p + W) = T(H_p) + T(\omega)$, hence the interpolated noise is simply $T(\omega)$.

In order to explore the effect of AWGN for various interpolation methods, we show the following observations:

Theorem 6.1: Suppose function $\phi(\omega)$ has expression (7) and its samplings are given by $\{H_p(m) := \phi(\frac{m}{N_p}) + W(m)\}_{m=0}^{N_p-1}$ (pilot tones) in $[0, 1]$, where $W(m)$ are i.i.d. complex Gaussian random variables with variance N_0 . Let ϕ_e denote the approximation function resulting from the interpolation method proposed in Section IV. Suppose we know the approximate time delay locations and thus the basis space \mathbf{B} , then $\phi_e(\frac{n}{N}) = \phi(\frac{n}{N}) + \epsilon_1(\frac{n}{N}) + \epsilon_2(\frac{n}{N})$, where ϵ_1 is the interpolation error and ϵ_2 is the interpolated noise with $E(\|\epsilon_2(\frac{n}{N})\|_2^2) = \frac{\tilde{m}}{N_p} N_0$, \tilde{m} is the number of basis functions in \mathbf{B} and $n = 0, 1, \dots, N - 1$.

From this theorem we can see, that if the number of basis functions is small we can get an additional reduction of the channel estimation error that is caused by AWGN.

To compare the interpolation error and interpolated noise for different methods, we introduce an example as follows.

Example 6.2: Consider the function $\phi(\omega) = \sum_{i=1}^{\tilde{r}} c_i e^{-2\pi j \omega \tau_i}$, where c_i is i.i.d. complex Gaussian random variable with variance N_0 , $\tilde{r} = 15$ is the number

of the multiple path and τ_i is the time delay for the i -th path. Assume we are given the $N_p = 128$ noisy samples $\{\phi(\frac{m}{N_p}) + W(m)\}_{m=0}^{N_p-1}$, where $W(m)$ is AWGN. We need to evaluate the values of function $\phi(\omega)$ at $\{\frac{k}{N}\}_{k=0}^{N-1}$ for $N = 1024$.

In the first experiment, we let the noise be zero and τ_i be uniformly distributed random variables in $[0, \alpha]$, where α changes from 0 to 66 for $i = 1, \dots, \tilde{r}$. When applying our interpolation method, we first run the algorithm from Section V to find the approximate locations of τ_i , then use the selected basis functions to approximate ϕ . Fig 2 shows the numerical results for interpolation error of different methods.

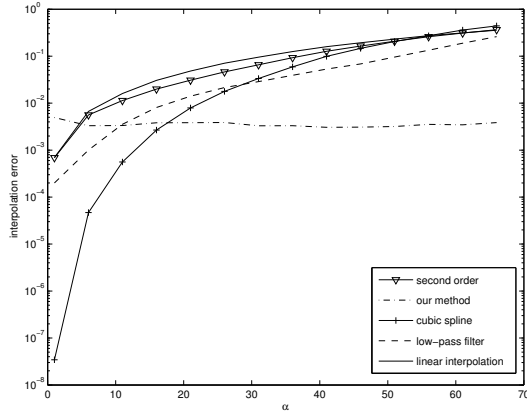


Fig. 2. Comparisons of interpolation error

In the second experiment, we let τ_i be uniformly distributed random variables in $[0, 10]$ which are given, for $i = 1, 2, \dots, \tilde{r}$ and the SNR changes from 0 to 40. Fig 3 shows the numerical results for interpolated noise of different methods.

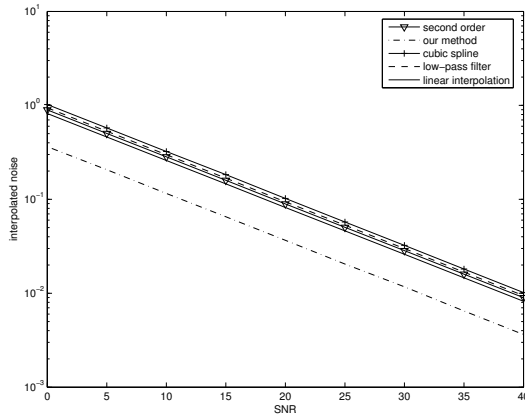


Fig. 3. Comparisons of interpolated noise

When α is small, the interpolation error of cubic spline interpolation is the smallest, followed by our interpolation method, lowpass filter, second order interpolation and linear interpolation. As α becomes large, the interpolation error of our method stays at the same level, while the interpolation error of the other interpolation methods grows rather quickly. When α is large, our method achieves the smallest interpolation error, as illustrated in Fig 3.

In OFDM system, the interpolation error and interpolated noise of an interpolation method correspond to how well it can preserve the channel information and reduce noise respectively. Fig 2 and Fig 3 show the efficiency of our interpolation method in these two aspects. The overall BER performance of an interpolation method depends on its interpolated noise and interpolation error together. In section VII, simulations for all kinds of channel models make more clear comparisons for different interpolation methods.

VII. SIMULATIONS AND COMPARISONS

The performance of our proposed channel estimation method is evaluated under fast fading channels. A 16QAM-OFDM system with carrier frequency of 1 GHz and bandwidth of 2 MHz is used. The total number of sub-carriers in one OFDM block is 1024, the number of uniformly distributed pilots sub-carriers is 128. The channel models are Rayleigh as recommended by GSM Recommendation 05.05 [10]. In the first channel model, our channel has 6 paths, the time delays are given by $0\mu s, 0.2\mu s, 0.5\mu s, 1.6\mu s, 2.3\mu s, 5.0\mu s$ and the average power of each path is given by $-3.0\text{ dB}, 0.0\text{ dB}, -2.0\text{ dB}, -6.0\text{ dB}, -8.0\text{ dB}, -10.0\text{ dB}$ respectively. In the second channel model, we have 10 paths and the normalized time delays are uniformly distributed in $[0, 40]$ with the same average power -1.0 dB . We assume that the guard intervals are longer than the maximum time delay to avoid ISI, and the ICI is affected only by Doppler spread.

The proposed interpolation method is compared to first and second order interpolation, cubic spline interpolation, and low-pass filtering. Furthermore we compare our method to the case when the time delay positions are known in (8), as well as to the ideal case when the receiver has exact channel knowledge. For our interpolation method, we first run the algorithm to find the approximate time delay locations then approximate the channel response function. Known time delay interpolation in (8) and known true channel case are of course simulated only for comparison purposes, as they are not applicable in real problems. For different noise and ICI levels, the simulation results are shown in Fig 4 and Fig 5.

For the first channel model, the maximum time delay is not large. From the analysis in the previous section we know that the interpolated noise is the dominant part for channel estimation errors. From Fig 4 we can find the overall BER of our interpolation method is smaller than linear interpolation, followed by second order interpolation, low pass filter and cubic spline, which coincides with the analysis in last section. When the time delays are large, as in our second channel model, the overall performance is determined by interpolation error and interpolated noise together. For high SNR, the dominant part is interpolation error, from Fig 5 we can find the BER of our interpolation method is smaller than low-pass filter, followed by cubic spline, second order interpolation and linear interpolation method, which confirms the error analysis in last section again.

In these two channel models, our method beats the other interpolation methods. In the following we compare the result of our interpolation method with the best result of linear

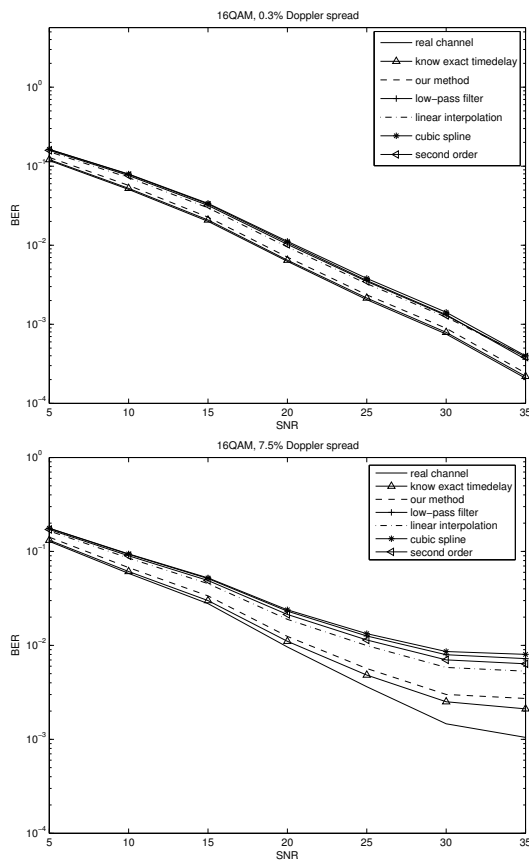


Fig. 4. Comparisons of BER in channel 1

interpolation, second order interpolation, cubic spline and low pass filtering in each experiment. When the maximum Doppler spread is 0.3% of the subcarrier spacing, our method can achieve improvement about 1.2 dB for the first channel model. For the second channel model, our method can achieve an improvement of about 1.2 dB when the SNR is 10 and about 2 dB when the SNR is 20, it increases when the SNR increases. When the maximum Doppler spread is 7.5% of the subcarrier spacing, our method can achieve 1.2 – 1.5 dB improvement for these two channel models when SNR is 10 and 2.1 – 2.3 dB improvement when SNR is 20. The improvement becomes much larger when the SNR increases as shown in Fig 4 and Fig 5.

VIII. CONCLUSIONS

We have proposed a promising interpolation method and shown its efficiency when applying it to OFDM channel estimation. This method outperforms all the typical interpolation methods introduced in this paper when comparing the BER performance. Numerical experiments for various channel models demonstrate that the proposed channel estimation method yields a BER performance that is only slightly worse than when the receiver has full knowledge of time delay positions. Another appealing property of our OFDM channel estimator is that we only need pilot information and do not require any knowledge of channel statistics and SNR. In addition,

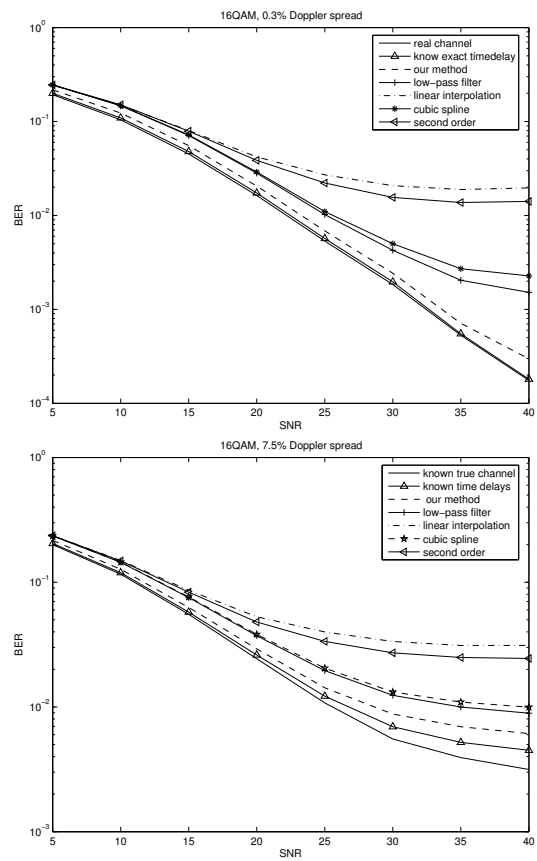


Fig. 5. Comparisons of BER in channel 2

this approach can also be applied when the pilots are non-uniformly spaced.

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