Maxwell's equations

(Redirected from Maxwells equations)

Maxwell's equations are the set of four equations, attributed to James Clerk Maxwell, that describe the behavior of both the electric and magnetic fields, as well as their interactions with matter.

Maxwell's four equations express, respectively, how electric charges produce electric fields (Gauss's law), the experimental absence of magnetic charges, how currents produce magnetic fields (Ampère's law), and how changing magnetic fields produce electric fields (Faraday's law of induction). Maxwell, in 1864, was the first to put all four equations together and to notice that a correction was required to Ampere's law: changing electric fields act like currents, likewise producing magnetic fields.

Furthermore, Maxwell showed that the four equations, with his correction, predict waves of oscillating electric and magnetic fields that travel through empty space at a speed that could be predicted from simple electrical experiments—using the data available at the time, Maxwell obtained a velocity of 310,740,000 m/s. Maxwell (1865) wrote:

> *This velocity is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.*

Maxwell was correct in this conjecture, though he did not live to see its vindication by Heinrich Hertz in 1888. Maxwell's quantitative explanation of light as an electromagnetic wave is considered one of the great triumphs of 19th-century physics. (Actually, Michael Faraday had postulated a similar picture of light in 1846, but had not been able to give a quantitative description or predict the velocity.) Moreover, it laid the foundation for many future developments in physics, such as special relativity and its unification of electric and magnetic fields as a single tensor quantity, and Kaluza and Klein's unification of electromagnetism with gravity and general relativity.

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Historical developments of Maxwell's equations and relativity

Maxwell's 1865 formulation was in terms of 20 equations in 20 variables, which included several equations now considered to be auxiliary to what are now called "Maxwell's equations" — the corrected Ampere's law (three component equations), Gauss's law for charge (one equation), the relationship between total and displacement current densities (three component equations), the relationship between magnetic field and the vector potential (three component equations, which imply the absence of magnetic charge), the relationship between electric field and the scalar and vector potentials (three component equations, which imply Faraday's law), the relationship between the electric and displacement fields (three component equations), Ohm's law relating current density and electric field (three component equations), and the continuity equation relating current density and charge density (one equation).

The modern mathematical formulation of Maxwell's equations is due to Oliver Heaviside and Willard Gibbs, who in 1884 reformulated Maxwell's original system of equations to a far simpler representation using vector calculus. (In 1873 Maxwell also published a quaternion-based notation that ultimately proved unpopular.) The change to the vector notation produced a symmetric mathematical representation that reinforced the perception of physical symmetries between the various fields. This highly symmetrical formulation would directly inspire later developments in fundamental physics.

In the late 19th century, because of the appearance of a velocity,