COURSE DESCRIPTION: In this class we define the *Derivative* of Calculus and teach Leibniz's notation for how to use it. This prepares the student for MAT 16B, in which we complete the picture of Calculus by defining the *Integral*. There is a startling connection between them—the derivative and the integral turn out to be integrally[1] connected through what has come to be known as the *Fundamental Theorem of Calculus*. Your goal in MAT 16AB is to learn this.

Differerential and integral calculus as we know it was set out by Gottfried Leibniz and Isaac Newton (independently) in the mid-17th century. The notation of Newton was discarded and the notation of Leibniz was universally taken over. The Leibniz notation alone is an act of genius, and essentially makes this deep subject accessible to first year students of mathematics.

Since Leibniz, the word "*derivative*" has become perhaps the single most important word in the language of science. Why? The answer is that **laws of science almost always come to us stated in terms of derivatives!** Why this is true is a big question with many partial answers–but the following important example suffices to show how it works.

Consider the velocity of your car as it moves down the highway. The *velocity*, or speed of the car, is simply the change of distance per time of the car's motion, or said more precisely, the rate of change of distance with respect to time. Similarly, the acceleration of the car is the rate of change of velocity with respect to time. In mathematics, the word for "rate of change" is "derivative", and we say the *velocity is the derivative of distance with respect to time*, and acceleration is the *derivative of velocity with respect to time.* But there is nothing special about velocity

[1] Pun intended!

and acceleration. Whenever you can graph how one quantity is related to another, you can talk about the *rate* at which the one quantity changes with respect to the other, and that is precisely its *derivative*. One of the most profound discoveries, discovered by the collective experience of great scientists over the past five centuries, is that if you want to *derive* (pun intended!) how one quantity is related to another in a scientific problem, (eg. so you can graph the relationship), you almost always must start with a law that describes the rate of change of the one quantity with respect to the other, (the *derivative*), and then use this to construct the graph. To get the graph from the derivative, you must *integrate* (MAT 16B), so that explains why the derivative and integral are so "integrally connected" .

For the example, consider the problem of predicting the graph of distance vs time for a car assuming you know the car's $Mass$ and the horsepower of the engine. For this the starting point is Newton's force law: $Force = Mass \times Acceleration$. Nevermind how to use horsepower minus drag to get a formula for $Force$ on the left hand side, the point is that the *Law* gives you the *Acceleration*, which is a *derivative* (acceleration is the derivative of velocity with respect to time). To get the graph of velocity vs time, we would have to *integrate up the acceleration*, (MAT 16B). And to get the graph of distance vs time we have to *integrate up the velocity*, (MAT 16B). You do not yet know how to solve this problem, but you can get the "big picture" now: The fundamental law, the starting point, is Newton's Force Law, and it comes to us stated in terms of a *derivative*! This happens again and again in science: **Fundamental laws come to us stated in terms of derivatives.** The purpose of MAT 16A is to define the derivative and learn the Leibniz notation to use it.