

Name: _____

Student ID#: _____

Section: _____

Midterm Exam 1

Friday, February 2

MAT 185A, Temple, Winter 2024

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Recall $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$.

(a) Prove that $\frac{d}{dz} \sin z = \cos z$.

(b) Find the real and imaginary parts of $f(z) = \sin z$, and verify directly they satisfy the Cauchy Riemann equations $u_x = v_y$, $u_y = -v_x$.

Problem #2 (20pts): The complex conjugate of $z = x + iy$ is $\bar{z} = x - iy$.

(a) Prove that $\bar{z}\bar{w} = \overline{zw}$ for all $z, w \in \mathbb{C}$.

(b) Let $f(z) = \frac{\bar{z}}{x^2+y^2}$. Prove that $f(z)$ is analytic, or else prove that it is not analytic, for $z \neq 0$.

(c) Let $f(z) = (\bar{z})^2$. Prove that $f(z)$ is entire, or else prove that it is not entire.

Problem #3 (20pts): Recall the Math 21D notation $\vec{\mathbf{G}} = \overrightarrow{(M, N)}$ for a vector field, assume $f(z) = u + iv$ is analytic (i.e. complex differentiable), and let $dz = dx + idy$.

(a) Use Leibniz calculus of differentials to derive vector fields $\vec{\mathbf{G}}_1$ and $\vec{\mathbf{G}}_2$ such that

$$\int_c f(z)dz = \int_c \vec{\mathbf{G}}_1 \cdot \vec{\mathbf{T}} ds + i \int_c \vec{\mathbf{G}}_2 \cdot \vec{\mathbf{T}} ds.$$

(b) Assume $\nabla U(x, y) = \vec{\mathbf{G}}_1$ and $\nabla V(x, y) = \vec{\mathbf{G}}_2$. Prove that $F(z) = U + iV$ is differentiable, and $F'(z) = f(z)$.

Problem #4 (20pts): Let \mathcal{C} denote the positively oriented circle of center $z_0 = i$ and radius $r = 2$. Evaluate the line integral

$$\int_{\mathcal{C}} \left\{ z^n + \frac{1}{z - i} \right\} dz$$

by *direct parameterization*, where n is any positive integer.

Problem #5 (20pts):

(a) Find all $w \in \mathbb{C}$ such that $w = i^{1/5}$.

(b) Find all r and θ such that $w = re^\theta$ and $w = i^{\pi+ie}$.