SECTION-7
The Global Solution of the Riemann Problem for the p-system

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Math-280: A Mathematical Introduction to

Blake Temple, UC-Davis

③ Riemann Problem for p-system: (Ch 17 Smoller)

- Lagrangian system for Compressible Euler:

$$\left.\begin{array}{ll} v_t - u_x = 0 & v = \frac{1}{\rho} = \text{spec vol} \\ u_t + p_x = 0 & u = \text{velocity} \\ \left\{\begin{array}{l} \mathcal{E}_t + (pu)_x = 0 \\ S_t = 0 \end{array}\right. & \begin{array}{l} p = p(v,s) = \text{pressure} \\ \mathcal{E} = \frac{1}{2}u^2 + e = \text{spec } \mathcal{E} \end{array} \end{array}\right. \qquad (L)$$

- The p-system: assume $p = p(v)$ with

$$p' < 0, \; p'' > 0 \qquad (P)$$

- Recall: polytropic eqn of state:

$$p = C_v(\gamma - 1)\frac{1}{v^\gamma}\exp\left(S/C_v\right) \qquad (1)$$

$$pv = RT \qquad (2)$$

Isothermal $\Rightarrow$ $p = \frac{K}{V}$ $\quad p(v) = \frac{K}{v}$

Isentropic $\Rightarrow$ $p = C_v(\gamma-1)\exp\frac{S_0}{C_v}\,\frac{1}{v^\gamma}$

$$p(v) = k\frac{1}{v^\gamma} \quad \gamma > 1$$

- Note: $S_t = 0$ for smooth solutions $\Rightarrow$ $s = \text{const.}$ is no approx for smooth soln's

Moreover: recall 1, 3 - eigenfamilies

$$\lambda_{1,3} = \mp\sqrt{-p_v}\,, \quad R_{1,3} = \begin{bmatrix} 1 \\ \pm\sqrt{-p_v} \\ 0 \end{bmatrix}$$

Thus integral curves $R_{1,3}$ have $s \equiv \text{const}$. Since shock-curves have 2nd order contact at U_L, $s = O(1)\varepsilon^3$ along shock-curves, so $s \equiv \text{const}$ is a good approx for small amplitude soln's or soln's with weak shocks $\Rightarrow$ isentropic is a reasonable assumption.

③

Riemann Problem for p-system:

$$\text{p-system } (p) \begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = 0 \end{cases} \qquad p' < 0 \quad p'' > 0$$

isentropic: $p = \frac{1}{v^\gamma}$ $\quad 1 < \gamma \leq \frac{5}{3}$

isothermal: $p = \frac{1}{v}$

- Eigenvalues:

$$f\begin{pmatrix} v \\ u \end{pmatrix} = \begin{bmatrix} -u \\ p(v) \end{bmatrix} \qquad A = df = \begin{bmatrix} \nabla f_1 \\ \nabla f_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ p' & 0 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & -1 \\ p' & -\lambda \end{vmatrix} = \lambda^2 + p' = 0$$

$$\lambda_{\mp} = \mp\sqrt{-p'(v)} = \text{"sound speed"}$$

$$\lambda_1 \equiv \lambda_- , \quad \lambda_2 \equiv \lambda_+$$

- Set $U = \begin{pmatrix} v \\ u \end{pmatrix}$, $f(U) = \begin{pmatrix} -u \\ p(v) \end{pmatrix}$ so p-system is $U_t + f(U)_x = 0$

④

- Eigenvectors: $R_1 = (1, a)$ satisfies

$$[A - \lambda_1 I] R_1 = 0 \iff \begin{bmatrix} \sqrt{-p'} & -1 \\ p' & \sqrt{-p'} \end{bmatrix} \begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$$\sqrt{-p'} - a = 0 \Rightarrow a = \sqrt{-p'}$$

$$R_1 = \begin{bmatrix} 1 \\ \sqrt{-p'} \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1 \\ -\sqrt{-p'} \end{bmatrix}$$

(as we got taking $s = \text{const}$ in (L))

- Integral curves $\mathcal{R}_1$ satisfy

$$\frac{du}{dv} = \sqrt{-p'(v)}$$

$$\Rightarrow \quad u = \int \sqrt{-p'(v)}\, dv + \text{const}$$

$\mathcal{R}_1 \equiv$ integral curves of R_1 satis

$$u = \int \sqrt{-p'(v)}\, dv + \text{const}$$

$\mathcal{R}_2 \equiv$ integral curves of R_2 satisfy

$$u = -\int \sqrt{-p'(v)}\,dv + \text{const}$$

Conclude: The integral curves through a state (v_L, u_L) satisfy

$$\mathcal{R}_1(\bar{U}_L):\quad u = \int_{v_L}^{v} \sqrt{-p'(v)}\,dv + u_L = \varphi_1(v)$$

$$\mathcal{R}_2(\bar{U}_L):\quad u = -\int_{v_L}^{v} \sqrt{-p'(v)}\,dv + u_L = \varphi_2(v)$$

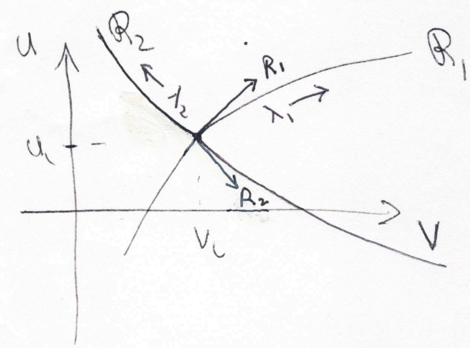
gives param. of 1,2-rarefaction curves wrt v

$\mathcal{R}_1(u_L)$: $\quad \dfrac{du}{dv} = \sqrt{-p'(v)} > 0$

$$\frac{d^2u}{dv^2} = \frac{-p''(v)}{2\sqrt{-p'(v)}} < 0$$

$\mathcal{R}_2(u_L)$: $\quad \dfrac{du}{dv} = -\sqrt{-p'(v)} < 0$

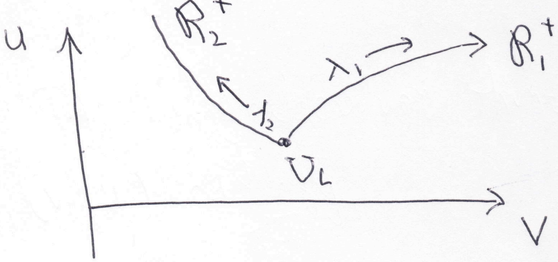
$$\frac{d^2u}{dv^2} = \frac{+p''}{2\sqrt{-p'}} > 0$$



- Note: $\dfrac{d\lambda_1}{dv} = -\dfrac{d\sqrt{-p'}}{dv} = \dfrac{p''}{2\sqrt{-p'}} > 0$, $\dfrac{d\lambda_2}{dv} < 0$

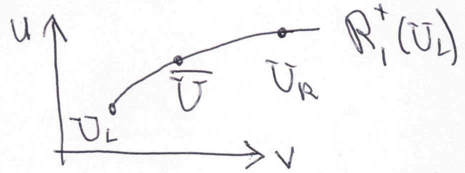
$\therefore$ if $R_i = (1, b_i)$, $\nabla\lambda_i \cdot R_i = \dfrac{d\lambda_i}{dv}$ $\begin{cases} > 0 & i=1 \\ < 0 & i=2 \end{cases}$

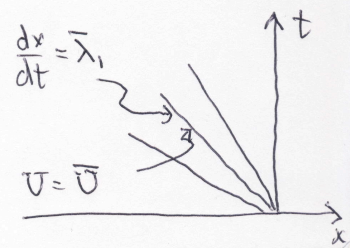
<u>Conclude</u>: The p-system is strictly hyperbolic & genuinely nonlinear in physical domain $V > 0$



<u>Defn</u>: For each $U_L = (V_L, u_L)$, the i-rarefaction curve R_i^+ is the portion of $R_i(U_L)$ along which $\lambda_i > \lambda_i(U_L)$

<u>We have</u>: for each $U_R \in R_i^+$ there is a centered i-simple wave (rarefaction wave) connecting U_L to U_R





Shock-waves for p-system:

$$s[U] = [f]$$

$$s\begin{bmatrix} V - V_L \\ u - u_L \end{bmatrix} = \begin{bmatrix} -u + u_L \\ P(V) - P(V_L) \end{bmatrix}$$

(1) $s(V - V_L) = -(u - u_L)$

(2) $s(u - u_L) = P(V) - P(V_L)$

$$\Rightarrow s^2 (V - V_L) = -(P(V) - P(V_L))$$

(3) $s = \mp\sqrt{-\dfrac{P(V) - P(V_L)}{V - V_L}}$

Substituting (3) into (1) we obtain

$$u = u_L \pm \sqrt{-\frac{P(V) - P(V_L)}{V - V_L}}\,(V - V_L)$$

± goes with ± shock speeds

(4)

$$u = u_L \pm \sqrt{(P(V_L) - P(V))(V - V_L)} \qquad V > V_L$$

$$u = u_L \mp \sqrt{(P(V_L) - P(V))(V - V_L)} \qquad V < V_L$$

☒ Consider: $u = u_L \pm \sqrt{(p(v_L) - p(v))(v - v_L)}$

$$\frac{du}{dv} = \mp \frac{p'(v)(v-v_L) + (p(v_L) - p(v))}{2\sqrt{(p(v_L) - p(v))(v-v_L)}}$$

$$= \mp \frac{v - v_L}{2\sqrt{(p(v_L)-p(v))(v-v_L)}} \underbrace{\left\{ p'(v) + \frac{p(v_L)-p(v)}{v_L - v} \right\}}_{p'(v) + p'(v^*) < 0}$$

$$= \begin{cases} \mp \dfrac{1}{2\sqrt{-\dfrac{p(v_L)-p(v)}{v_L - v}}} \left\{ p'(v) + \dfrac{p(v_L)-p(v)}{v_L - v} \right\} & v > v_L \\[2ex] \pm \dfrac{1}{2\sqrt{-\dfrac{p(v_L)-p(v)}{v_L - v}}} \left\{ p'(v) + \dfrac{p(v_L)-p(v)}{v_L - v} \right\} & v < v_L \end{cases} \qquad (*)$$

• Note: (*) implies:

as $v \to v_L^-$, $\dfrac{du}{dv} \to \pm \dfrac{2p'(v_L)}{2\sqrt{-p'(v_L)}} = \mp \sqrt{-p'(v_L)}$

as $v \to v_L^+$, $\dfrac{du}{dv} \to \mp \dfrac{2p'(v_L)}{2\sqrt{-p'(v_L)}} = \pm \sqrt{-p'(v_L)}$

Thus define:

$$(5)\ \mathcal{S}_1(U_L):\ u = \begin{cases} u_L - \sqrt{(p(v_L)-p(v))(v-v_L)} & v < v_L \\ u_L + \sqrt{(p(v_L)-p(v))(v-v_L)} & v > v_L \end{cases}$$

$$(6)\ \mathcal{S}_2(U_L):\ u = \begin{cases} u_L + \sqrt{(p(v_L)-p(v))(v-v_L)} & v < v_L \\ u_L - \sqrt{(p(v_L)-p(v))(v-v_L)} & v > v_L \end{cases}$$

So that

$\mathcal{S}_1(U_L)$ is tangent to $\mathcal{R}_1$ @ $U = U_L$

$\mathcal{S}_2(U_L)$ is tangent to $\mathcal{R}_2$ @ $U = U_L$

Lemma: The soln of (R-H) $s[u]=[f]$ for $[u]=u-u_L$ consists of $u\in\mathcal{S}_1(u_L)\cup\mathcal{S}_2(u_L)$. Moreover, for $u\in\mathcal{S}_1(u_L)$ we must take

$$s=-\sqrt{-\frac{p(v)-p(v_L)}{v-v_L}}$$

and for $u\in\mathcal{S}_+(u_L)$ take

$$s=+\sqrt{-\frac{p(v)-p(v_L)}{v-v_L}}$$

Pf. This follows directly from (3),(4),(5),(6).

- As a consequence of Lax's general theorem, we know $\mathcal{S}_i$ has C^2-contact with $\mathcal{R}_i$
- Define the wave curves

$$\mathcal{W}_1\equiv\mathcal{W}_1(\bar{U}_L)=\begin{cases}\mathcal{R}_1^+(\bar{U}_L) & v>v_L\\ \mathcal{S}_1^-(\bar{U}_L) & v<v_L\end{cases}$$

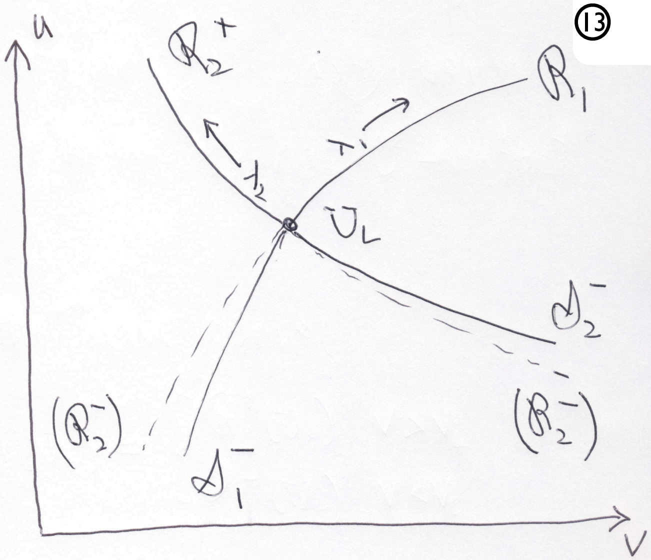
$$\mathcal{S}_1^-(\bar{U}_L):\ u=u_L-\sqrt{(p(v_L)-p(v))(v-v_L)},\quad v<v_L$$

$$\mathcal{W}_2\equiv\mathcal{W}_2(\bar{U}_L)=\begin{cases}\mathcal{R}_2^+(\bar{U}_L) & v<v_L\\ \mathcal{S}_2^-(\bar{U}_L) & v>v_L\end{cases}$$

$$\mathcal{S}_2^-(\bar{U}_L):\ u=u_L-\sqrt{(p(v_L)-p(v))(v-v_L)},\quad v>v_L$$

Note: on $\mathcal{W}_1$ & $\mathcal{W}_2$ u is a fn of v, so wave curves never "hit" $v=0$ $\Rightarrow$ $v\geq 0$ is "invariant region"

Picture:



Principle: Pick out the shock curves $\mathcal{S}_2^-$ that move in the directions of $\mathcal{R}_2$ in which λ decreases, since there are directions in which we cannot obtain simple wave solutions

I.e. "put in the shocks only when $\not\exists$ a smoother solution ..."

Lemma 1: s decreases on $\mathcal{S}_i^-(\bar{U}_L)$ going away from $\bar{U}_L$, and moreover

$$(4) \qquad \lambda_i(\bar{U}) < s(\bar{U}) < \lambda_i(\bar{U}_L)$$

for $\bar{U} \in \mathcal{S}_i^-(\bar{U}_L)$, and (4) fails on $\mathcal{S}_i^+(\bar{U}_L)$.

Homework: Prove this.

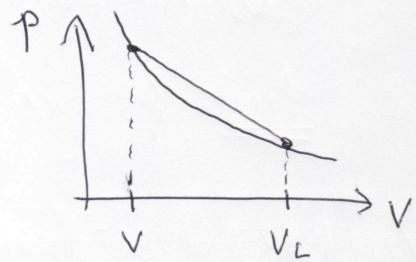
Cor: shock waves $[\bar{U}_L, \bar{U}]$ for $\bar{U} \in \mathcal{S}_i(\bar{U}_L)$ give the admissible i-shocks with left state $\bar{U}_L$.

Homework: How many characteristics impinge on an i-shock? for P-system? in general?

Soln: From (3), $i=1$

$$s = -\sqrt{-\frac{P(V)-P(V_L)}{V-V_L}}$$

$\left.\begin{array}{l} P \text{ convex up} \\ V < V_L \text{ on } \mathcal{S}_2^- \end{array}\right\} \Rightarrow$

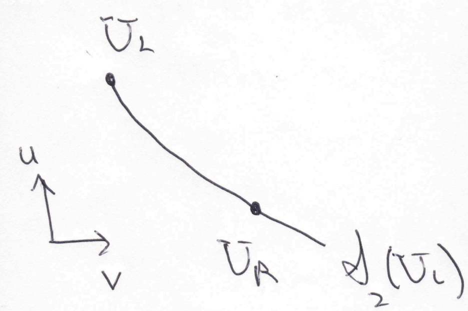


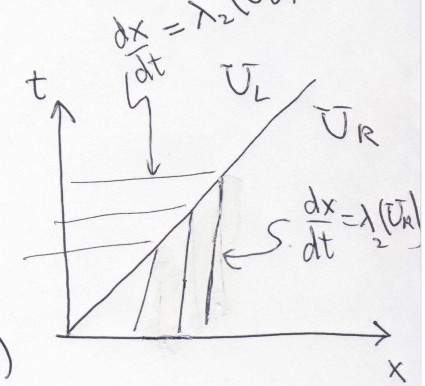
$$P'(V) < \frac{P(V)-P(V_L)}{V-V_L} < P'(V_L)$$

and put in minus signs...

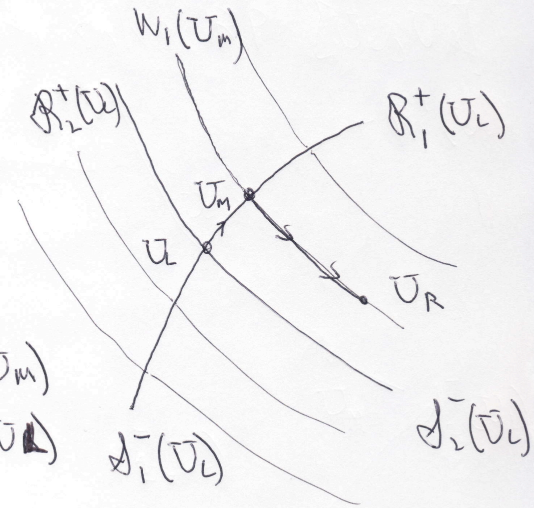
• From (4) we see that i-shocks satisfy the condition that characteristics in the family of the shock impinge on the shock, & characteristics in the opposite family cross the shock.

Note: this is global (shock & rarefaction waves can be arbitrarily strong) while Lax's thm is local.

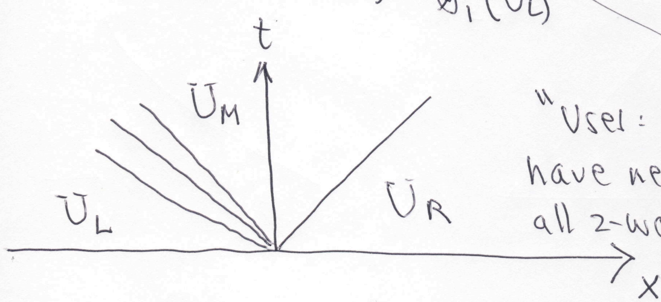




- To solve the R.P., given $\bar{U}_L$, for any $\bar{U}_R$: ① For each $\bar{U}_M \in W_1(\bar{U}_L)$, draw in 2-wave curve $W_2(\bar{U}_M)$. ② given $\bar{U}_R$, find $\bar{U}_M$ s.t. $\bar{U}_R \in W_2(\bar{U}_M)$ ③ The R.P. is solved by the (neg speed) 1-wave taking $\bar{U}_L \to \bar{U}_M$ followed by the (positive speed) 2-wave taking $\bar{U}_M \to \bar{U}_R$.



Picture: $\bar{U}_R \in W_2(\bar{U}_M)$
$\bar{U}_M \in W_1(\bar{U}_L)$



"Uses: all 1-waves have neg speed < all 2-waves have pos speed"

- Missing Steps:

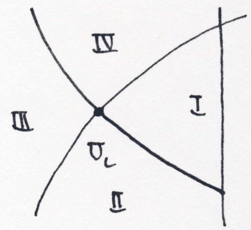
(A) We need that the curves $W_2(\bar{U}_M)$ for $\bar{U}_M \in W_1(\bar{U}_L)$ fill up all of $-\infty < u < +\infty$, $v \geq 0$ in order that each $\bar{U}_R$ has a $\bar{U}_M$ st $\bar{U}_R \in W_2(\bar{U}_M)$

(B) We need that $W_2(\bar{U}_M) \cap W_2(\bar{U}_M') = \emptyset$ for each $\bar{U}_M \neq \bar{U}_M'$, $\bar{U}_M, \bar{U}_M' \in W_1(\bar{U}_L)$.

(A) $\equiv$ required for existence of RP soln $[\bar{U}_L, \bar{U}_R]$ for every $\bar{U}_R$

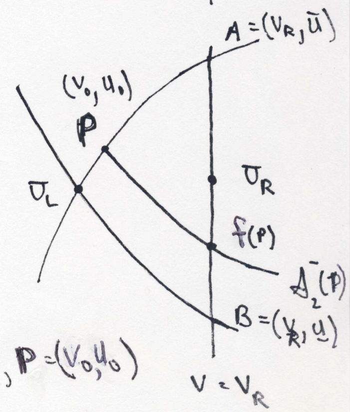
(B) $\equiv$ required for uniqueness of RP soln $[\bar{U}_L, \bar{U}_R]$ for every $\bar{U}_R$

- We verify (A) in region I, similar in II, III, fails in IV.



- wlog do region I
- choose $\mathcal{U}_R$ in I, $\mathcal{U}_R = (V_R, u_R)$.
- recall $\mathcal{S}_2^-$ given by

(☆) $$u = u_0 - \sqrt{(p(v_0) - p(v))(v - v_0)}$$
$$= \Psi(u_0, v_0, v).$$



We need that for some $P \in \widehat{U_L A}$,

$$\Psi(u_0, v_0, V_R) = u_R = f(P), \quad P = (v_0, u_0)$$

- But f is cont $\widehat{U_L A} \to [\underline{u}, \bar{u}]$,

$$B = f(U_L) \le u_L \le f(A) = A$$

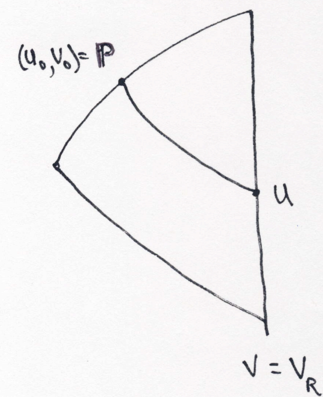
$\Rightarrow$ by IVT that $f(P) = u_L$ for some $P \in \widehat{U_L A}$ ✓

Intermediate Value Theorem

- we verify (B)
- assume region (I).

$$u = u_0 - \sqrt{(p(v_0) - p(v_R))(v_R - v_0)}$$
$$= \Psi(u_0, v_0, V_R)$$

$$u_0 = u_L + \int_{v_L}^{v_0} \sqrt{-p'(\xi)}\, d\xi$$



Fix v_L, v_R:

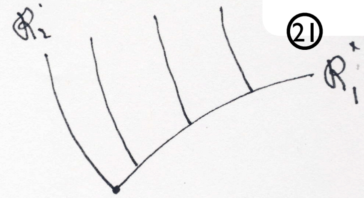
$$\Rightarrow \quad u = u_L + \int_{v_L}^{v_0} \sqrt{-p'(\xi)}\, d\xi - \sqrt{(p(v_0) - p(v_R))(v_R - v_0)} = \varphi(v_0)$$

For uniqueness, need only $\dfrac{du}{dv_0} = \varphi'(v_0) \ne 0$.

$$\varphi'(v_0) = \sqrt{-p'(v_0)} - \frac{p'(v_0)(v_R - v_0) - (p(v_0) - p(v_R))}{\sqrt{(p(v_0) - p(v_R))(v_R - v_0)}}$$

$$= \sqrt{-p_0'} - \frac{\left(p_0' - \frac{p_0 - p_R}{v_R - v_0}\right)\cancel{(v_R - v_0)}}{\sqrt{-\frac{p_0 - p}{v_0 - v_R}}\ \cancel{(v_R - v_0)}}$$

$$= \sqrt{-p_0'} + \frac{-p_0' - \frac{p_0 - p_R}{v_0 - v_R}}{\sqrt{}} > 0 \quad \text{as } p_0' < 0 \ ✓$$



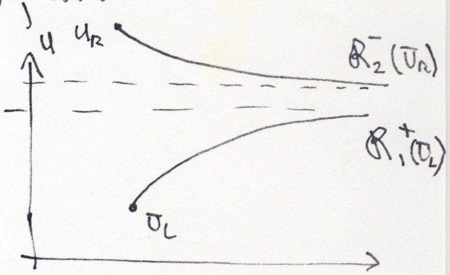
- In region I, uniqueness is clear since $R_2^+(\bar v)$ are integral curves of an (autonomous) vector field & hence are nonintersecting:

<u>problem</u>: R_1^+ may not reach all values of u!

$$R_1^+ : \quad u = u_L + \int_{v_L}^{v} \sqrt{-p'(\xi)}\, d\xi$$

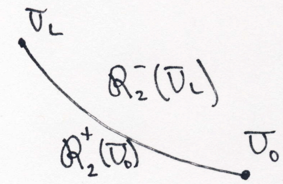
if $\sqrt{-p'(\xi)}$ is integrable, then $u \to \tilde u$ as $v \to \infty$

- In this case, $R_2^-(\bar v_R)$ does not reach all values of u either:



$$R_2^- : \quad u = u_L - \int_{v_L}^{v} \sqrt{-p'(\xi)}\, d\xi$$

FOR EXAMPLE:



$R_2^-(\bar v_L)$:

$$u = u_L - \int_{v_L}^{v} \sqrt{-p'(\xi)}\, d\xi$$

$R_2^+(\bar v_0)$:

$$u = u_0 + \int_{v_0}^{v} \sqrt{-p'(\xi)}\, d\xi \qquad v < v_0$$

$$= u_L + (u_0 - u_L) - \int_{v_L}^{v} \sqrt{-p'(\xi)}\, d\xi + \int_{v_L}^{v} - \int_{v}^{v_0} \sqrt{-p'(\xi)}$$

$$= u_L + (u_0 - u_L) - \int_{v_L}^{v} \sqrt{-p'(\xi)}\, d\xi + \int_{v_L}^{v_0} \sqrt{-p'(\xi)}$$

$$-\int_{v}^{v+a} \sqrt{-p'(\xi)}\, d\xi$$

$$= u_L - \int_{v_1}^{v+a} \sqrt{-p'(\xi)}\, d\xi \quad ✓$$

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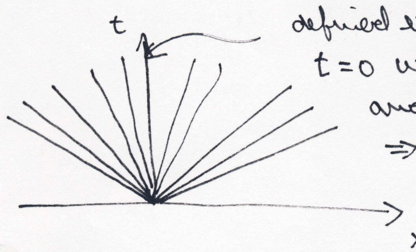
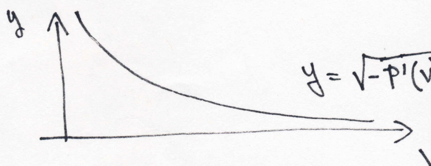
i.e., the soln of the R.P. is given as follows:



defined everywhere except $t=0$ where $V=\frac{1}{\rho}=\infty$ and u is undefined $\Rightarrow$ vacuum

$p'<0$, $p''>0$ $\Rightarrow$ p' increasing

$\Rightarrow -p'$ decreasing $\Rightarrow \sqrt{-p'}$ decreasing



$y=\sqrt{-p'(V)}$

Thus $\sqrt{-p'(V)}$ integrable $\Rightarrow \sqrt{-p'(V)} \to 0$ as $V\to\infty$

$\Rightarrow \lambda_1(V), \lambda_2(V) \to 0$ as $V\to\infty$

$\therefore$ picture is justified

H.W. Recall polytropic $p = C_v(\gamma-1)\, e^{S/C_v}\, \frac{1}{V^\gamma}$, $\gamma>1$, $\gamma\le 5/3$

If isentropic $\Rightarrow S=\text{const} \Rightarrow p=\frac{a^2}{V^\gamma}$

Also $pV=kT$ so isothermal $\Rightarrow p=\frac{a^2}{V}$

Show: Isentropic $\Rightarrow$ vacuum, isothermal $\Rightarrow$ no vacuum

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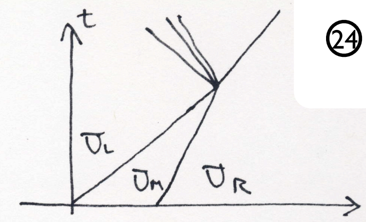
☐ WAVE INTERACTIONS:

2-2 shocks interact:

what waves will come out?



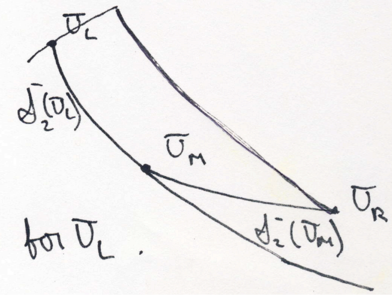
Ans (real physics!) a 2-shock & a 1-rarefaction

We prove this:

- We need: if

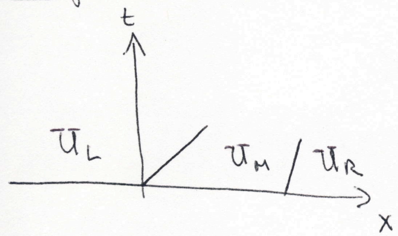
$U_M \in S_2^-(U_L)$,

then $S_2^-(U_M) \in$ region I for U_L.



<u>Theorem</u>. The interaction of two 2-shocks produces a 1-rarefaction wave and a 2-shock wave.

Proof: assume that at $t=0$ we have

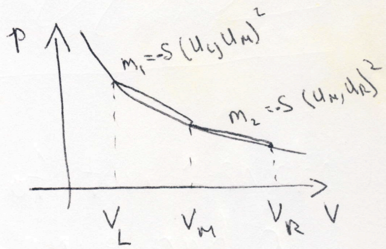


Then $U_M \in S_2^-(U_L)$, $U_R \in S_2^-(U_M)$

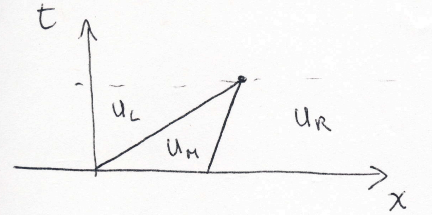
$$S(U_L,U_M) = +\sqrt{-\frac{P(V_M)-P(V_L)}{V_M-V_L}}$$

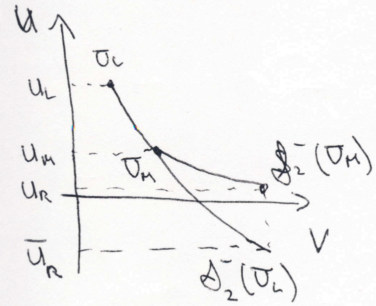
$$> S(U_M,U_R) = +\sqrt{-\frac{P(V_R)-P(V_M)}{V_R-V_M}}$$

because P is convex up



∴ the two shocks must interact at some later time "Ie the abs magnitude of the slope gives square of the pos speed"





Need: $S_2^-(\bar U_M)$ lies above $S_2^-(\bar U_L)$ for $V > V_M$. I.e.

Need: $\bar u_R < u_R$

$S_2^-(\bar U_L)$: $u = u_L - \sqrt{(P(V_L)-P(V))(V-V_L)}$ $\quad V > V_L$

$S_2^-(\bar U_M)$: $u = u_M - \sqrt{(P(V_M)-P(V))(V-V_M)}$ $\quad V > V_M$

$$u_R = u_M - \sqrt{(P(V_M)-P(V_R))(V_R-V_M)}$$

$$u_M = u_L - \sqrt{(P(V_L)-P(V_M))(V_M-V_L)}$$

$$\bar u_R = u_L - \sqrt{(P(V_R)-P(V_L))(V_L-V_R)}$$

$$u_R - \bar u_R = u_R - u_M + u_M - u_L + u_L - \bar u_R = -H(R,M) - H(M,L) + H(R,L)$$

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$$u_R - \bar{u}_R = \sqrt{(P(V_R) - P(V_L))(V_L - V_R)}$$

$$- \sqrt{(P(V_R) - P(V_M))(V_M - V_R)} - \sqrt{(P(V_L) - P(V_M))(V_M - V_L)}$$

Let $H(a,b) = \sqrt{(P(a) - P(b))(b-a)} = H(b,a)$

Homework

Need: $\{H(a,c) - H(a,b) - H(b,c)\} > 0 \;\forall\; 0 < a < b < c < \infty$

H.W. Show that $H(a,c) - H(a,b) - H(b,c) > 0$ for $0 < a < b < c$

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Homework Soln Let

$$H(x,y) = \sqrt{(x-y)(P(y) - P(x))}$$

Show that $H(x,z) > H(x,y) + H(y,z) \quad \forall\; x > y > z$.

For proof, reverse the following steps:

$$\sqrt{(x-z)(P(z)-P(x))} > \sqrt{(x-y)(P(y)-P(x))} + \sqrt{(y-z)(P(z)-P(y))}$$

$$\underbrace{(x-z)(P(z)-P(x))}_{\substack{\| \\ P(z)-P(y)+P(y)-P(x)}} > (x-y)(P(y)-P(x)) + (y-z)(P(z)-P(y)) + 2\sqrt{(x-y)(y-z)(P(y)-P(x))(P(z)-P(y))}$$

$$\Leftrightarrow (x-y)(P(z)-P(y)) + (y-z)(P(y)-P(x))$$

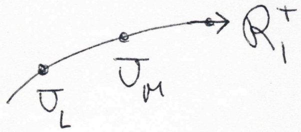
$$> (x-y)(y-z)\, 2\sqrt{\left(\frac{P(y)-P(x)}{x-y}\right)\left(\frac{P(z)-P(y)}{y-z}\right)}$$

$$\Leftrightarrow \frac{P(z)-P(y)}{y-z} + \frac{P(y)-P(x)}{x-y} > 2\sqrt{\alpha B}$$

$$\Leftrightarrow \alpha + B > 2\sqrt{\alpha B}$$

$$\Leftrightarrow (\sqrt{\alpha} - \sqrt{B})^2 > 0 \quad \checkmark$$

<u>Note</u>: the rarefaction curves are integral curves: $R_i^+(U_M) \subset R_i^+(U_L)$ if $U_M \in R_i^+(U_L)$



The shock curves are <u>not</u> integral curves of vector field: $S_i^-(U_M) \not\subset S_i^-(U_L)$ for

$$U_M \in S_i^-(U_L)$$

<u>This</u> is what causes most of the theoretical problems:

<u>OPEN PROBLEM</u>: it is not known that I-value problem is well-posed for p-system, even for local soln:

EXISTENCE ✓ (GLIMM)

UNIQUENESS (~~OPEN~~)

CONT DEP (~~OPEN~~)

Bressan Solved All these

Global Conv. of Glimm Scheme ~TV bound OPEN

Global Existence (Diperna / Compensated Compactness)