Name: _____

Student ID#: _____

Section: _____

Midterm Exam 1
MAT 25–Temple
Wednesday, February 8, 2017

Show your work on every problem. Correct answers with no supporting work
will not receive full credit. Be organized and use notation appropriately. No
calculators, notes, books, cellphones, etc. may be used on this exam. Please
write legibly. Please have your student ID ready to be checked when you
turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Short Answer: (**a**) Find the Least Upper Bound and Greatest Lower Bound of the set

$$S = \left\{ (-1)^n - \frac{1}{n^2} : n \in \mathbf{N} \right\}.$$

(**b**) Let a and b be real numbers. Use the triangle inequality to bound $|a + b|$ *above* and *below* in terms of $|a|$ and $|b|$. (You need not justify your answer.)

(**c**) Is the following statement true or false? "If $\sqrt{-1} > 0$, then *every real number is rational.*" Justify your answer.

(**d**) Without using the negation sign or the word "not", write the negation of the following sentence employing the *For Every* and *There Exists* symbols:

There exists $s_0 \in \mathbf{R}$ such that for every $\epsilon > 0$ there exists $N \in \mathbf{N}$ such that for every $n > N$, $|s_n - s_0| < \epsilon$.

Problem #2 (20pts): Using only the field axioms for the real numbers, prove that $a \cdot 0 = 0$ for every real number $a \in \mathbf{R}$. Justify every step.

Problem #3 (20pts): Using the definition, prove that if $s_n \to s_0$ and $t_n \to t_0$ then $s_n t_n \to s_0 t_0$.

Problem #4 (20pts): Let r be a real number such that $|r| < 1$, and let s_n denote the sequence of partial sum $s_n = \sum_{k=2}^{n} r^k = r^2 + r^3 + \cdots + r^n$. Derive a formula for s_n that does not involve a summation, and use it to evaluate $\lim_{n\to\infty} s_n$.

Problem #5 (20pts): Assume that $0 < s_n < 2t_n$ and that $t_n \to 0$. Use the definition of convergence to give a careful proof that $s_n \to 0$.