Name: ______________________________

Student ID#: ________________________

Section: ________________________

Midterm Exam 2

Friday, March 10

MAT 21D, Temple, Fall 2023

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Let $\overrightarrow{\mathbf{r}} = \sin t\,\mathbf{i} + 2t\,\mathbf{j} + \cos t\,\mathbf{k}$. Find:

(**a**) The velocity $\overrightarrow{\mathbf{v}}(t)$.

(**b**) The acceleration $\overrightarrow{\mathbf{a}}(t)$.

(**c**) The unit tangent vector $\overrightarrow{\mathbf{T}}(t)$.

(**d**) The principle normal vector $\overrightarrow{\mathbf{N}}(t)$.

(**e**) The speed $\frac{ds}{dt}$.

(**f**) The arc length of the curve from $t = 0$ to $t = \frac{\pi}{2}$.

Problem #2 (20pts): Let $\overrightarrow{\mathbf{F}} = (\sin y + z\cos x)\,\mathbf{i} + x\cos y\,\mathbf{j} + \sin x\,\mathbf{k}$.
(**a**) Show that $\mathrm{Curl}(\overrightarrow{\mathbf{F}}) = 0$.

(**b**) Use the method of partial integration to find an f such that $\overrightarrow{\mathbf{F}} = \nabla f$.

Problem #3 (20pts): Recall Green's Theorem:

$$\int_{\mathcal{R}} N_x - M_y \, dA = \int_{\mathcal{C}} M dx + N dy$$

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(**a**) Derive the Stokes Theorem version of Green's Theorem.

(**b**) Derive the Divergence Theorem form of Green's Theorem.

Problem #4 (20pts): Assume a mass m moves along a path $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $a \leq t \leq b$, through a conservative force field $\mathbf{F} = \nabla f$.

(**a**) Show that $\int_{\mathcal{C}} \mathbf{F} \cdot \mathbf{T}\, ds = f(B) - f(A)$ for any curve taking A to B.

(**b**) Show that if further, $\mathbf{F} = m\mathbf{a}$, then also $\int_{\mathcal{C}} \mathbf{F} \cdot \mathbf{T} ds = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$. (Here $v_A = \|v(a)\|^2$, $v_B = \|v(b)\|^2$.)

Problem #5 (20pts): Assume a substance of mass density $\delta(x, y, z) = 2z\frac{kg}{m^3}$ is flowing through a surface $\mathcal{S}$ at velocity $v = y(2\mathbf{i} + \mathbf{j} - \mathbf{k})$. Assume $\mathcal{S}$ is parameterized by $\mathbf{r}(u, v) = (2u + v)\mathbf{i} + (u - v)\mathbf{j} + (u + v)\mathbf{k}$, $0 \leq u \leq 2$, $0 \leq v \leq 1$. Find a unit normal $\mathbf{n}$ to $\mathcal{S}$, the amplification factor for area, and the rate and direction at which mass is passing through $\mathcal{S}$, (i.e find the Flux).