Solutions

MIDTERM EXAM I
Math 16A
Temple-Winter 2012

–Print your name, section number and put your signature on the upper right-hand corner of this exam. Write only on the exam.

–Show all of your work, and justify your answers for full credit.

SCORES

#1

#2

#3

#4

#5

#6

#7

TOTAL:

1. Let $f(x) = \left(\frac{x^2-16}{x-4}\right)^{12}$. Evaluate the following limits: (Do not simplify)

(a) (6 pts) $\lim_{x\to 2}\left(\frac{x^2-16}{x-4}\right)^{12}$ $= \left(\frac{2^2-16}{-2}\right)^{12}$

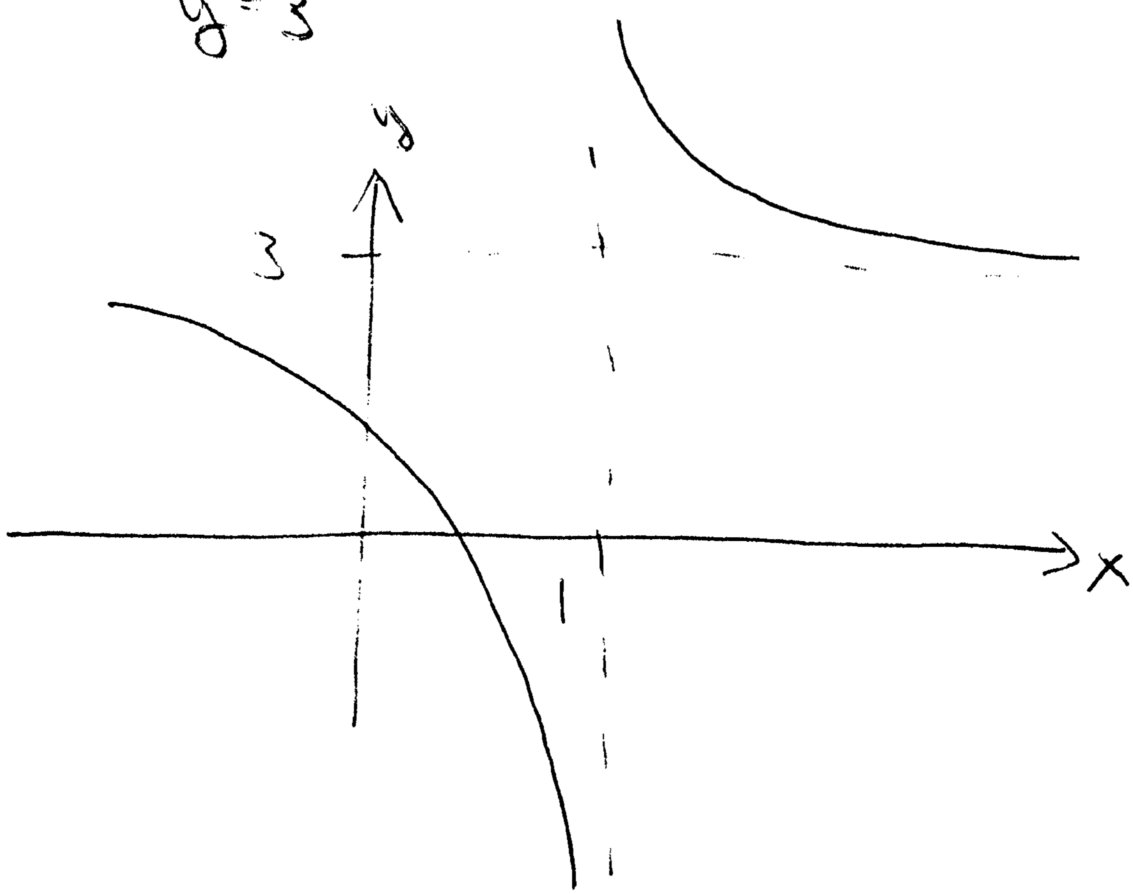
(b) (6 pts) $\lim_{x\to 4}\left[\left(\frac{x^2-16}{x-4}\right)^{12}\right] = \left[\left(\frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}}\right)^{12}\right] = \left(\frac{8}{1}\right)^{12}$

2. (7 pts) Find all vertical and horizontal asymptotes and sketch the graph of the function $f(x) = \frac{3x+1}{x-1}$. Justify your answer.

Vertical $x = 1$

Horizontal $\lim_{x \to \infty} \frac{3x+1}{x-1} = 3$, $\lim_{x \to -\infty} \frac{3x+1}{x-1} = 3$.

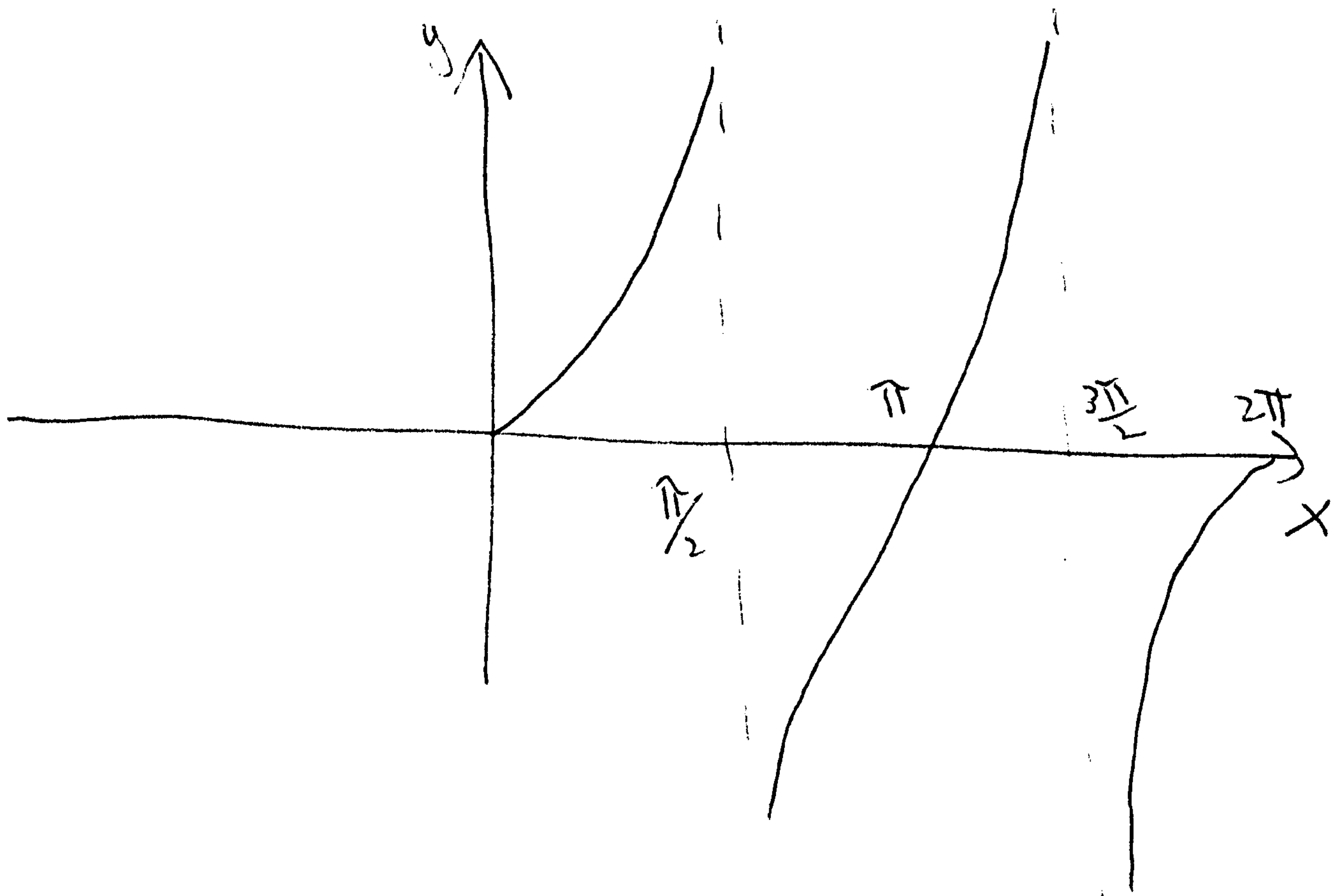
$y = 3$



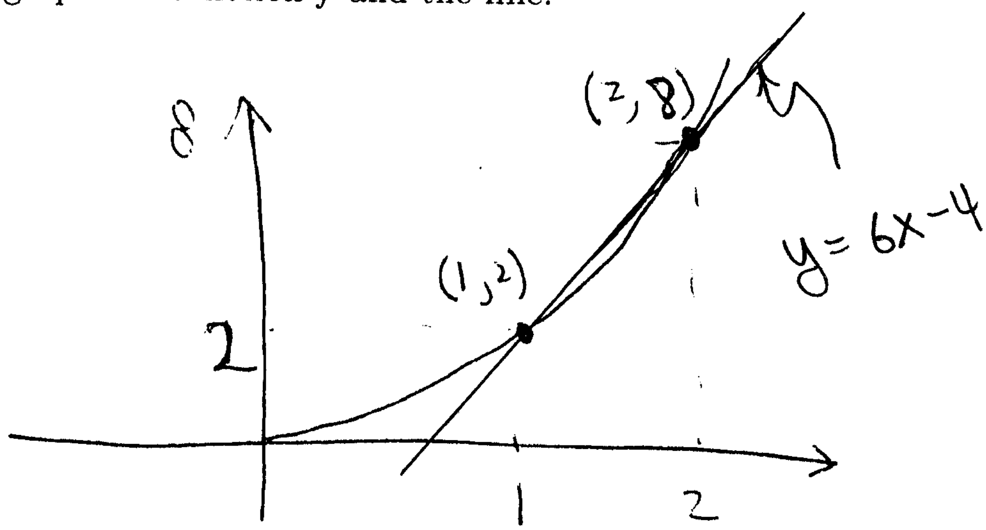
3. (7 pts) Find *all* vertical asymptotes and sketch the graph of the function $f(x) = \tan(x)$ for $0 \leq x \leq 2\pi$.

$f(x) = \tan x = \dfrac{\sin x}{\cos x}$ $\quad \cos x = 0 \iff x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Vert asymptotes $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$



4. (7 pts) Let $f(x) = 2x^2$. Find the equation of the line passing through the two points on its graph $(1, f(1))$ and $(2, f(2))$. Sketch a graph the function f and the line.



$$m = \frac{8-2}{2-1} = 6$$

Eqn $\quad \frac{y-2}{x-1} = 6 \qquad y-2 = 6x-6$

$$\boxed{y = 6x-4}$$

5. (7 pts) Use the definition of derivative to find the slope $\frac{dy}{dx} = f'(2)$ of the line tangent to the graph of $f(x) = 2x^2$ at the point $(2, f(2))$. Sketch the graph and the tangent line.

$$\left(\text{Recall}: \; f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.\right)$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left[\frac{2(2+\Delta x)^2 - 2(2)^2}{\Delta x} = \frac{2(2)^2 + 4 \cdot 2\Delta x + 2\Delta x^2 - 2(2)^2}{\Delta x} \right]$$

$$= \lim_{\Delta x \to 0} \frac{8\cancel{\Delta x} + 2\Delta x^2}{\cancel{\Delta x}} = 8$$

[No credit for $f'(x) = 4x$]

6. (a) (7 pts) Give a formula for a function $f(x)$ that is not continuous at $x = 2$, but such that $\lim_{x \to 2} f(x) = 3$.

$$f(x) = 3\frac{x-2}{x-2}$$

(b) (4 pts) Give a formula for a function $f(x)$ such that f is continuous at $x \neq 2$, but $\lim_{x \to 2} f(x)$ does not exist.

$$f(x) = \frac{1}{x-2}$$

7. Let $f(x) = \frac{1}{4-x}$.

(a) (5 pts) What is the Domain of f. $\boxed{x \neq 4}$

(b) (5 pts) Find a formula for the inverse f^{-1}.

$$y = \frac{1}{4-x} \Rightarrow x = \frac{1}{4-y} \Rightarrow 4-y = \frac{1}{x}$$

$$\boxed{y = 4 - \frac{1}{x} = f^{-1}(x)}$$

(c) (6 pts) Find the Domain of $(f \circ f)(x) = f(f(x))$.

$$f(f(x)) = \frac{1}{4 - \left(\frac{1}{4-x}\right)} = \frac{4-x}{4(4-x)-1}$$

$$= \frac{4-x}{16-4x-1} = \frac{4-x}{15-4x}$$

Domain $15-4x \neq 0$

$$\boxed{x \neq \frac{15}{4}}$$