Name: ______________________________

Student ID#: ________________________

Section: ________________________

Midterm Exam 1

Friday, February 3

MAT 21D, Temple, Winter 2023

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): (a) Sketch the region of integration $\mathbf{R}_{xy}$ determined by the iterated integral

$$\int_0^1 \int_{x^5}^{x^2} dy dx. \tag{1}$$

(b) Write (1) as an iterated integral with limits reversed.

Problem #2 (20pts): For (1) of Problem #1, assume $\mathbf{R}_{xy}$ is a metal plate of density $\delta(x, y) = x^2y^3$.

(a) Write the coordinates of the center of mass in terms of iterated integrals. (Do not evaluate.)

(b) Write the Kinetic Energy of rotation of R_{xy} about the axis $y = 1$ in terms of iterated integrals assuming constant angular rotation rate $\frac{d\theta}{dt} = \omega$. (Do not evaluate.)

Problem #3 (20pts): Recall the *amplification factor* for the area change $dA_{xy} = J\, dA_{uv}$ associated with the coordinate change $(u, v) \to (x, y)$ is

$$J = det \left| \frac{\partial(x,y)}{\partial(u,v)} \right|, \quad x = g(u,v),\ y = h(u,v). \tag{2}$$

(a) Use (2) to derive the formula for J in the case of polar coordinates $x = r\cos\theta$, $y = r\sin\theta$.

(b) Use (2) to show that (r, θ) is positively, and (θ, r) is negatively oriented with respect to (x, y), *respectively*. (That is, show $(r, \theta) \to (x, y)$ is *orientation preserving*, and $(\theta, r) \to (x, y)$ is *orientation reversing*.) Explain.

Problem #4 (20pts): (a) Iterate the integral $I = \int\int\int_D f(x, y, z)dV$ in cylindrical coordinates, where D is the region bounded below by the plane $z = 0$, laterally by the circle $(x - 1)^2 + y^2 = 1$, and above by the surface $z = \sqrt{x^2 + y^2}$. Sketch the region, labeling the sides.

(b) Evaluate the iterated integral in the case when $f(x, y, z) = \frac{1}{x^2+y^2+z^2}$.

Problem #5 (20pts): (a) Use spherical coordinates to set up the iterated integral for the mass of the ice-cream cone shaped region D with $0 \leq \phi \leq \frac{\pi}{6}$, but with variable top given by $\rho = e^{\theta}$, $0 \leq \theta \leq 2\pi$, assuming constant density δ. Sketch the region.

(b) Assuming distance measured in meters m and mass in kilograms kg, evaluate the integral and determine the total mass assuming $\delta = \frac{1}{\sin^2 \phi} \frac{kg}{m^3}$.