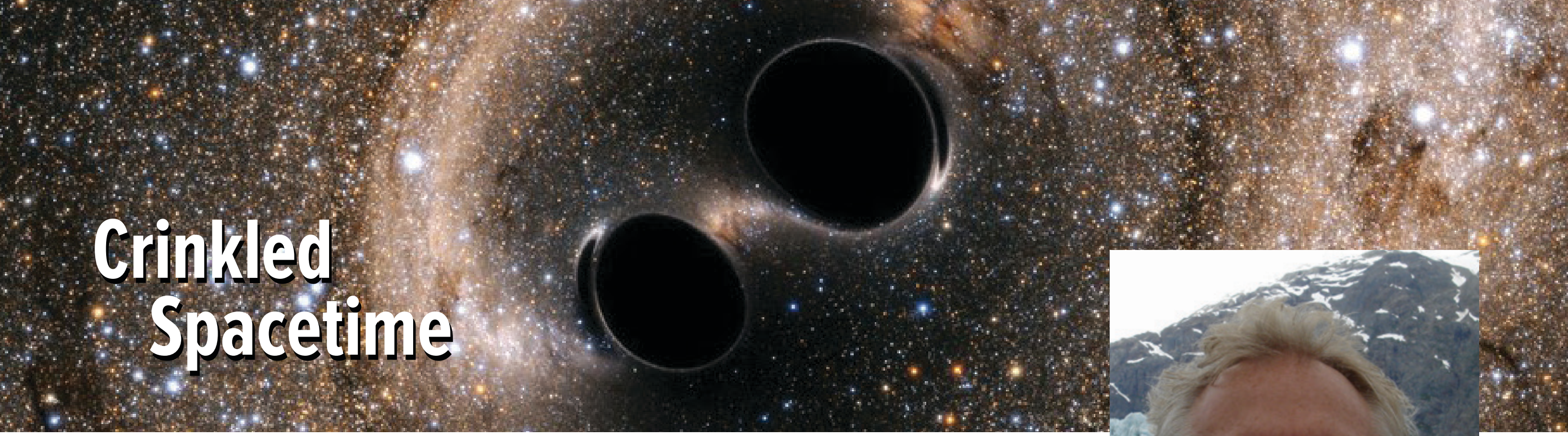

Crinkled Spacetime

by Blake Temple

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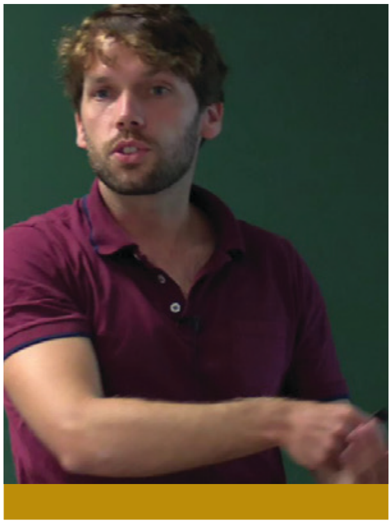
In the early 20th century, Einstein proposed a set of equations that related the gravitational force to the curvature of space time. Understanding the mathematical theory of these questions is a central challenge that is far from complete. Recent work in this area by Professor Blake Temple with coauthor Moritz Reintjes has attracted attention in the Math and Physics communities. They have developed a new theory of "metric smoothing" in General Relativity, based on what they call *Regularity Transformation Equations*, or RT-equations.

Their studies began by investigating whether the interaction of shock waves in General Relativity (GR) could create a new kind of *regularity singularity* where spacetime itself is not smooth. Earlier work on shock waves was only able to prove existence of shock wave solutions for metrics that are Lipschitz continuous. A more regular spacetime, one order smoother, is required to make the correspondence between Einstein's theory of General Relativity and the physics of Special Relativity, the case when there is no gravitational curvature. What was unclear was whether the lack of smoothness was inherent in the spacetime itself, or in contrast, whether it was a feature of the way that space was being modeled, or *parametrized*. That is, was the lack of smoothness due only to a bad choice of map, or coordinate system, used to describe the spacetime? Reintjes and Temple set out to investigate this phenomenon.

In a series of five papers (two of which have appeared), they have now succeeded in characterizing the mechanism for smoothing out the wrinkles in a wrinkled map. Specifically, by a wrinkled map, they mean a map in which the gravitational metric is one, not the usual two derivatives more regular than the curvature; and unwrinkling the map means lifting the regularity of the metric up one order, to two full derivatives above the curvature. They prove that to construct a new smoothed out map from the original, it is necessary and sufficient to solve the RT-equations. In their final paper, they establish the general theory by proving solutions of the RT-equations always exist for wrinkled maps at least one order smoother than the shock wave case. In other words, for wrinkled maps above a threshold smoothness, the lack of smoothness is *never* caused by the nature of spacetime, but always by the way it is being modeled. As they put it, "a crinkled map of spacetime can always be smoothed out by a coordinate transformation." The theory still leaves open the problem of regularity singularities at GR shock waves, but reduces it to the problem of finding solutions of the RT-equations at the lowest level of smoothness.

In fact, their methods apply beyond Einstein's theory of relativity, to the general problem of smoothing the spaces that arise in analysis and differential geometry. A rather surprising aspect of their argument is that it relies on the theory of *elliptic* partial differential equations to find coordinate systems which smooth out solutions to the equations of General Relativity, which are *hyperbolic*. This is unexpected because hyperbolic PDE's, which govern sound waves and give rise to shocks, behave very differently than elliptic PDE's, which typically apply to more regular settings like electric fields or the distribution of heat. The RT-equations reduce the problem of regularity singularities at shock waves in GR, to the existence of well-studied *Calderon-Zygmund* type singularities in elliptic PDE theory, establishing a connection between two different kinds of singularities from two (apparently) different subjects.

The new theory enlarges the space of solutions to the Einstein equations, puts the problem of regularity singularities at GR shock waves on a solid mathematical foundation, and introduces a new direction for geometrical analysis.



Moritz Reintjes completed his Ph.D. with Professor Temple at UC Davis. Afterwards he was a postdoc at the University Regensburg, at the Max Planck Institute for Gravitational Physics in Potsdam and at the University of Michigan in Ann Arbor. From 2013 until 2016 Moritz was a postdoc at IMPA in Rio de Janeiro, Brazil. Since 2017 he is a postdoc at the Instituto Superior Tecnico in Lisbon, Portugal. Moritz received a "Research Scholarship" from the German Research Foundation (2013-2014), a "Postdoc of Excellence" scholarship at IMPA (2015-2016). His research concerns General Relativity, Shock Waves, Fluid Dynamics and the Dirac equation of Quantum Mechanics.