Expanding spacetimes which characterize the radial instability of critical and under-dense Friedmann Spacetimes

Blake Temple
UC-Davis

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Collaborators: Christopher Alexander, Zeke Vogler

(Dedicated to Joel Smoller)

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The Friedmann Spacetimes describe a uniformly expanding 3-space of constant density and curvature evolving in time according to Einstein's equations of General Relativity.

Point of departure is our paper 2017 RSPA…

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Research

Check for updates

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An instability of the standard model of cosmology creates the anomalous acceleration without dark energy

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We identify the condition for smoothness at the centre of spherically symmetric solutions of Einstein's original equations without the cosmological constant or dark energy. We use this to derive a universal phase portrait which describes general, smooth, spherically symmetric solutions near the centre of symmetry when the pressure $p=0$. In this phase portrait, the critical $k=0$ Friedmann space–time appears as a saddle rest point which is unstable to spherical perturbations. This raises the question as to whether the Friedmann space–time is observable by redshift versus luminosity measurements looking outwards from any point. The unstable manifold of the saddle rest point corresponding to Friedmann describes the evolution of local uniformly expanding space–times whose accelerations closely mimic the effects of dark energy. A unique simple wave perturbation from the radiation epoch is shown to trigger the instability, match the accelerations of dark energy up to second order and distinguish the theory from dark energy at third order. In this sense, anomalous accelerations are not only consistent with Einstein's original theory of general relativity, but are a prediction of it without the cosmological constant or dark energy.

1. Introduction

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Doing Without Dark Energy

Mathematicians Propose Alternative Explanation for Cosmic Acceleration

By Andy Fell on December 13, 2017 in Science & Technology

"Dark energy," a mysterious force that counters gravity, has been proposed to explain why the universe is expanding at an accelerating rate. Mathematicians at UC Davis and the University of Michigan, Ann Arbor, argue for an alternative. Galaxy cluster image from the Hubble Space Telescope.

Three mathematicians have a different explanation for the accelerating expansion of the universe that does without theories of "dark energy." Einstein's original equations for General Relativity actually predict cosmic acceleration due to an

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…the resulting stability analysis stands on its own.

…here we complete the stability analysis for case $p=0$ by incorporating the non-critical $k\neq 0$ Friedmann spacetimes into the theory.

We recently posted our new paper…

arXiv:2412.00643 (gr-qc)

[Submitted on 1 Dec 2024]

Cosmic Accelerations Characterize the Instability of the Critical Friedmann Spacetime

Christopher Alexander, Blake Temple, Zeke Vogler

Introduction

A Brief History of Friedmann Spacetimes

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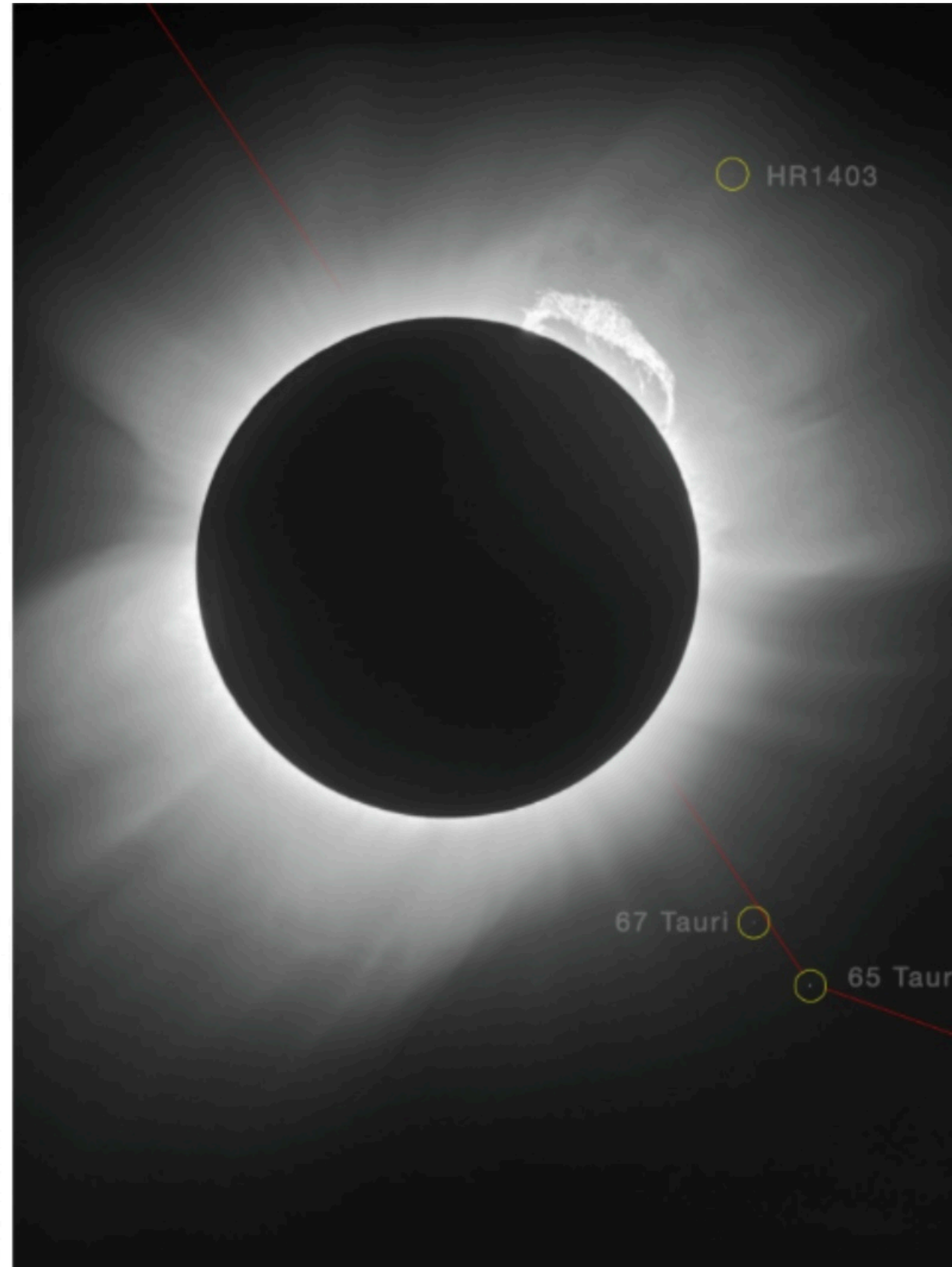
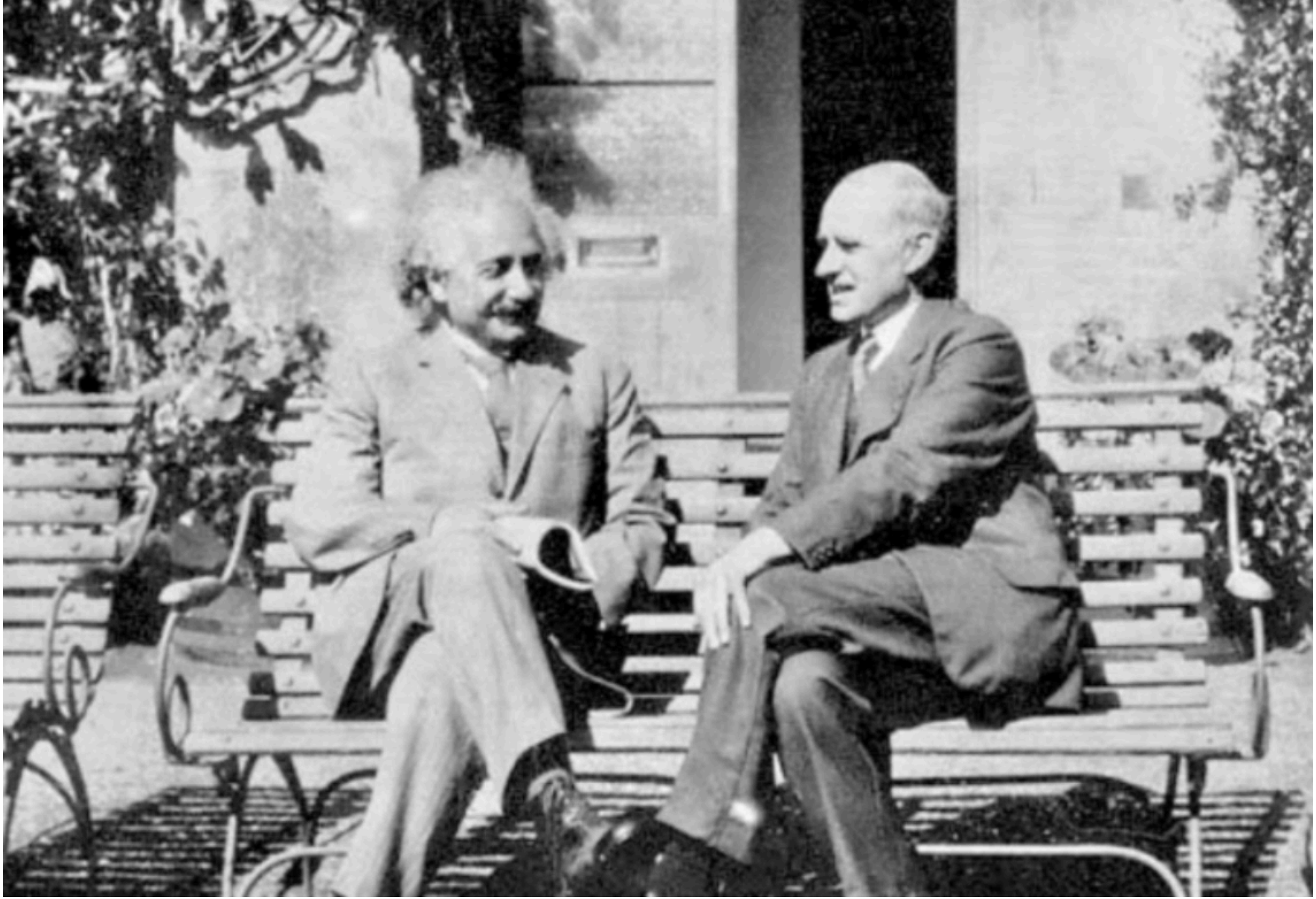
…and Einstein became an international celebrity.



Sir Arthur Eddington-1919



Albert Einstein and Sir Arthur Eddington

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…but accepted Friedmann's work was correct following Friedmann's appeal.

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1929- Hubble's experiment relating redshift vs luminosity confirmed the universe was expanding.

This was accepted as a second confirmation of Einstein's theory of General Relativity…

…that the universe on the largest scale was evolving according to a Friedmann spacetime.

Edwin Hubble (1889-1953)

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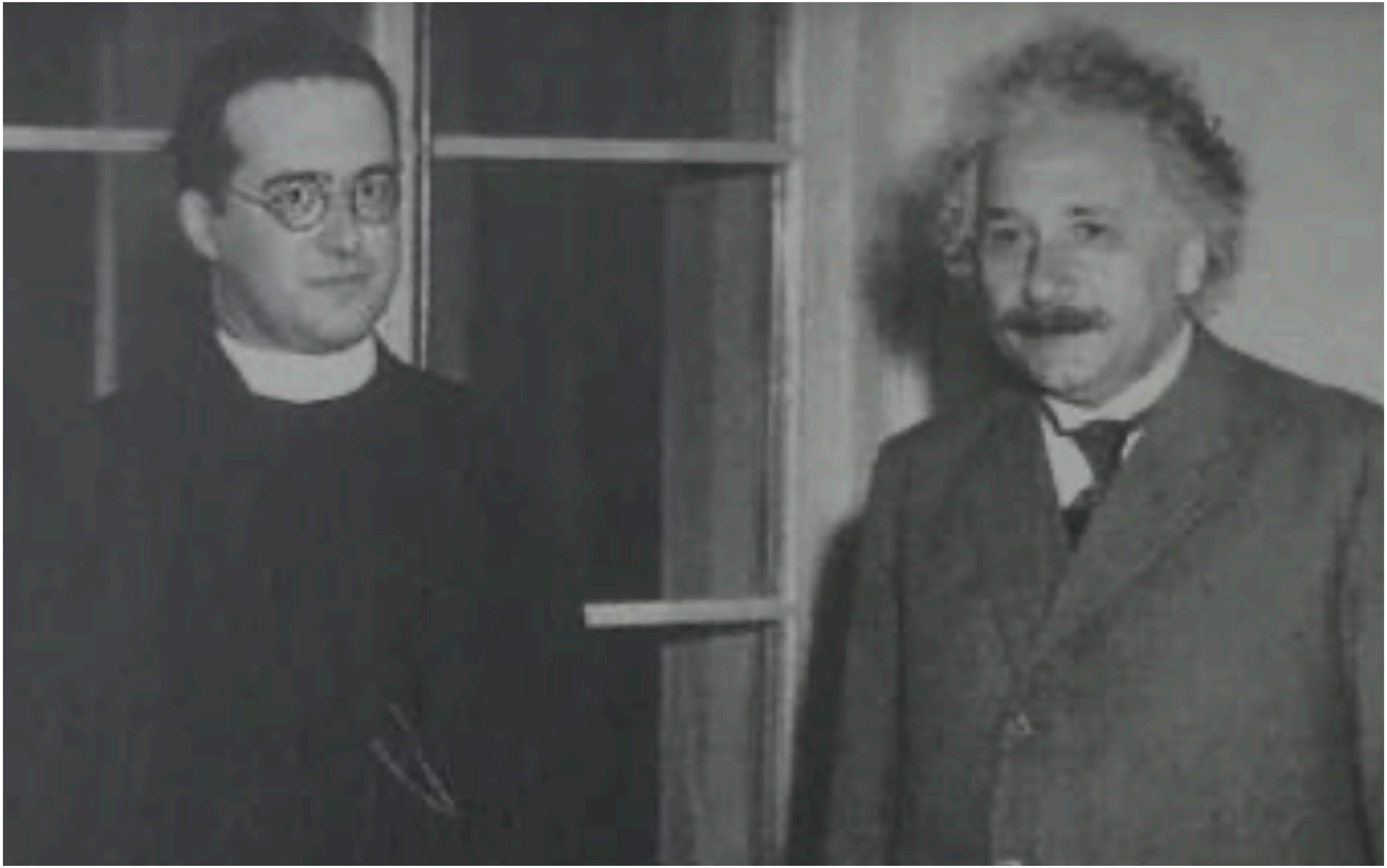
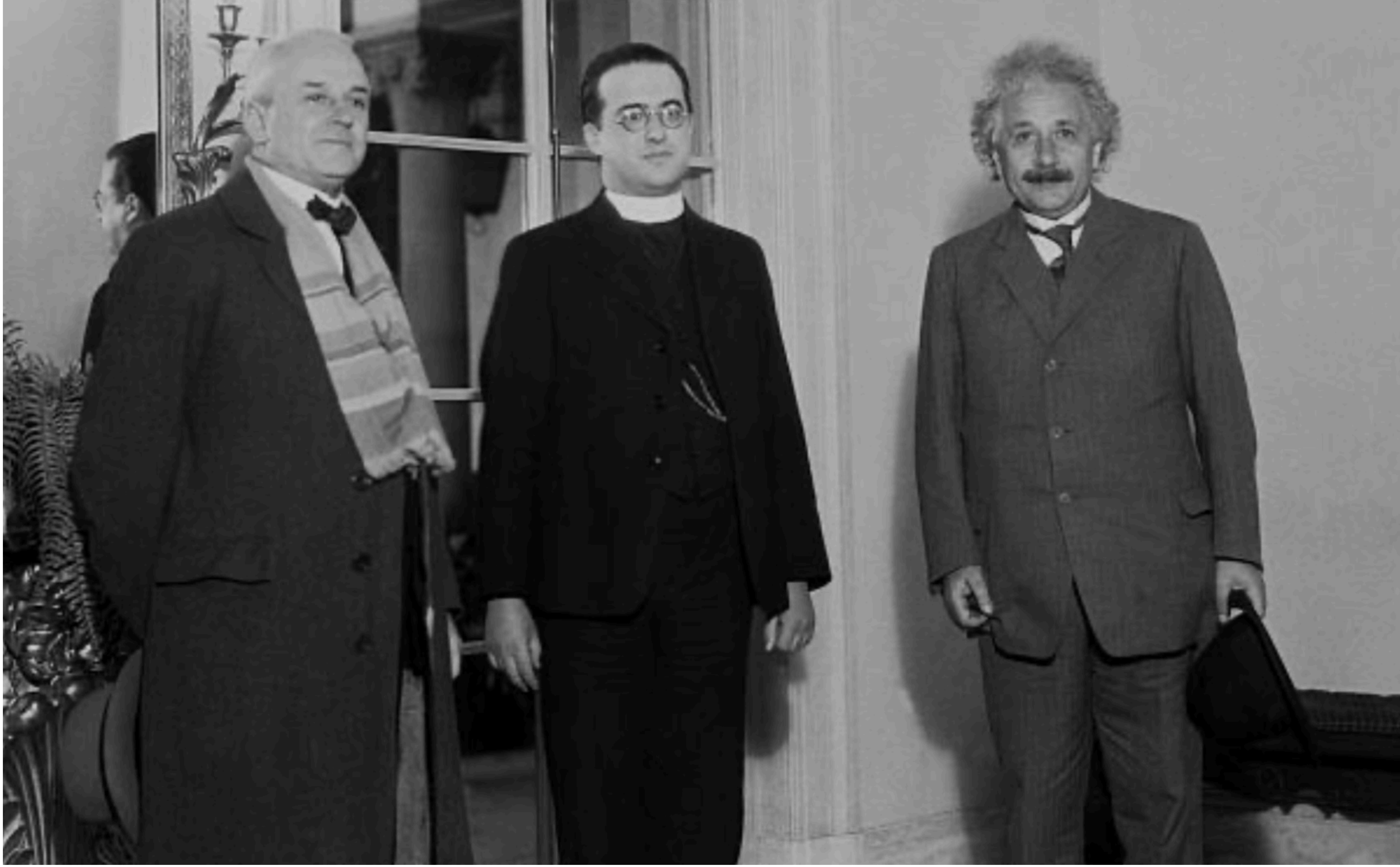
…the static model was shown to be unstable to radial perturbation.



Georges Lemaitre and Albert Einstein

Georges Lemaitre- Mendel Medal (1934)
- Francqui Prize (1934)



Robert Millikan, G. Lemaitre and A. Einstein

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Copernican Principle: If the 3-space at each fixed time is uniform and isotropic about every point, then the spacetime is Friedmann.

This connected Friedmann space-times with pre-scientific notions of earth not being in a special place in the universe, and stuck as a sort of principle in physics more or less accepted by established cosmologists.

Howard Robertson

Arthur Walker

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Often referred to as the FLRW spacetimes.

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$\Lambda \approx .67$	70% Dark Energy
$\mathrm{CDM} \approx .27$	25% Cold Dark Matter

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The $\Lambda\mathrm{CDM}$ model appears to account for most of the cosmological data.

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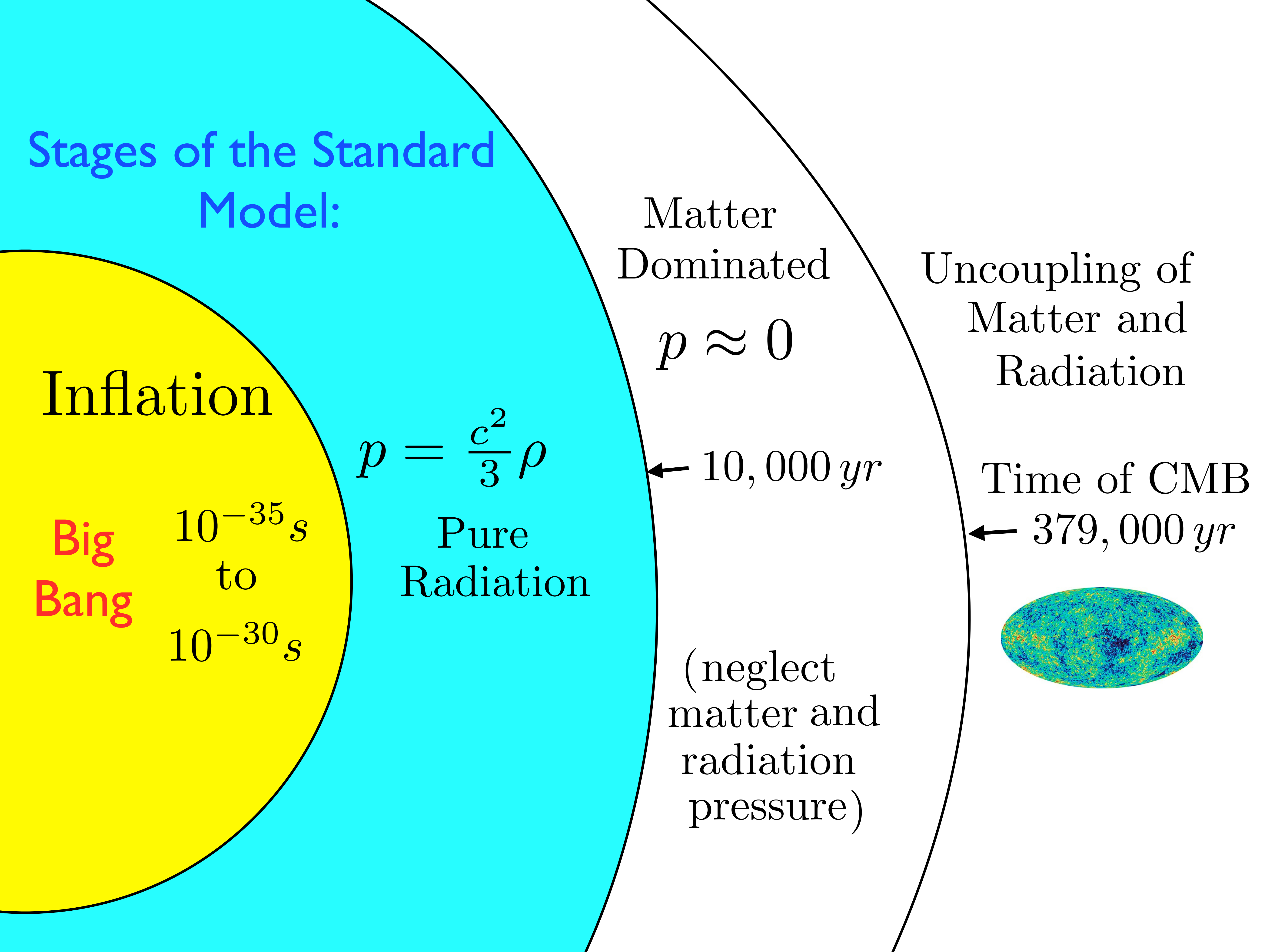
We now have extended this to a complete picture of the local and global instability of critical and under-dense Friedmann spacetimes in the matter dominated regime (p=0). $(\sim \Lambda > 0)$

This characterizes all under-dense solutions which are smooth at the center of symmetry.


Stages of the Standard Model:
Inflation
Big Bang
$10^{-35}s$
to
$10^{-30}s$
$p = \frac{c^2}{3}\rho$
Pure Radiation
Matter Dominated
$p \approx 0$
$10,000\,yr$
(neglect matter and radiation pressure)
Uncoupling of Matter and Radiation
Time of CMB
$379,000\,yr$

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Problem: Radial perturbations create a center which violates the Copernican Principle.

Friedmann Spacetimes

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One parameter $-\infty < k < \infty$

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The Friedmann spacetimes describe a 3-dimensional space of constant curvature evolving in time.

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Note the gauge freedom $t \to t - t_* \ldots$

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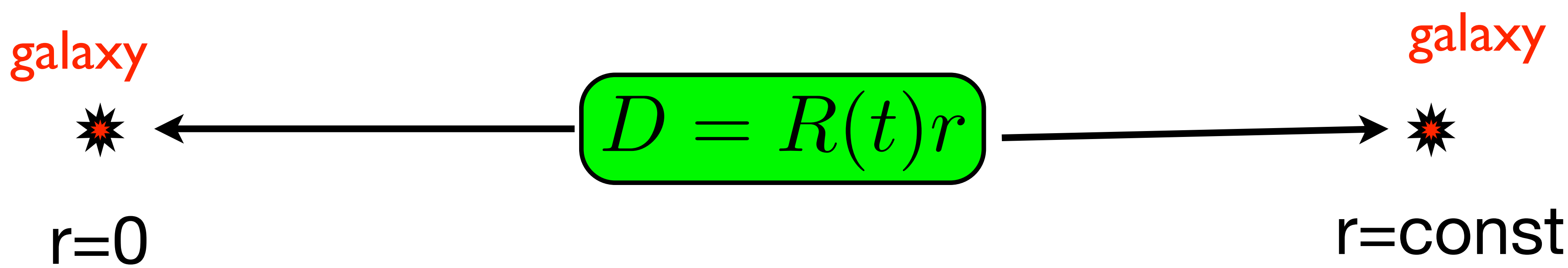
$R(t)$ is determined by the Einstein equations…

Standard Model of Cosmology

- FRW metric $k = 0$:

$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

- $D = Rr$ Measures distance between galaxies at each fixed t



- Conclude: $$\dot{D} = \dot{R}r = \frac{\dot{R}}{R}Rr = HD$$

$$\dot{D} = HD$$

Hubble's Law

$$\text{Hubble's Constant} \equiv H \equiv \frac{\dot{R}}{R}$$

Friedmann Spacetimes

Einstein Equations (1915): $G_{ij} = \kappa T_{ij}$

G_{ij}=Einstein Curvature Tensor

$T_{ij} = (\rho + p)u_i u_j + p g_{ij}$=Stress Energy Tensor (perfect fluid)

Einstein Equations for Friedmann metrics:

$$H^2 = \frac{\kappa}{3}\rho - \frac{k}{R^2}$$

$$\dot{\rho} = -3(\rho + p)H$$

Equations close with equation of state: $p = p(\rho)$

Friedmann Spacetimes

Einstein equations imply $\lim_{t \to 0} R(t) = 0$

$$ds^2 = -dt^2 + R(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right\}$$

$R(0) = 0$ ➡ Big Bang

Space degenerates to a point at t=0.

$k = 0$	"Critical"	$R'(t) > 0$	$0 < t < \infty$
$k < 0$	"Underdense"	$R'(t) > 0$	$0 < t < \infty$
$k > 0$	"Overdense"	$R'(t) > 0$	$0 < t < t_{max}$

Standard Model of Cosmology

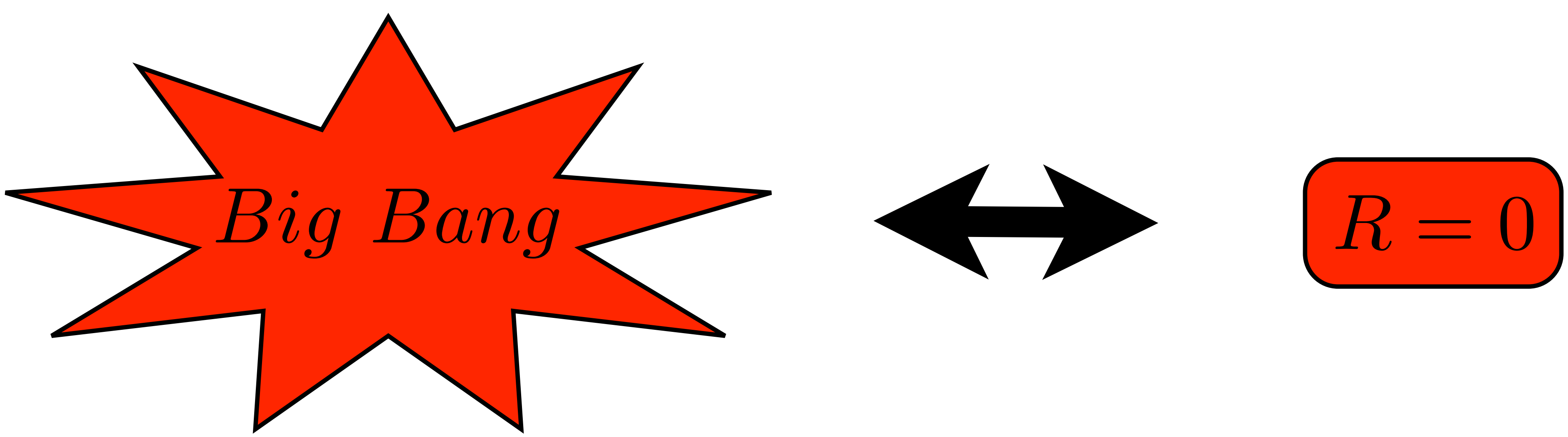
$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

Hubble's Law:

$$\dot{D} = HD$$

Conclude--

``The universe is expanding like a balloon''



The Hubble "Constant" at present time

The inverse Hubble Constant estimates the Age of the Universe

$$\frac{1}{H_0} \approx 10^{10} \text{ years} \approx \text{age of universe}$$

$\frac{c}{H_0}$ is the distance of light travel since the Big Bang, a measure of the size of the visible universe

$$\frac{c}{H_0} = \text{Hubble Radius} \approx 10^{10} lightyears$$

Incorporating Dark Energy into Friedmann

Assume Einstein equations with a cosmological constant:

$$G_{ij} = 8\pi T_{ij} + \Lambda g_{ij}$$

Assume $k = 0$ FRW: $$ds^2 = -dt^2 + R(t)^2 \left\{dr^2 + r^2 d\Omega^2\right\}$$

Leads to: $$H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda$$

Divide by $H^2 = \frac{\kappa}{3}\rho_{crit}$ $$1 = \Omega_M + \Omega_\Lambda$$

Best data fit leads to $\Omega_\Lambda \approx .7$ and $\Omega_M \approx .3$

Implies: The universe is 70 percent dark energy

Q: How to characterize the instability of Friedmann spacetimes to radial perturbation?

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We carry out a stability analysis in Standard Schwarzschild Coordinates (SSC) when p=0…

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We carry out a stability analysis in Standard Schwarzschild Coordinates (SSC) when $p=0$…

SSC is better aligned with the physics than co-moving coordinates.

Standard Schwarzschild Coordinates

A General Spherically Symmetric metric

$$\mathrm{ds}^2 = -D(t,\bar{r})d\bar{t}^2 + E(\bar{t},\bar{r})d\bar{t}d\bar{r} + F(\bar{t},\bar{r})d\bar{r}^2 + G(\bar{t},\bar{r})d\Omega^2$$

Transforms to SSC form: $(\bar{t},\bar{r}) \rightarrow (t,r)$

$$\boxed{ds^2 = -B(t,r)dt^2 + \frac{1}{A(t,r)}dr^2 + r^2 d\Omega^2} \qquad \text{(SSC)}$$

SSC Gauge Freedom

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SSC invariant under time change

$$t \to \phi(t)$$

NG gauge fixes geodesic time at r=0.

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Substituting SSC into the Einstein equations

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yields four equations…

Standard Schwarzschild Coordinates

Four PDE's

$$\left\{-r\frac{A_r}{A}+\frac{1-A}{A}\right\} = \frac{\kappa B}{A}r^2T^{00} \quad (1)$$

$$\frac{A_t}{A} = \frac{\kappa B}{A}rT^{01} \quad (2)$$

$$\left\{r\frac{B_r}{B}-\frac{1-A}{A}\right\} = \frac{\kappa}{A^2}r^2T^{11} \quad (3)$$

$$-\left\{\left(\frac{1}{A}\right)_{tt}-B_{rr}+\Phi\right\} = 2\frac{\kappa B}{A}r^2T^{22}, \quad (4)$$

where

$$\Phi = \frac{B_tA_t}{2A^2B}-\frac{1}{2A}\left(\frac{A_t}{A}\right)^2-\frac{B_r}{r}-\frac{BA_r}{rA} + \frac{B}{2}\left(\frac{B_r}{B}\right)^2-\frac{B}{2}\frac{B_r}{B}\frac{A_r}{A}.$$

(1)+(2)+(3)+(4) ⟺ (1)+(3)+div T=0

(weakly)

Theorem: (Gr-Te) The equations close in the "locally inertial" formulation (1), (2) & Div T=0:

$$\left\{T_M^{00}\right\}_{,0} + \left\{\sqrt{AB}T_M^{01}\right\}_{,1} = -\frac{2}{r}\sqrt{AB}T_M^{01}, \quad (1)$$

$$\left\{T_M^{01}\right\}_{,0} + \left\{\sqrt{AB}T_M^{11}\right\}_{,1} = -\frac{1}{2}\sqrt{AB}\left\{\frac{4}{r}T_M^{11} + \frac{(1-A)}{Ar}(T_M^{00} - T_M^{11}) + \frac{2\kappa r}{A}(T_M^{00}T_M^{11} - (T_M^{01})^2) - 4rT^{22}\right\}, \quad (2)$$

$$rA_r = (1-A) - \kappa r^2 T_M^{00}, \quad (3)$$

$$rB_r = \frac{B(1-A)}{A} + \frac{B}{A}\kappa r^2 T_M^{11}. \quad (4)$$

$$T_M^{00} = \frac{\rho c^2 + p}{1 - \left(\frac{v}{c}\right)^2} \qquad T_M^{01} = \frac{\rho c^2 + p}{1 - \left(\frac{v}{c}\right)^2}\frac{v}{c}$$

$$v = \frac{1}{\sqrt{AB}}\frac{u^1}{u^0}$$

$$T_M^{11} = \frac{p + \left(\frac{v}{c}\right)^2}{1 - \left(\frac{v}{c}\right)^2}\rho c^2 \qquad T^{22} = \frac{p}{r^2}$$

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Theorem: When $t-t_*$ is "time since Big Bang", (k=0) Friedmann in SSC-NG depends only on

$$\xi = r/t$$

(k=0) Friedmann in self-similar coordinates

Lemma: Assume $R(t) = 0$ at $t = 0$ and $p = \sigma\rho$.

Then in co-moving coordinates: $\vec{u} = (1, 0, 0, 0)$

$$R(t) = t^{\frac{2}{3(1+\sigma)}}$$

$$H(t) = \frac{2}{3(1+\sigma)t}$$

$$\rho(t) = \frac{4}{3\kappa(1+\sigma)^2 t^2}$$

(k=0) Friedmann in self-similar coordinates

The following coordinate transformation takes k=0 Friedmann to SSC coordinates $(\bar{t}, \bar{r})$.

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where $\eta=\frac{\bar{r}}{t}$, ,

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where $\eta = \frac{\bar{r}}{t}$, $\xi = \frac{\bar{r}}{\bar{t}} = \frac{\eta}{F(\eta)}$, $\bar{t} = F(\eta)t$

$$\mathcal{F}(\eta) = \left(1 + \frac{\alpha(2-\alpha)}{4}\eta^2\right)^{\frac{1}{2-\alpha}}, \qquad \alpha = \frac{4}{3(1+\sigma)}$$

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…so η is a function of ξ alone!

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(k=0) Friedmann in self-similar coordinates

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…which depend only on

$$\xi = \frac{\bar{r}}{\bar{t}} = \frac{\eta}{F(\eta)}$$

(k=0) Friedmann in self-similar coordinates

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$$\kappa \rho_\sigma r^2 = \frac{3}{4}\alpha^2\eta^2 \qquad v_\sigma = \frac{\alpha}{2}\eta$$

…so $z = \kappa \rho r^2$ and $w = \dfrac{v}{\xi}$

…which depend only on $\xi = \dfrac{\bar{r}}{\bar{t}} = \dfrac{\eta}{F(\eta)}$

(k=0) Friedmann in self-similar coordinates

Moreover…

$$\kappa \rho_\sigma r^2 = \frac{3}{4}\alpha^2\eta^2 \qquad v_\sigma = \frac{\alpha}{2}\eta$$

…so $z = \kappa \rho r^2$ and $w = \dfrac{v}{\xi}$

are natural density and velocity variables…

…which depend only on $\xi = \dfrac{\bar{r}}{\bar{t}} = \dfrac{\eta}{F(\eta)}$

Conclude: To represent k=0 Friedmann as stationary, transform SSC-NG equations to unknowns (z, w)...

...using $\xi = r/t$ as a spacelike coordinate.

$ct \approx$ distance of light travel since the Big Bang.

$\xi =$ "Fractional distance to Hubble Radius".

$0 \leq \xi < 1$ represents the visible universe.

$(t, r) \to (t, \xi)$ is regular coordinate transformation.

STV-PDE

Lemma: Transforming the SSC-NG equations to variables $z(t,\xi), w(t,\xi)$ yields equivalent system…

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$$tw_t + (-1+Dw)\xi w_\xi - w + Dw^2$$
$$-\frac{(1+\sigma^2)\sigma^2}{\xi z}\left(D\frac{(1-v^2)v^2}{(1+\sigma^2 v^2)^2}z\right)_\xi$$
$$+\frac{\sigma^2\xi}{z}\left(D\frac{1-v^2}{1+\sigma^2 v^2}\frac{z}{\xi^2}\right)_\xi = \mathrm{RHS}$$

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Lemma: Transforming the SSC-NG equations to variables $z(t,\xi), w(t,\xi)$ yields equivalent system...

$$t z_t + \xi\Big((-1 + Dw)z\Big)_\xi = -Dwz$$

$$t w_t + (-1 + Dw)\xi w_\xi - w + Dw^2$$
$$- \frac{(1+\sigma^2)\sigma^2}{\xi z}\left(D\frac{(1-v^2)v^2}{(1+\sigma^2 v^2)^2} z\right)_\xi$$
$$+ \frac{\sigma^2 \xi}{z}\left(D\frac{1-v^2}{1+\sigma^2 v^2}\frac{z}{\xi^2}\right)_\xi = \mathrm{RHS}$$

$$\mathrm{RHS} = -\frac{1}{\xi^2}\frac{1-v^2}{1+\sigma^2 v^2}\frac{D}{2A}\left((1-\sigma^2)(1-A) + 2\sigma^2\frac{1-v^2}{1+\sigma^2 v^2} z\right)$$

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$$D = \sqrt{AB} \qquad z = \frac{\kappa \rho r^2}{1-v^2} \qquad w = \frac{1+\sigma^2}{1+\sigma^2 v^2}\frac{v}{\xi}$$

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(Turns out…. $z = \kappa\rho r^2 + O(\xi^6)$)

STV-PDE

Restricting to the case of zero pressure $\sigma = 0$

...we get the $p = 0$ STV-PDE :

$\sigma = 0$

STV-PDE

$$tz_t + \xi\big((-1 + Dw)z\big)_\xi = -Dwz$$

$$tw_t + \xi(-1 + Dw)w_\xi = w - D\left(w^2 + \frac{1}{2\xi^2}(1 - \xi^2 w^2)\frac{1 - A}{A}\right)$$

$$\xi A_\xi = -z + (1 - A)$$

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$$z = \frac{\kappa \rho r^2}{1 - v^2} \qquad w = \frac{v}{\xi}$$

$$p = 0$$

$\sigma = 0$

STV-PDE

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$$z = \frac{\kappa\rho r^2}{1 - v^2} \qquad w = \frac{v}{\xi}$$

$$p = 0$$

We know k=0 Friedmann is a rest point.

STV-PDE

Note: The STV-PDE are NOT the Einstein Equations in self-similar coordinates $z(t,\xi), w(t,\xi)$

STV-PDE

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…they are the SSC-NG equations expressed in terms of $z(t,\xi), w(t,\xi)$

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Expanding solutions about the center in even powers of ξ the resulting ODE's close at every order…

…even powers iff smooth at the center restricts and simplifies the solution space …

Ansatz:

STV-ODE

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$$z(t,\xi) = z_2(t)\xi^2 + z_4(t)\xi^4 + \ldots + z_{2n}(t)\xi^{2n} + \ldots$$

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Substituting into STV-PDE and collecting like powers yields the STV-ODE of order n=1,2,3…

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$$\frac{d}{d\tau}\boldsymbol{v}_n = \begin{pmatrix} (2n+1)(1-w_0)-1 & -(2n+1)z_2 \\ -\frac{1}{2(2n+1)} & 2n(1-w_0)-1 \end{pmatrix}\boldsymbol{v}_n + \boldsymbol{q}_n$$

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STV-ODE

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…nested ODE's… linear at leading order.

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…autonomous ODE in $\tau = \ln t$

STV-ODE

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…autonomous means…

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$$t\frac{d}{dt} = \frac{d}{d\tau}$$

$$\boldsymbol{q}_n = \boldsymbol{q}_n(\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{n-1})$$

...autonomous means...

...amenable to phase portrait analysis.

STV-ODE (n=1)

Setting n=1 we obtain:

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STV-ODE (n=1)

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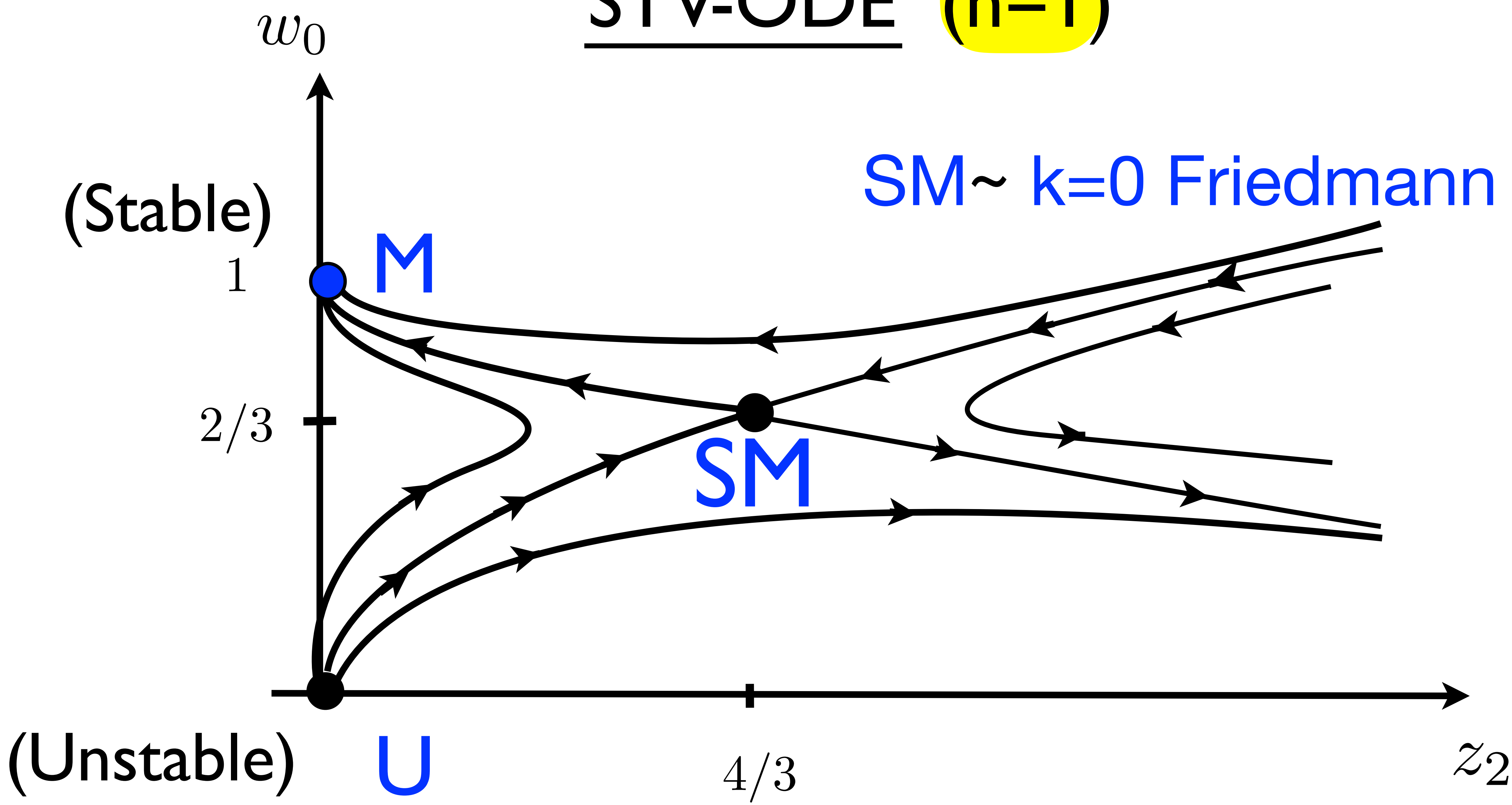
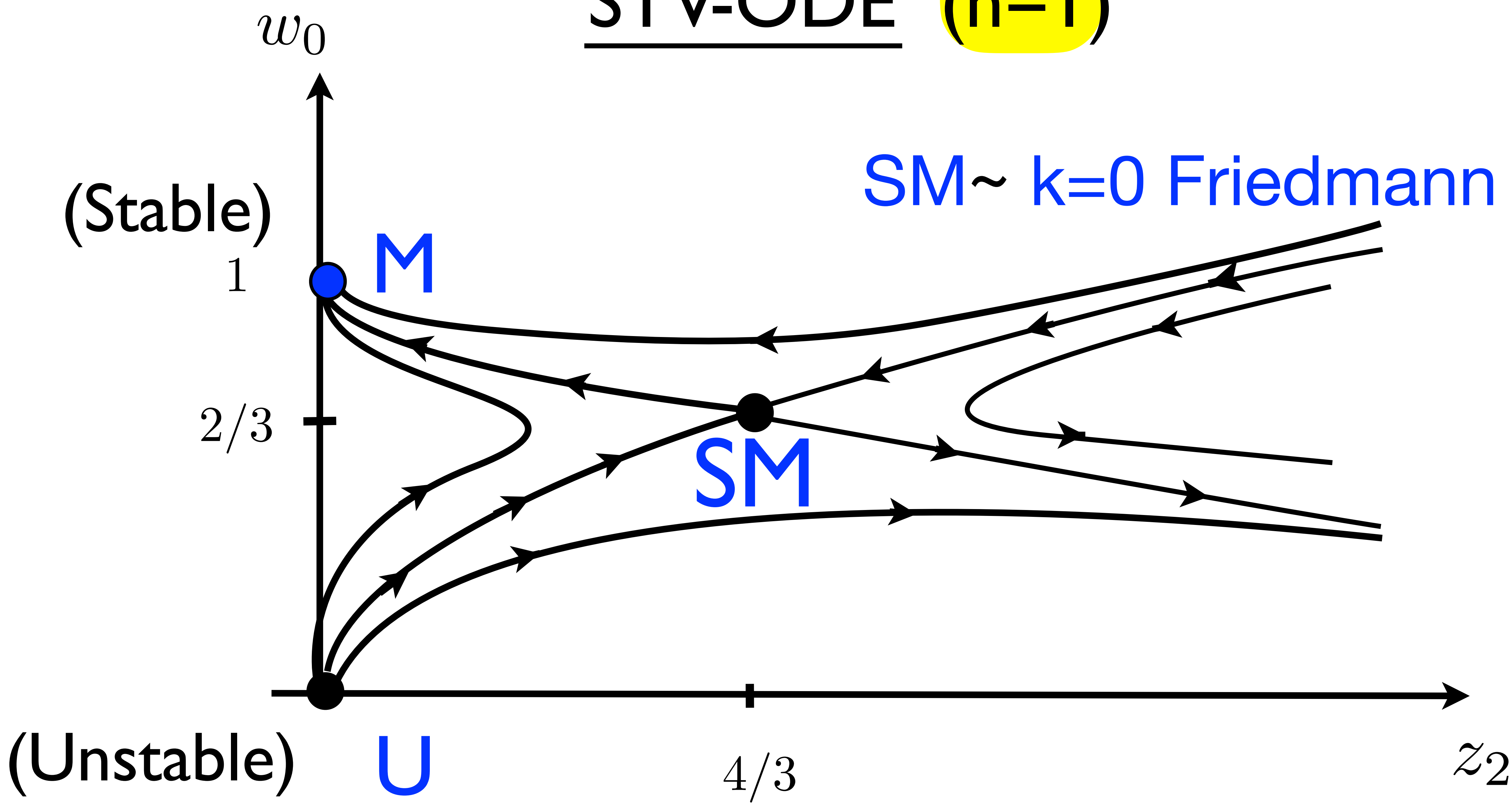
…a 2x2 autonomous system with 3 rest points

STV-ODE (n=1)

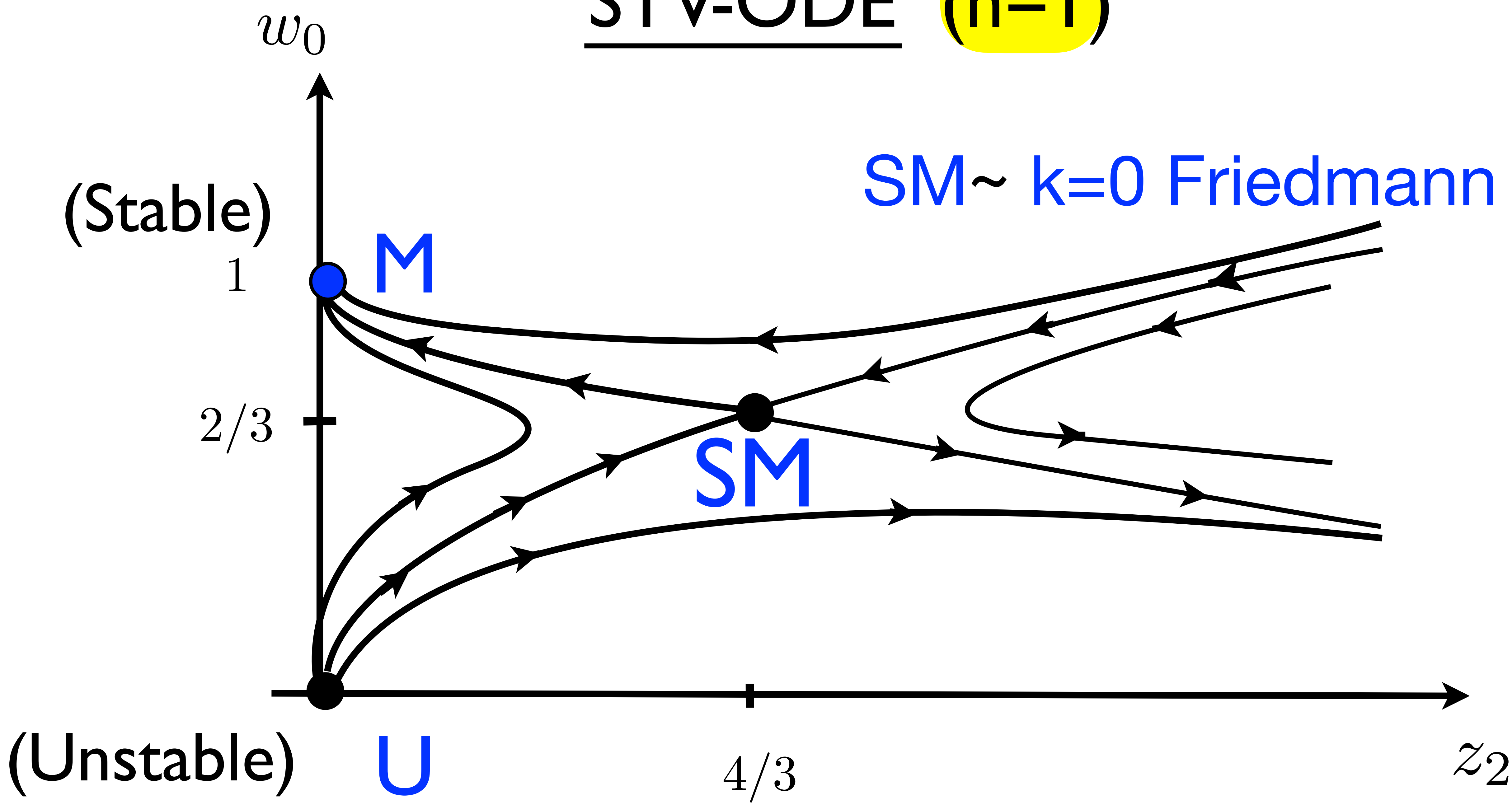


STV-ODE (n=1)



M degenerate stable node: $\lambda_M = -1 \quad \boldsymbol{R}_M = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

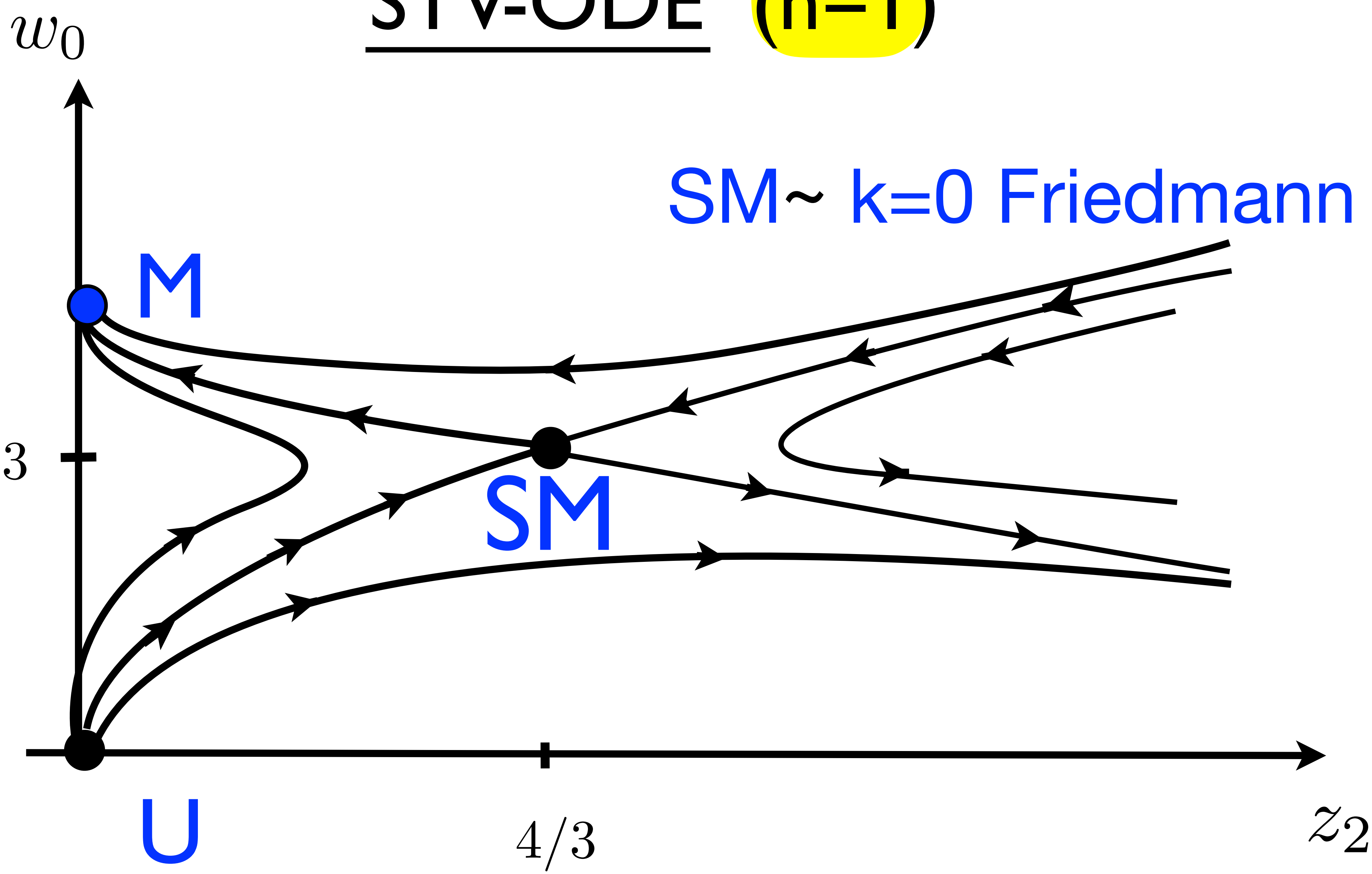
STV-ODE (n=1)



M degenerate stable node: $\lambda_M = -1 \quad \boldsymbol{R}_M = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

U degenerate unstable node: $\lambda_U = 1 \quad \boldsymbol{R}_U = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

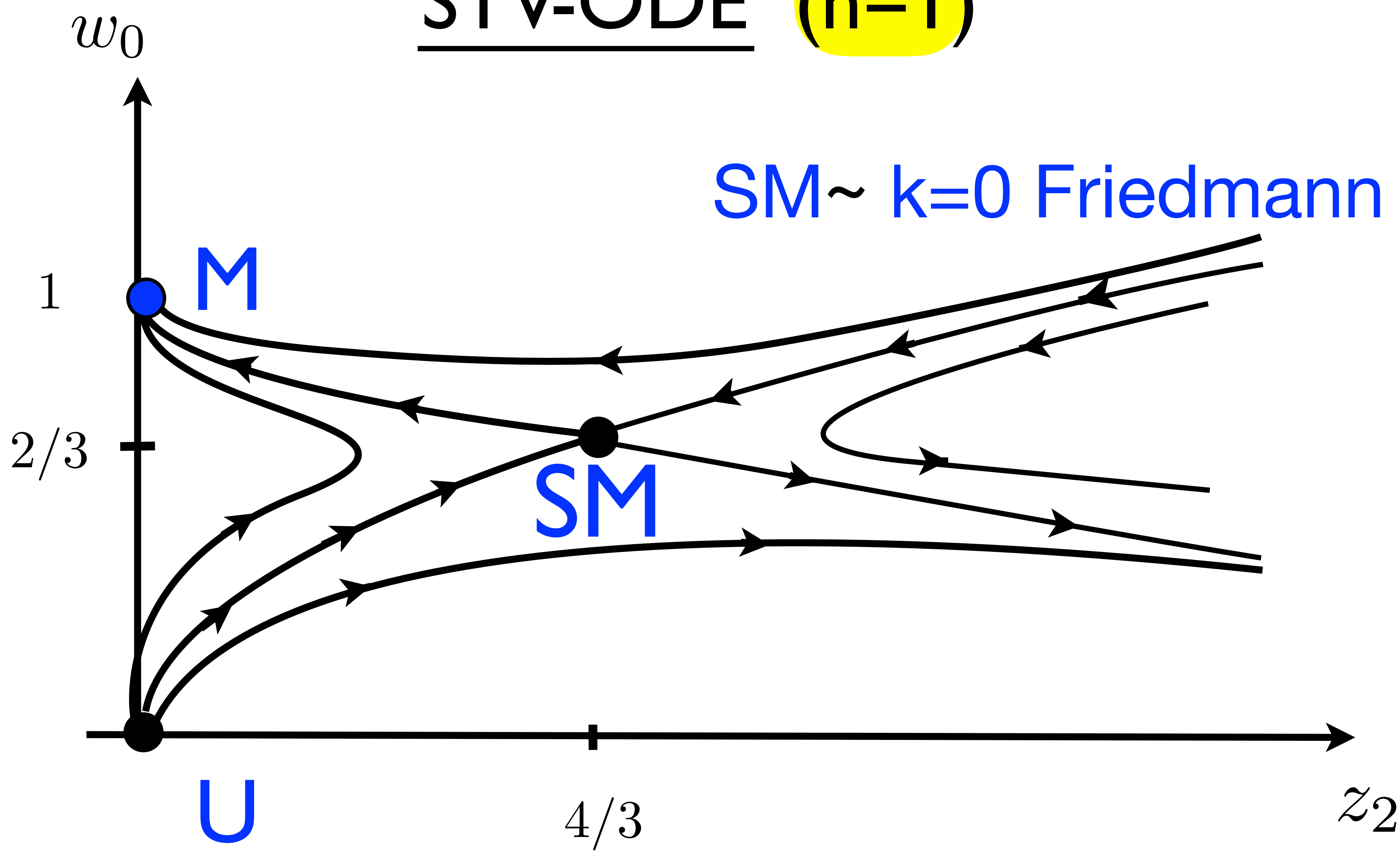
STV-ODE (n=1)



SM $= \left(\frac{4}{3}, \frac{2}{3}\right)$ regular unstable saddle:

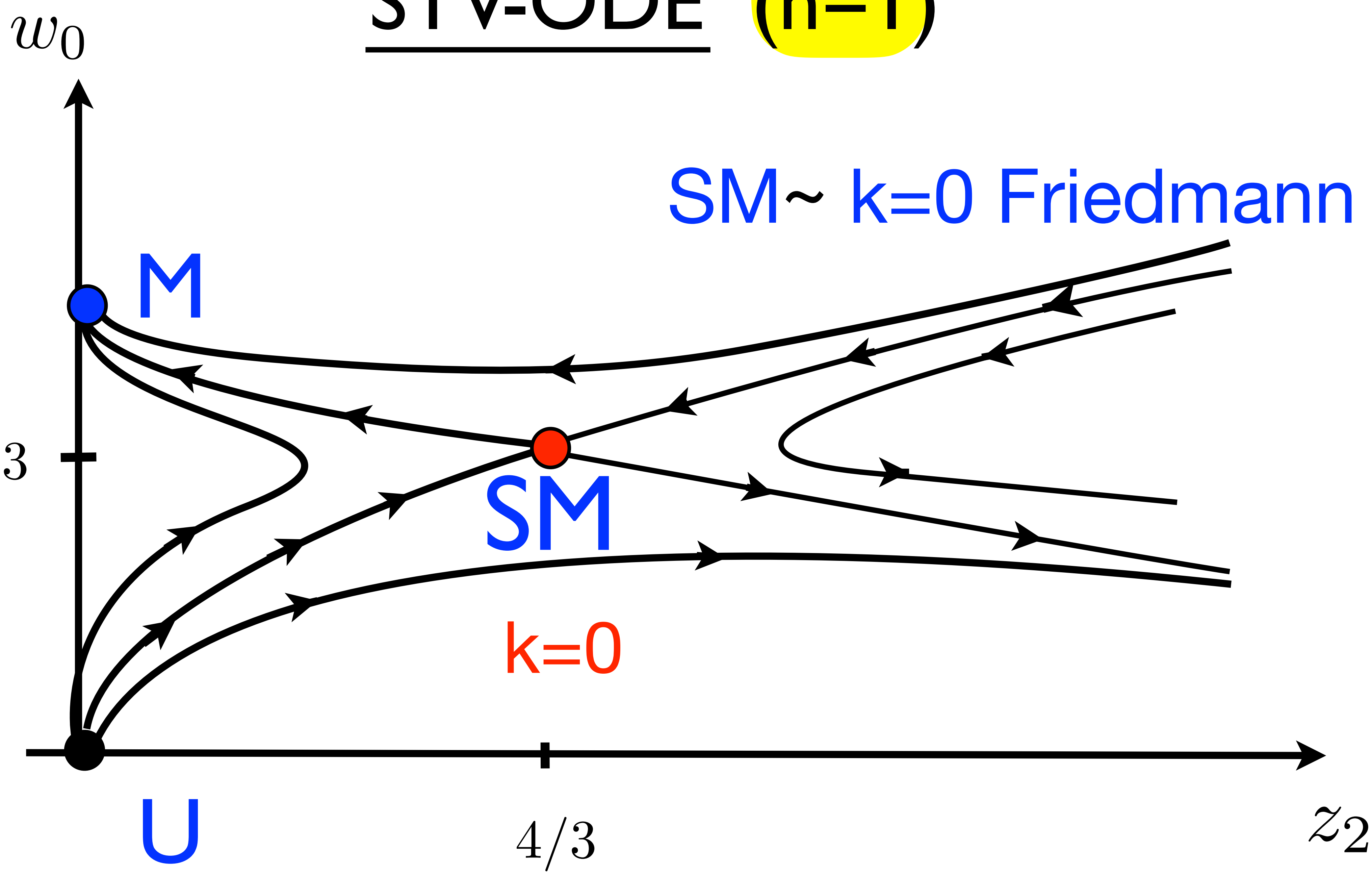
$\lambda_{A1} = \frac{2}{3}$ $\boldsymbol{R}_{A1} = \begin{pmatrix} -9 \\ \frac{3}{2} \end{pmatrix}$, $\lambda_{B1} = -1$ $\boldsymbol{R}_{B1} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

STV-ODE (n=1)



Theorem: The Friedmann spacetimes are SM and unstable manifold of SM.

STV-ODE (n=1)



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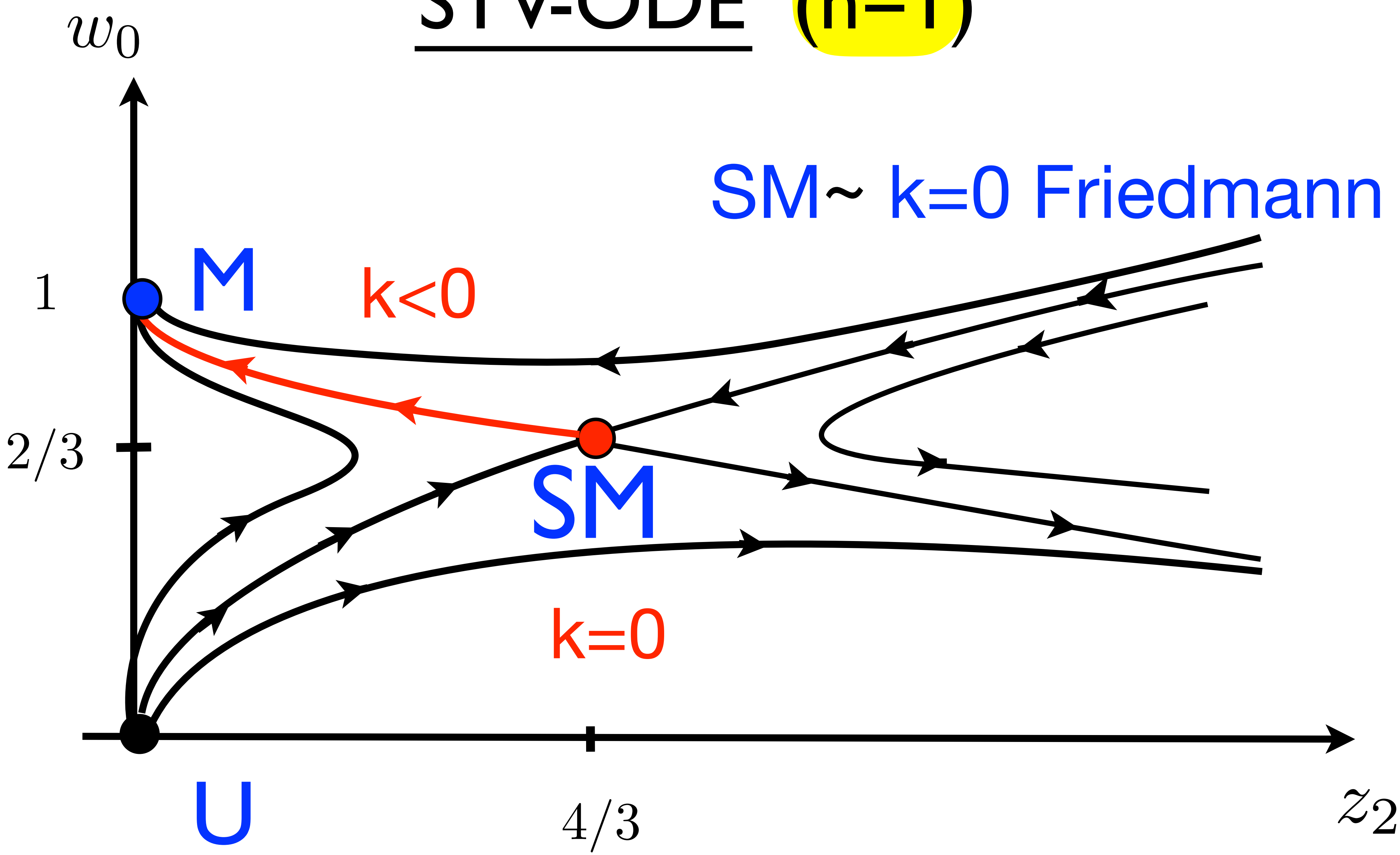

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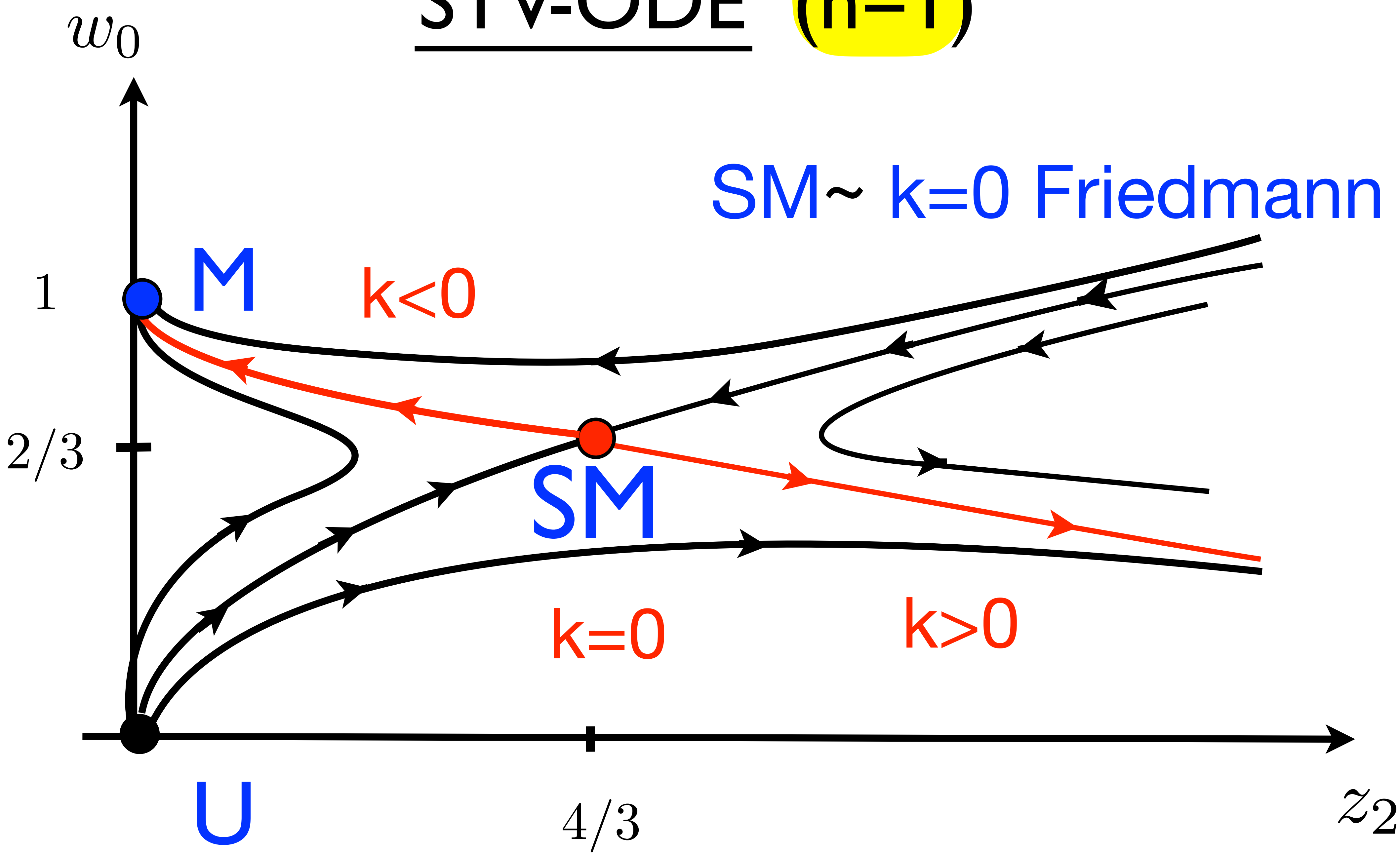

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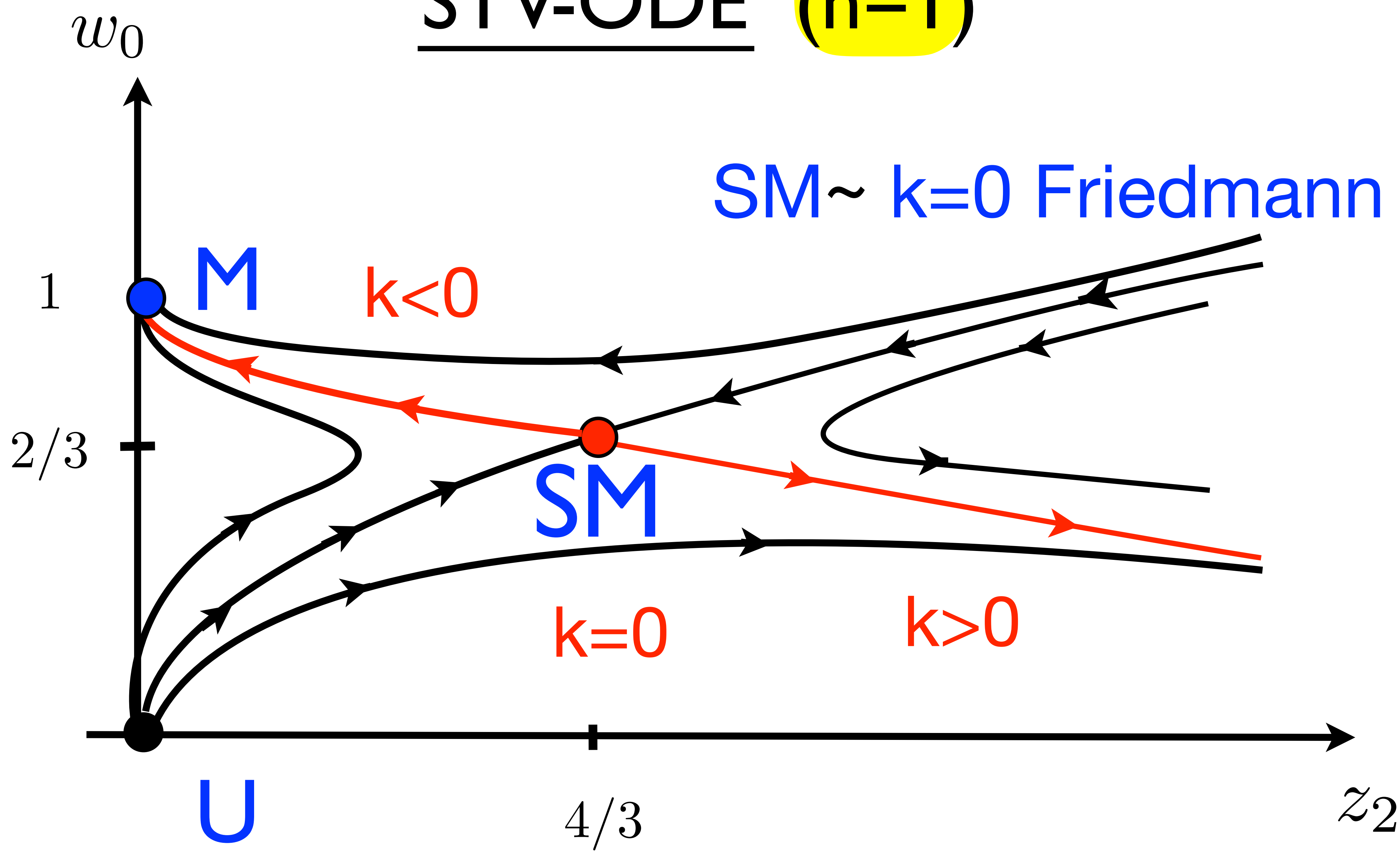
STV-ODE (n=1)



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Proof: Expand exact formulas in powers of ξ

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…we know Friedmann is some orbit, so this is it!

STV-ODE (n=1)

Theorem: There exists a solution dependent SSC time translation

$$t \to t - t_*$$

which maps each trajectory to SM or the unstable manifold of SM at order n=1.

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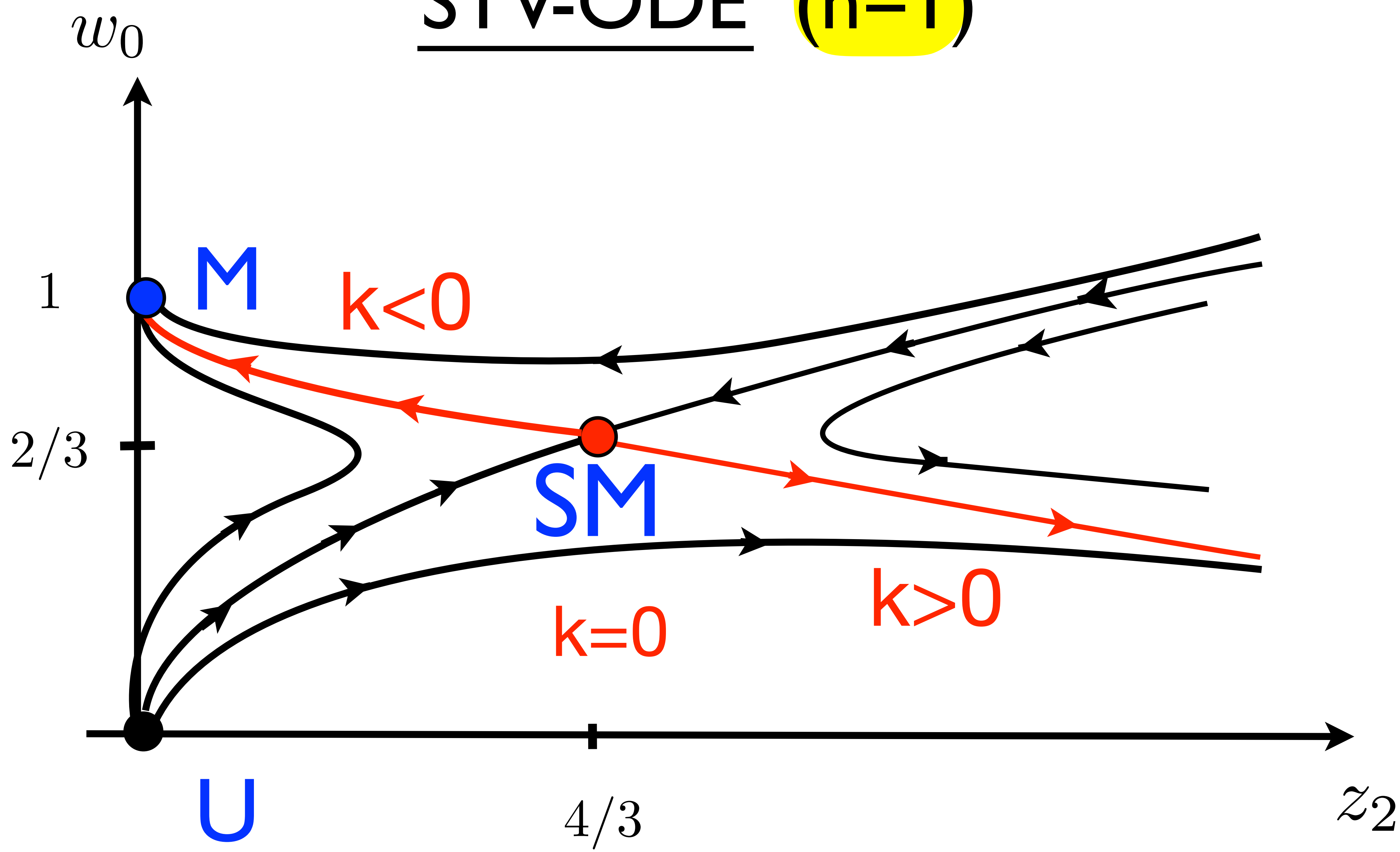
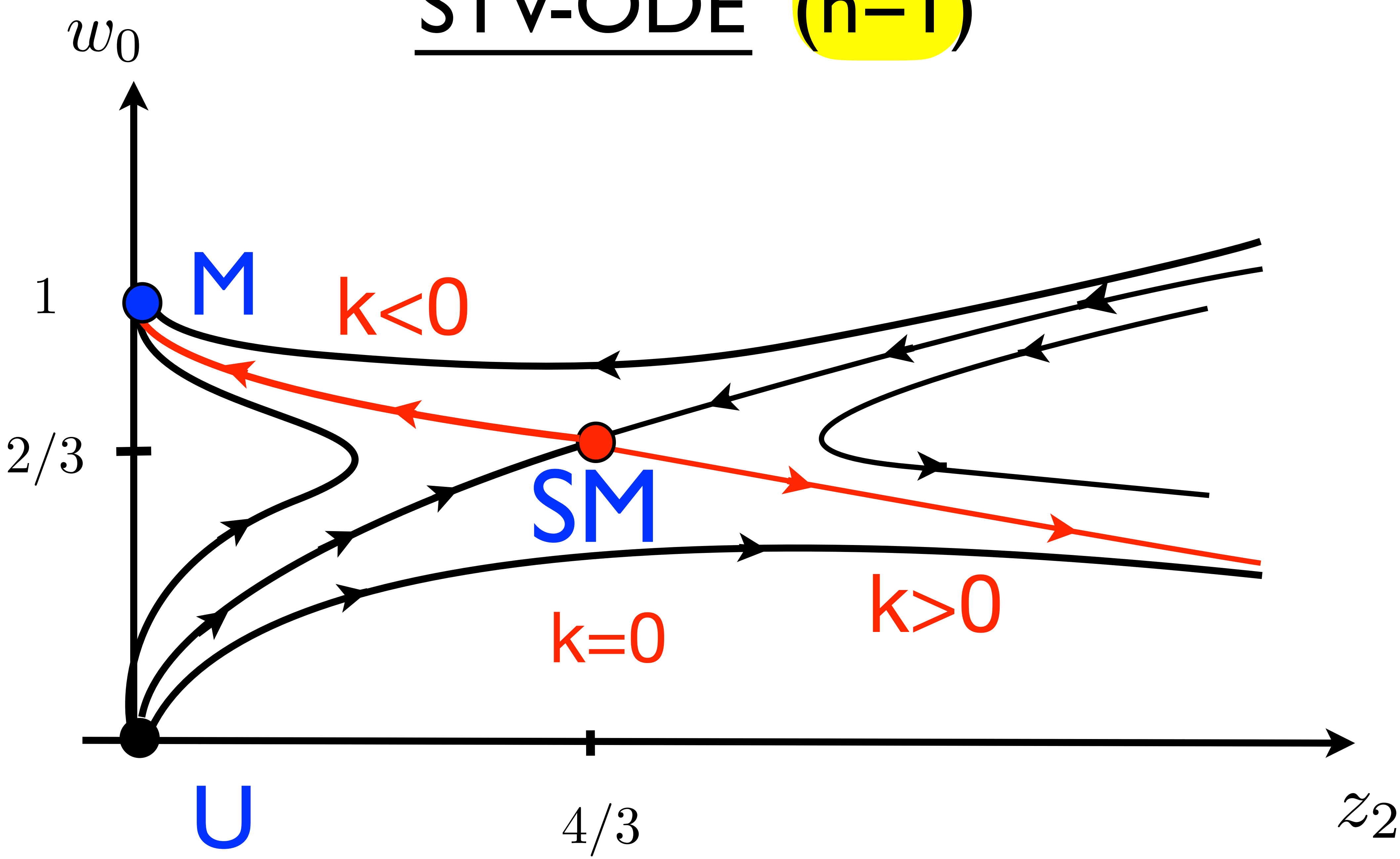
Cor: Every solution of the STV-PDE agrees with Friedmann at leading order n=1.

STV-ODE (n=1)



The final gauge freedom in the STV-ODE

STV-ODE (n=1)



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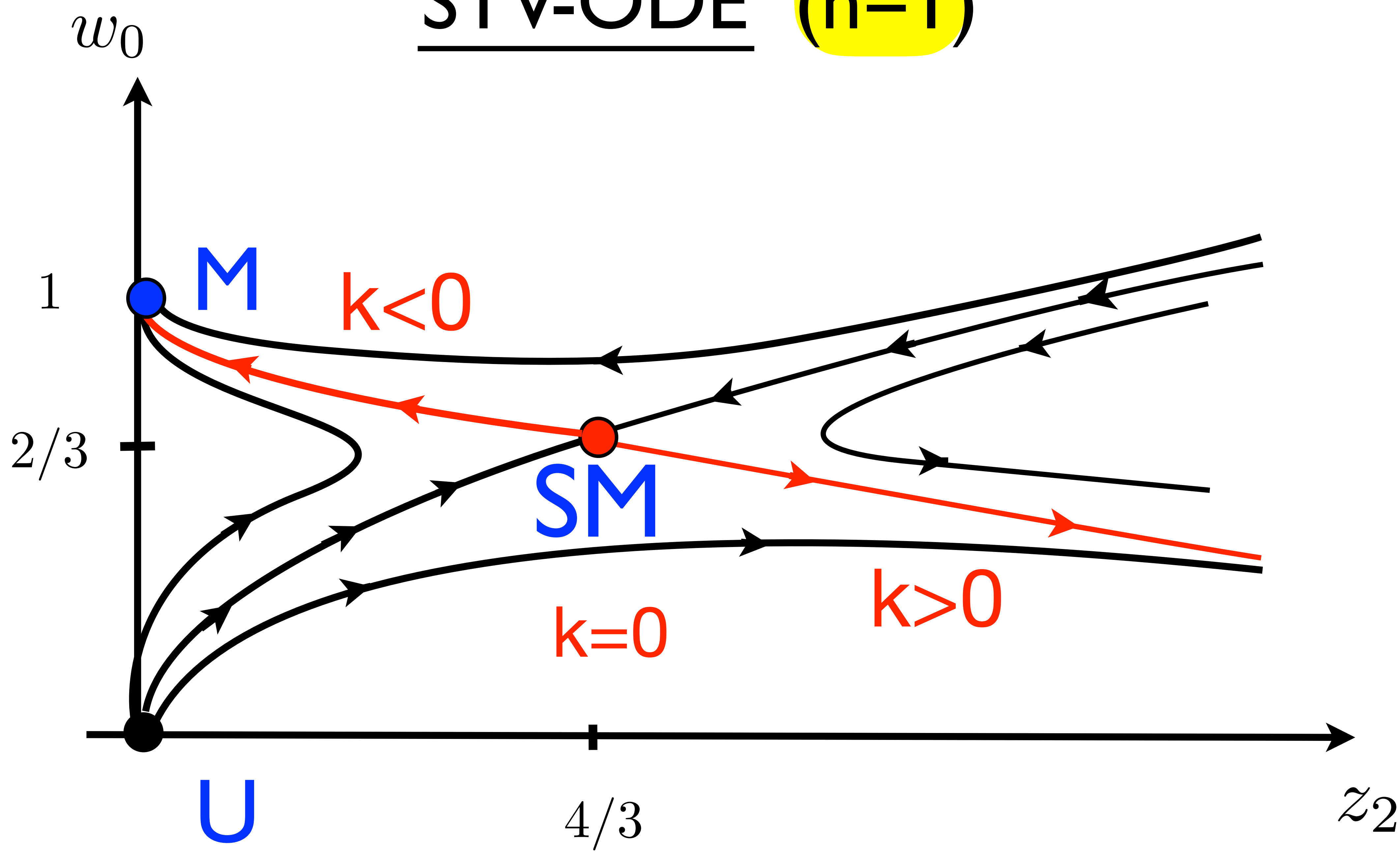

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$$\xi = \frac{r}{t} \to \frac{r}{t - t_*}$$

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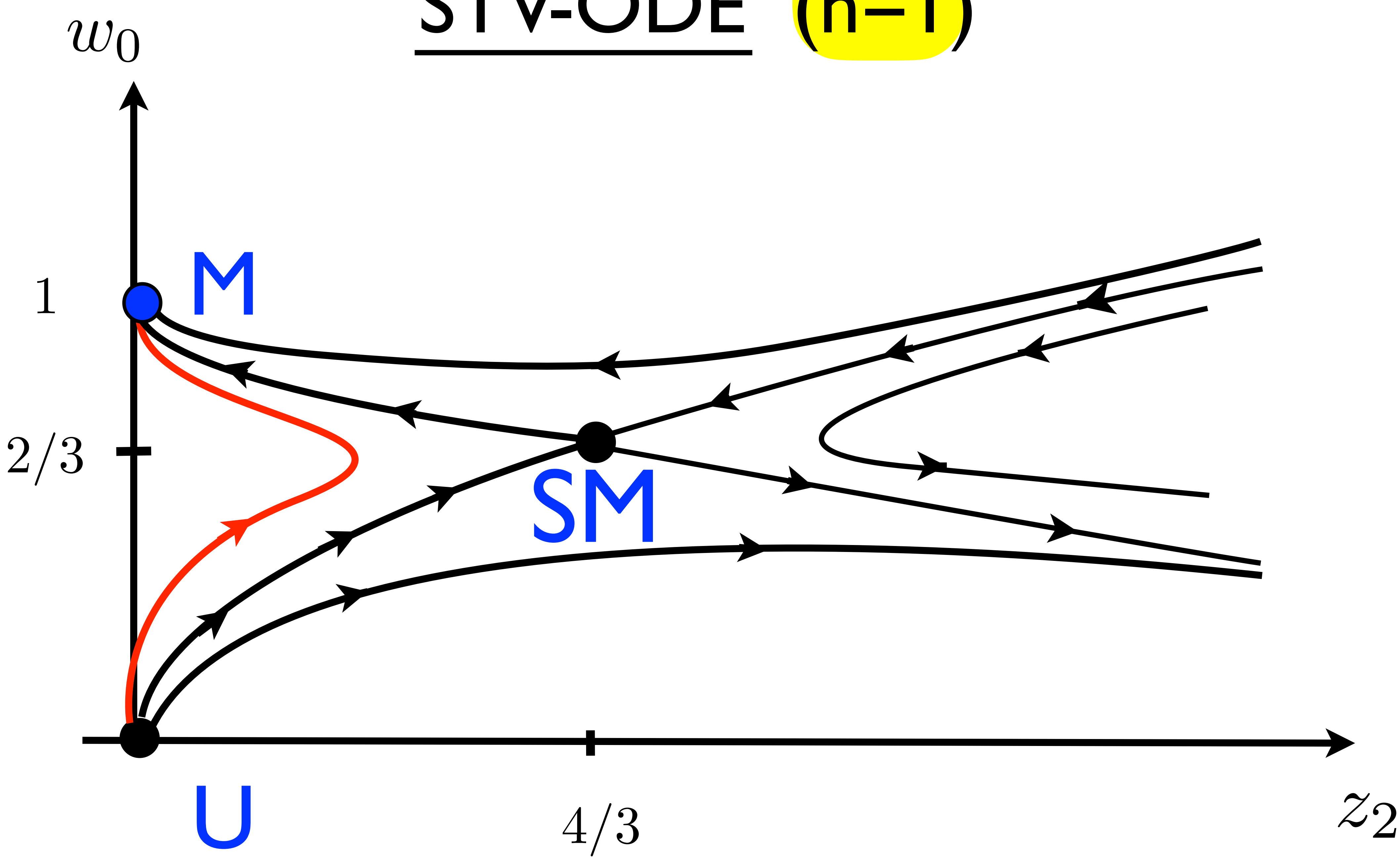

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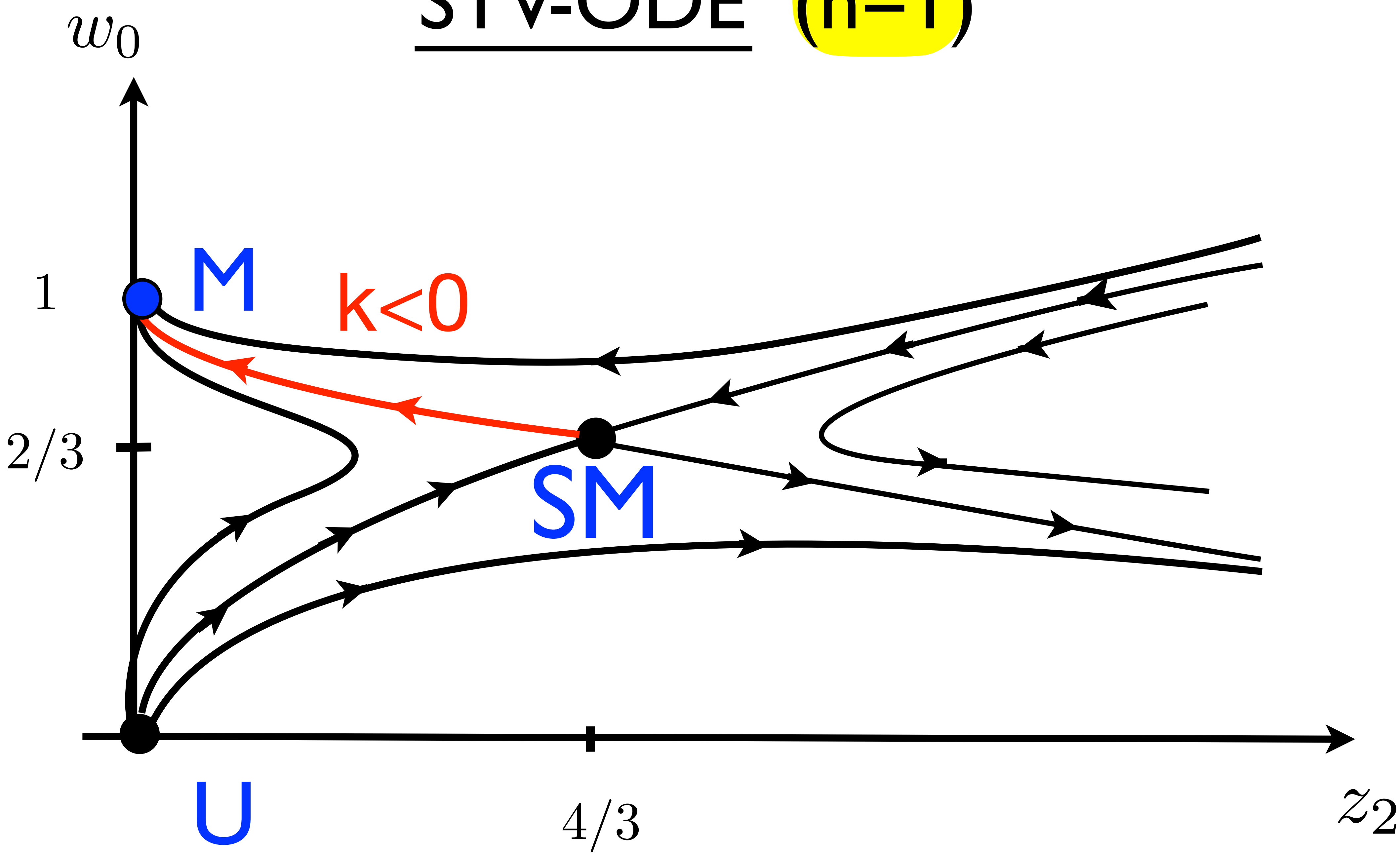

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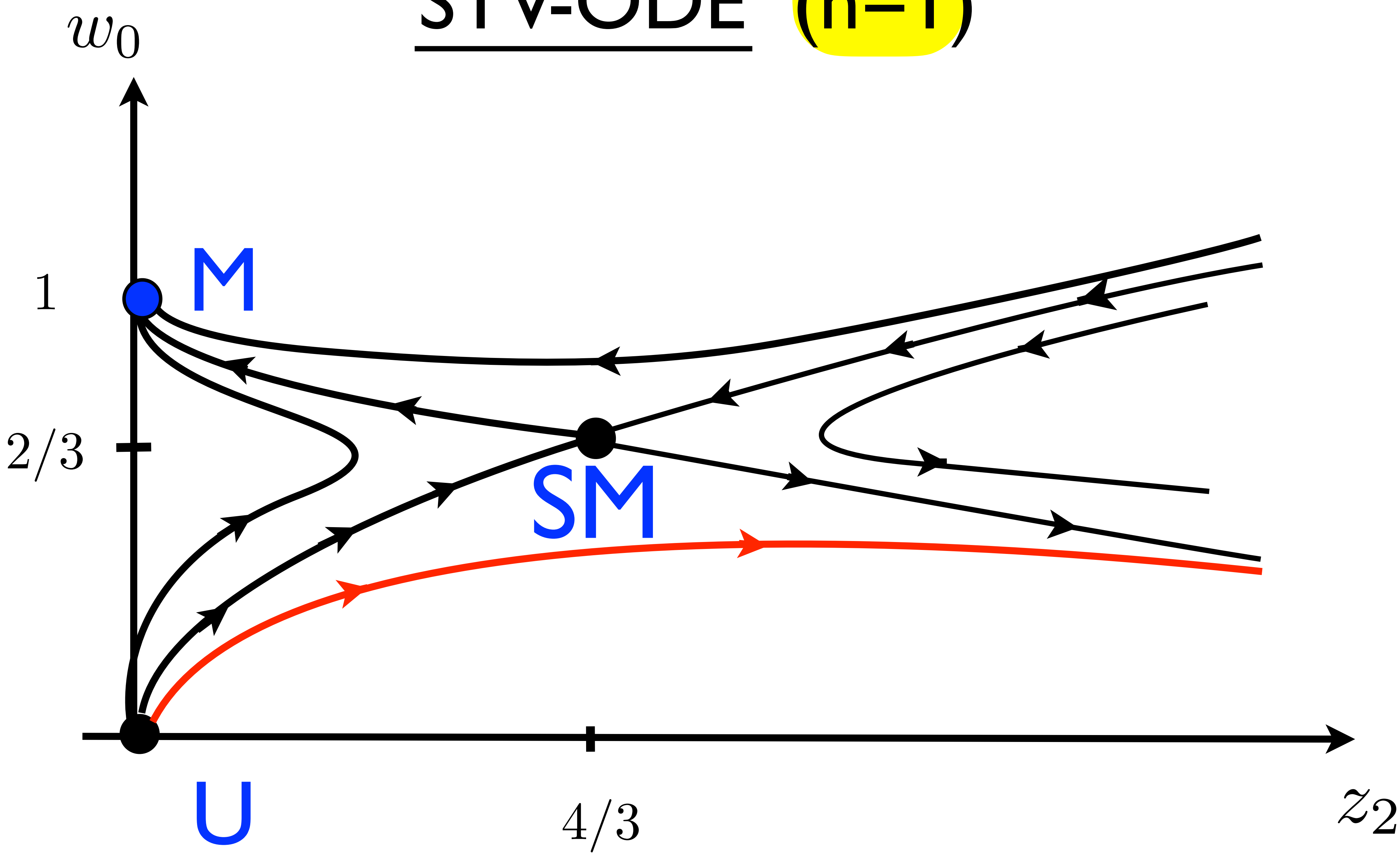

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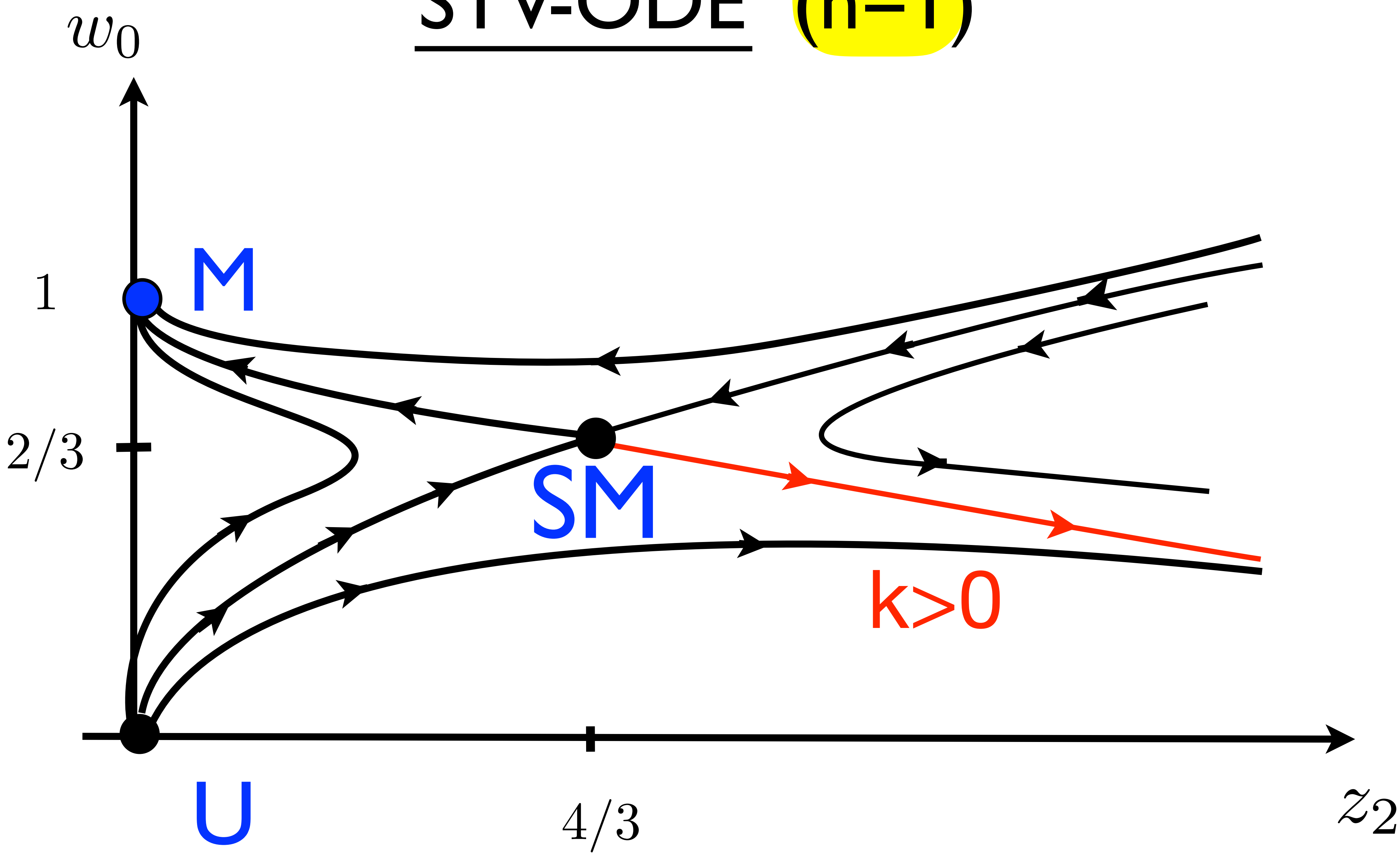

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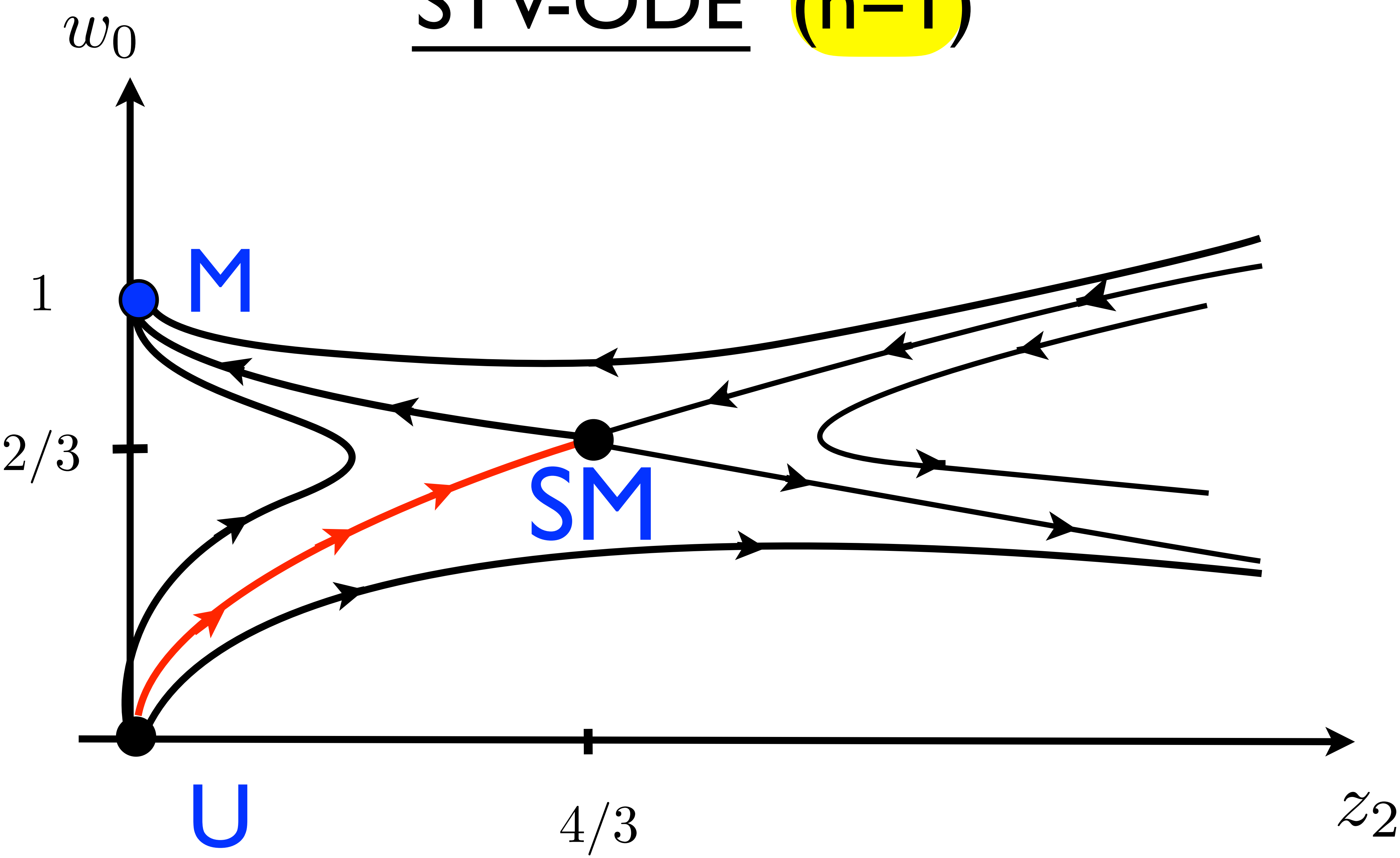

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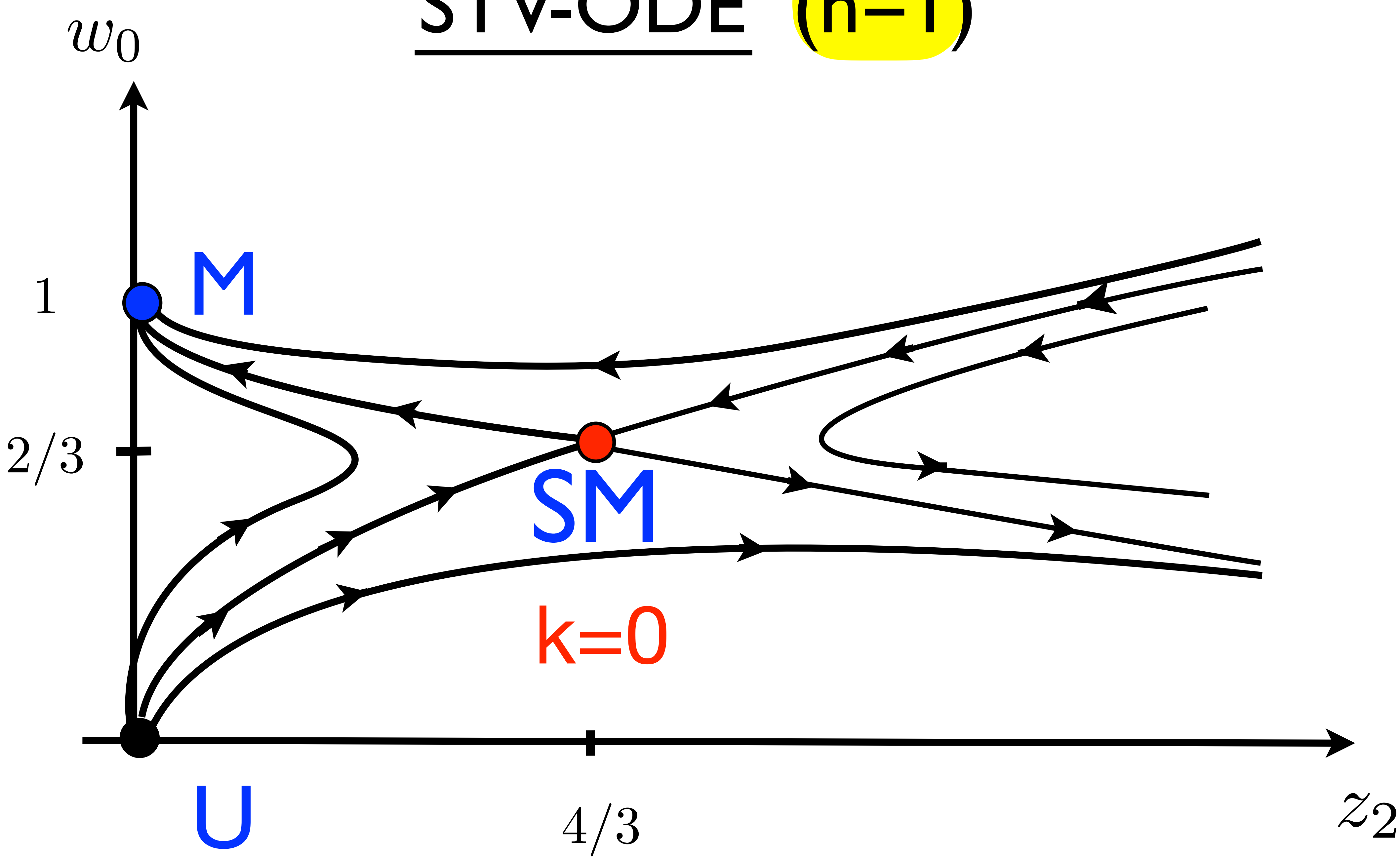

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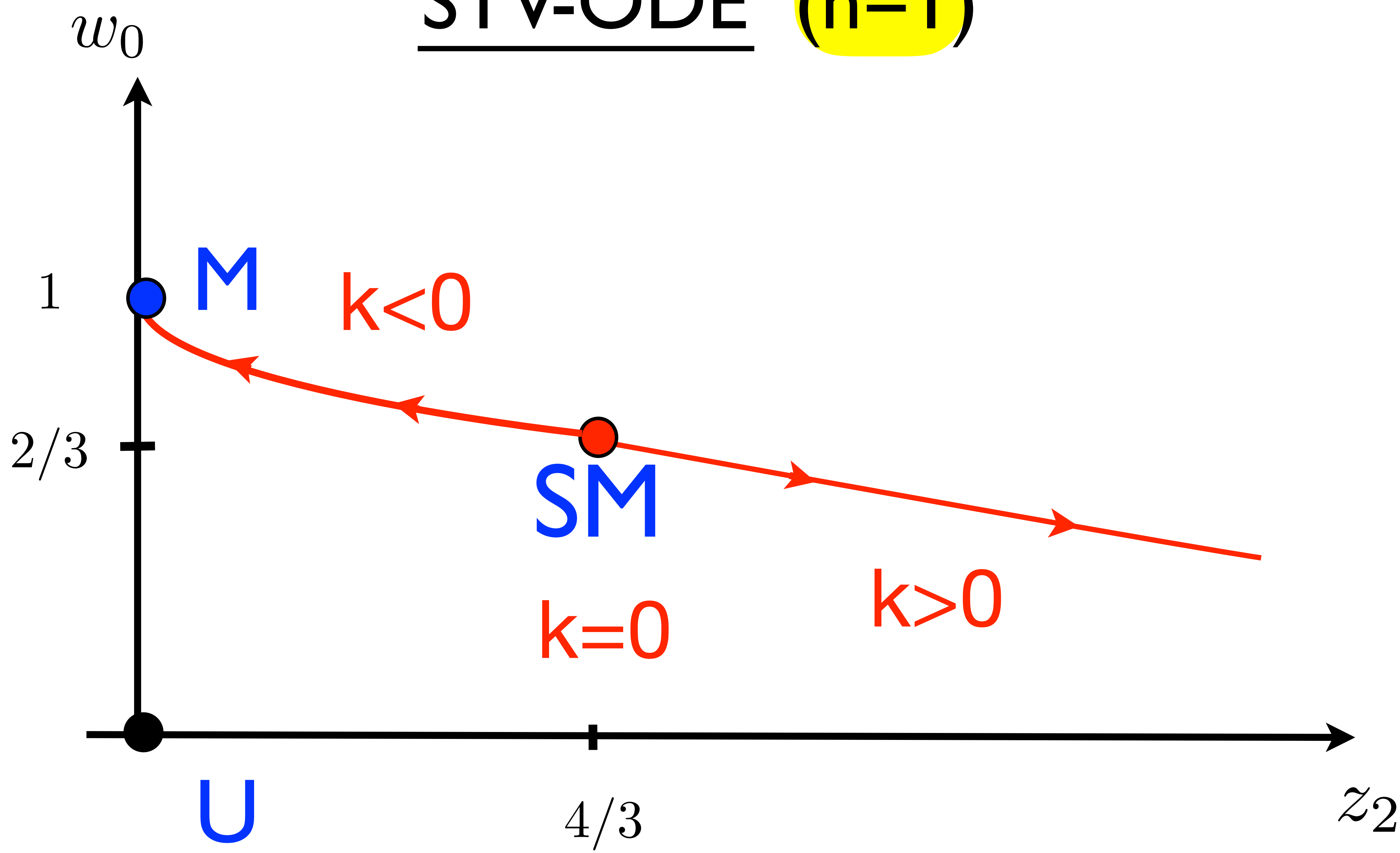
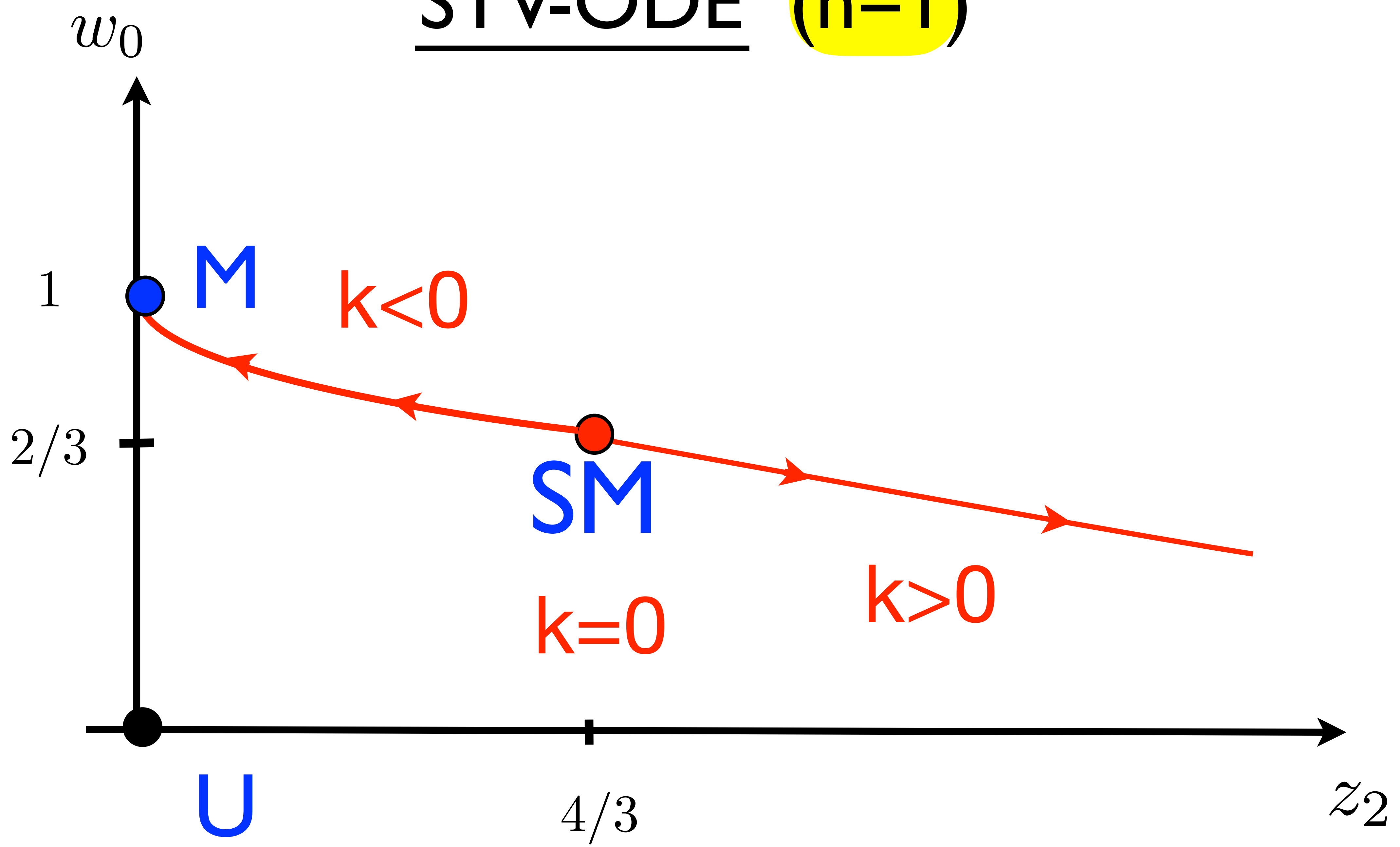


Imposing time since the Big Bang collapses the n=1 phase portrait to just three trajectories…

STV-ODE (n=1)



This eliminates a degree of freedom in STV-ODE

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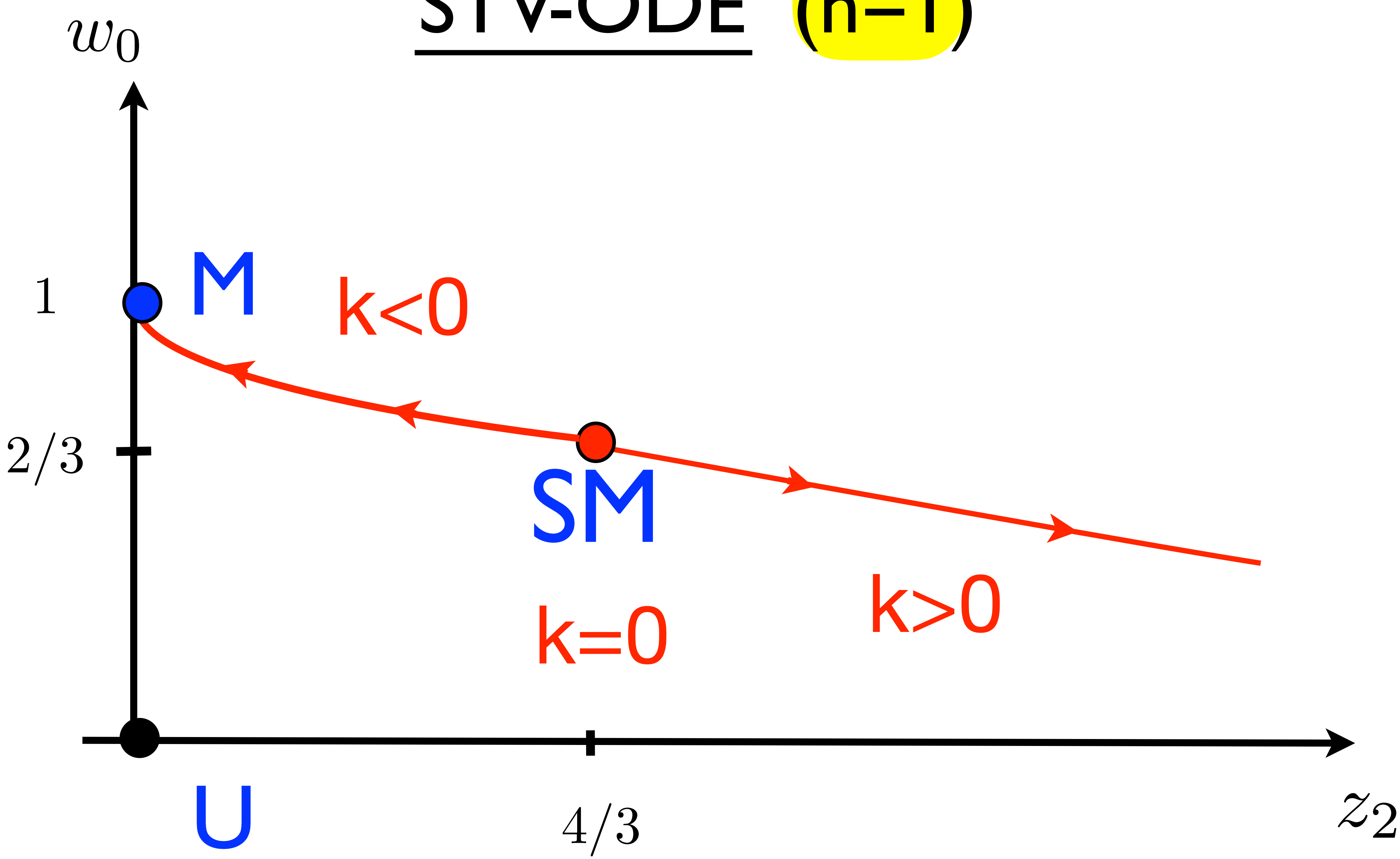

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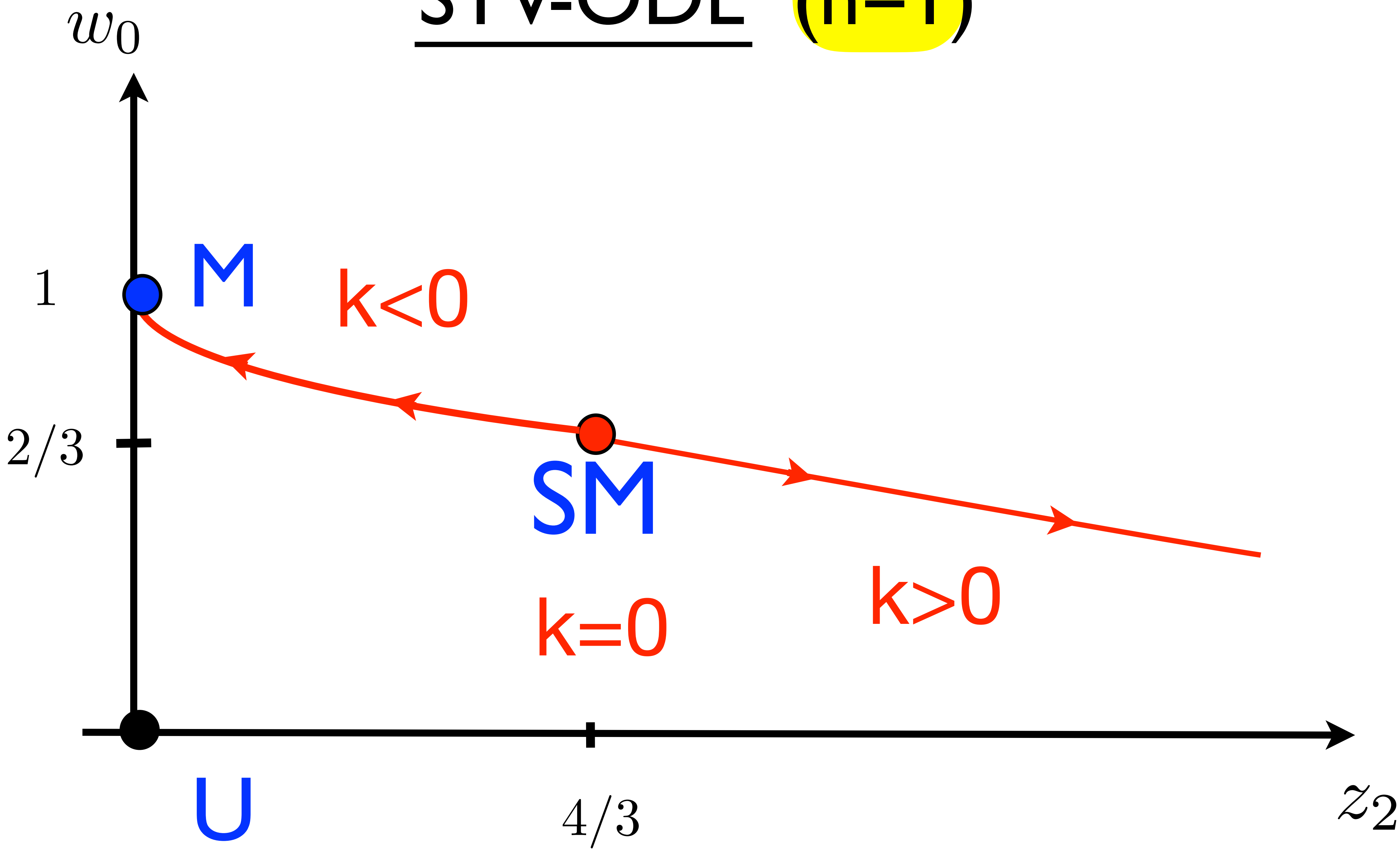

This eliminates a degree of freedom in STV-ODE

…eliminates the rest point U

…eliminates the negative eigenvalue at SM

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- Similarly for k=0 and k>0…

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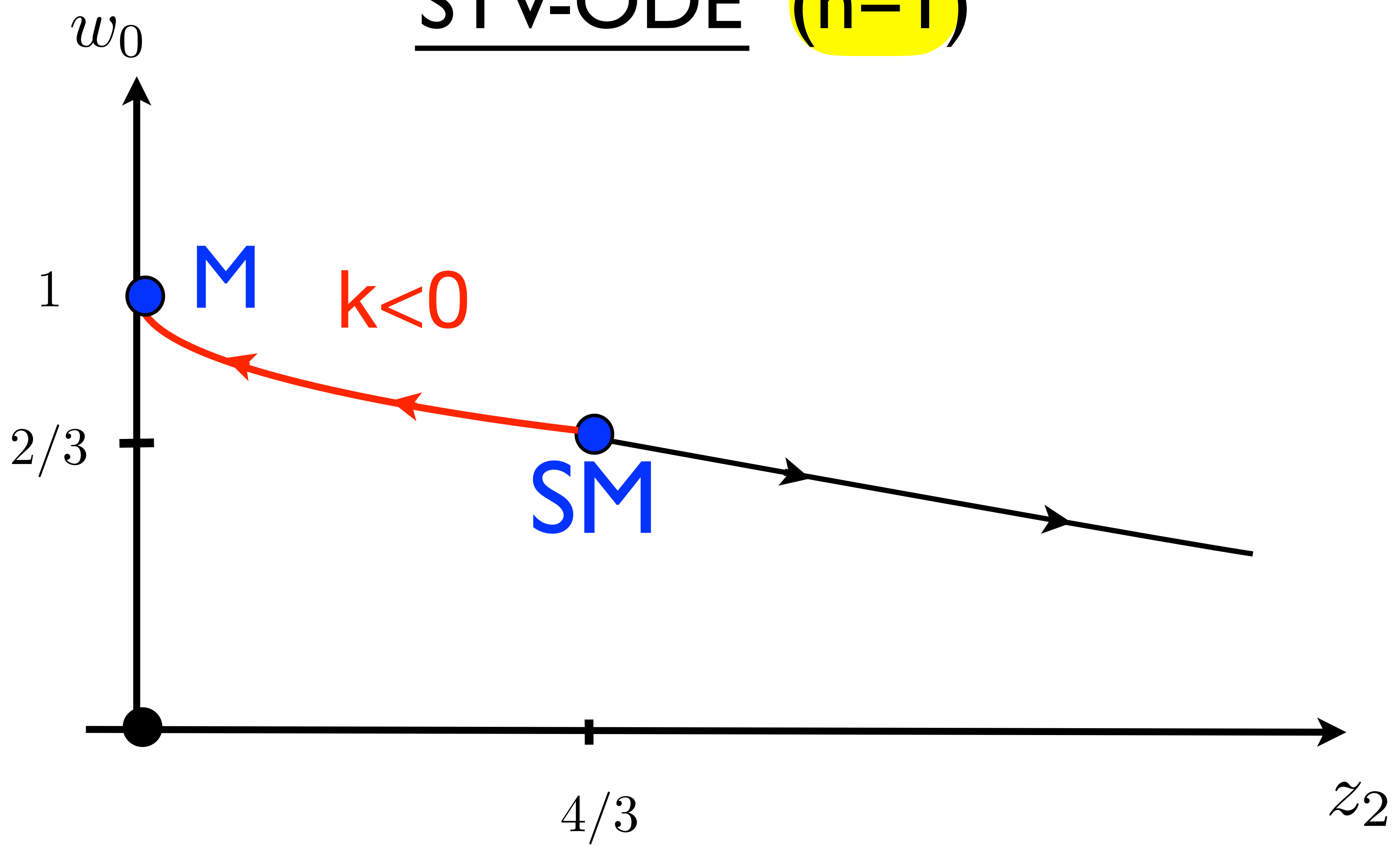

We say a solution is underdense iff k<0 at n=1…

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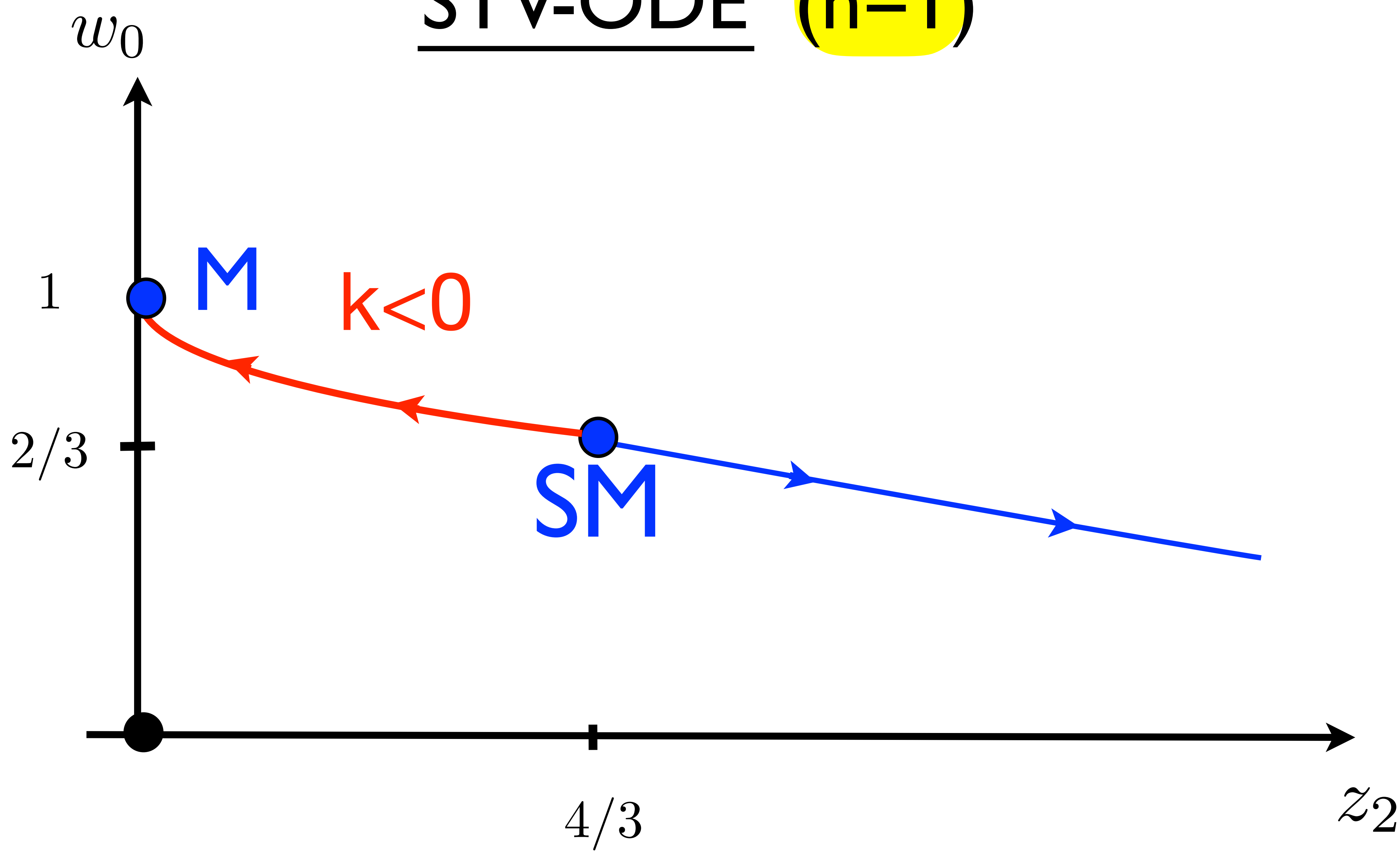

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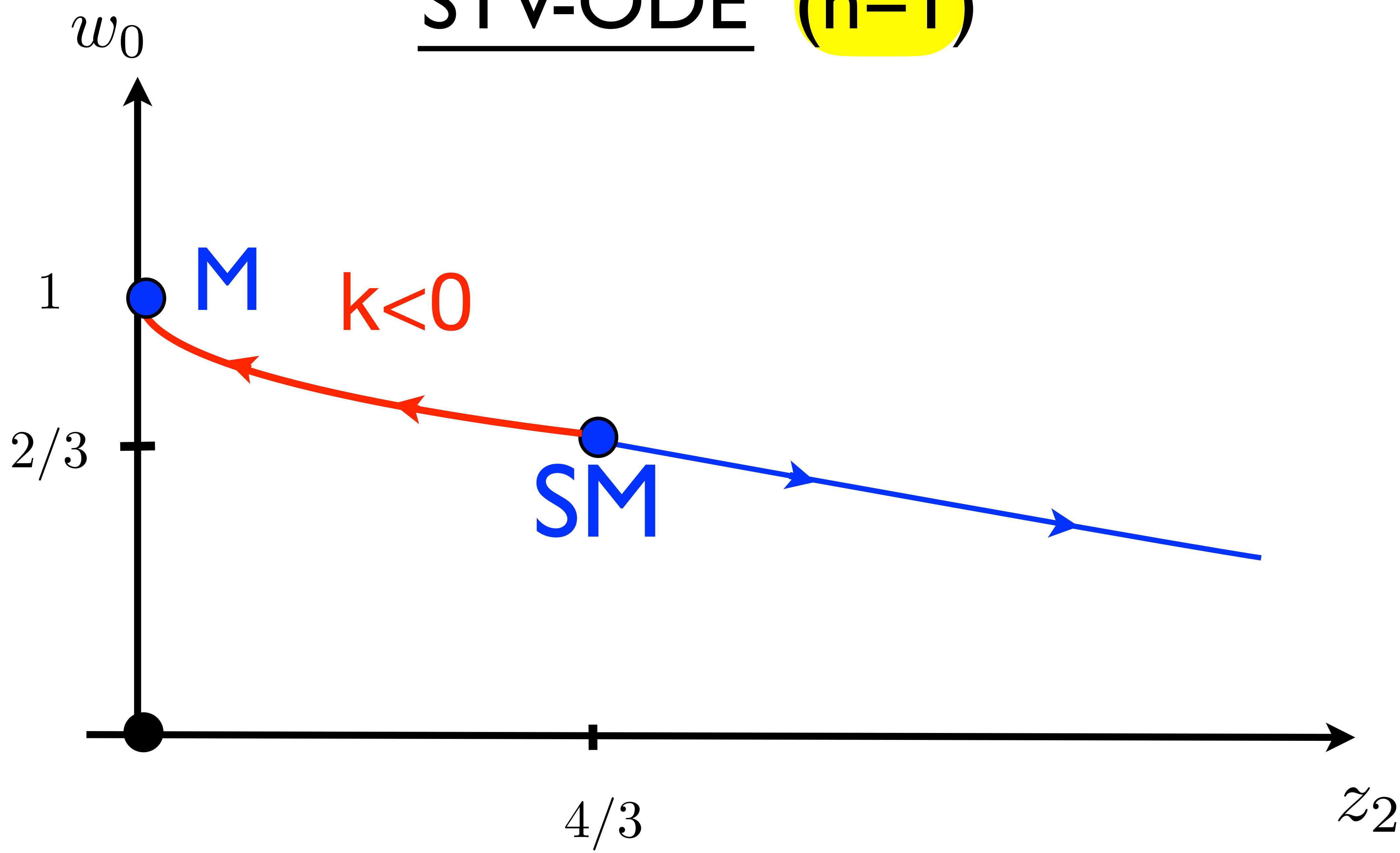

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but we are still free to impose i.c. at each n>1.

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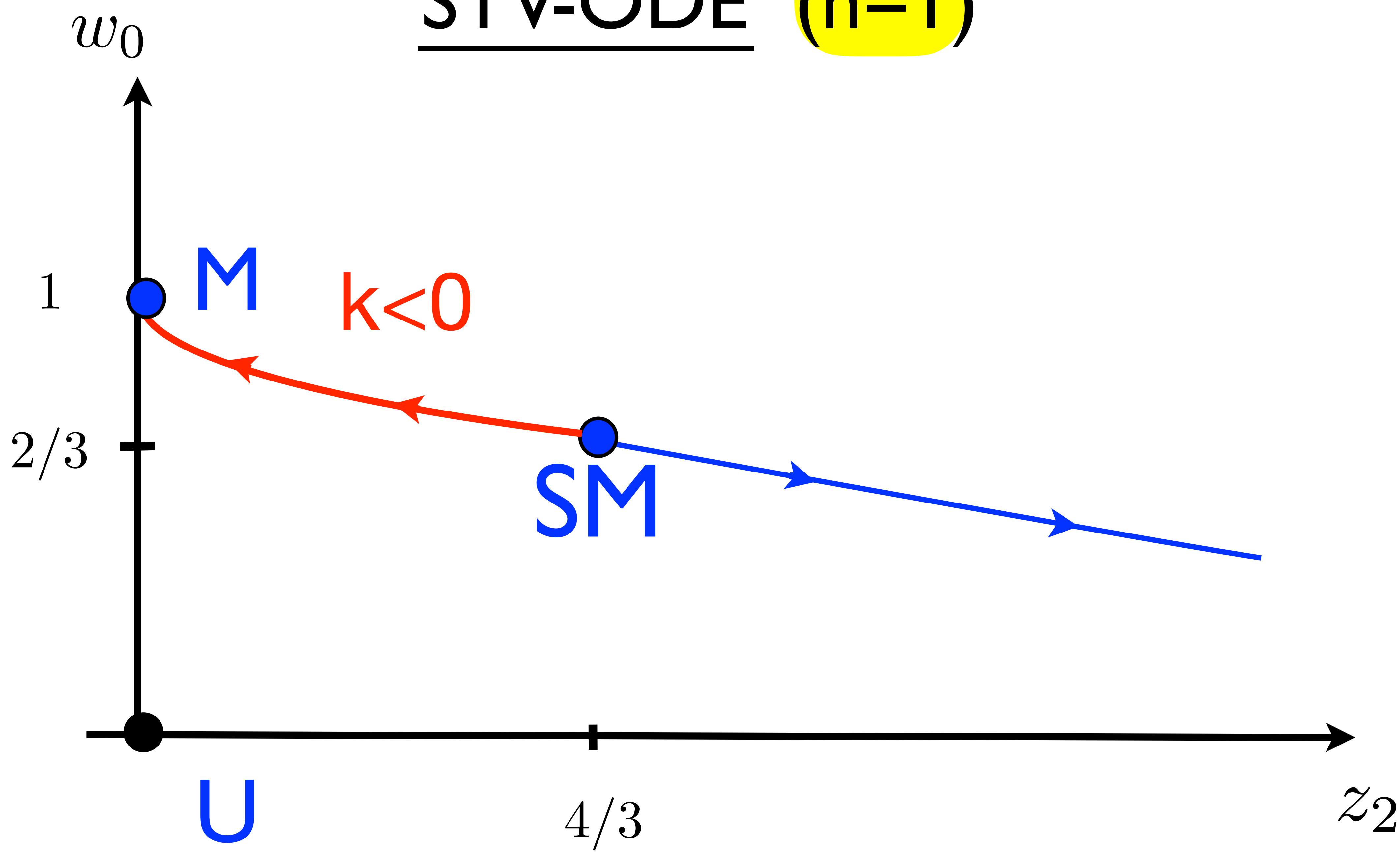

(Theorem: If a solution tends to M at order n=1, then it tends to M at every order n>1 as well.)

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…a 4x4 autonomous system with 3 rest points

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wlog assume time since bb

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STV-ODE (n=2)

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M: $\lambda_M = -1, \quad \boldsymbol{R}_M = (0,1,0,1)^T$

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Degenerate stable node…
…orbits enter along w-axis

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$$\lambda_{A1} = \frac{2}{3}, \qquad \boldsymbol{R}_{A1} = \left(-9, \frac{3}{2}, -\frac{10}{3}, -1\right)^T$$

$$\lambda_{B1} = -1, \qquad \boldsymbol{R}_{B1} = \left(4, 1, \frac{80}{9}, 1\right)^T$$

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$$\lambda_{B1} = -1, \qquad \boldsymbol{R}_{B1} = \left(4, 1, \frac{80}{9}, 1\right)^T$$

$$\lambda_{A2} = \frac{4}{3}, \qquad \boldsymbol{R}_{A2} = (0, 0, -10, 1)^T$$

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SM $= (\frac{4}{3}, \frac{2}{3}, \frac{40}{27}, \frac{2}{9})$, M $= (0, 1, 0, 0)$, U $= (0, 0, 0, 0)$

"time since big bang"

STV-ODE (n=2)

The Rest Points:

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SM is a regular saddle rest point…

…with 2-dimensional unstable manifold…

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Theorem: The Friedmann spacetimes are the trajectory of the leading order e-value $\lambda_{A1} = \frac{2}{3}$

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Theorem: The Friedmann spacetimes are the trajectory of the leading order e-value $\lambda_{A1} = \frac{2}{3}$

Proof: Expand exact formulas in powers of ξ.

STV-ODE (n=2)

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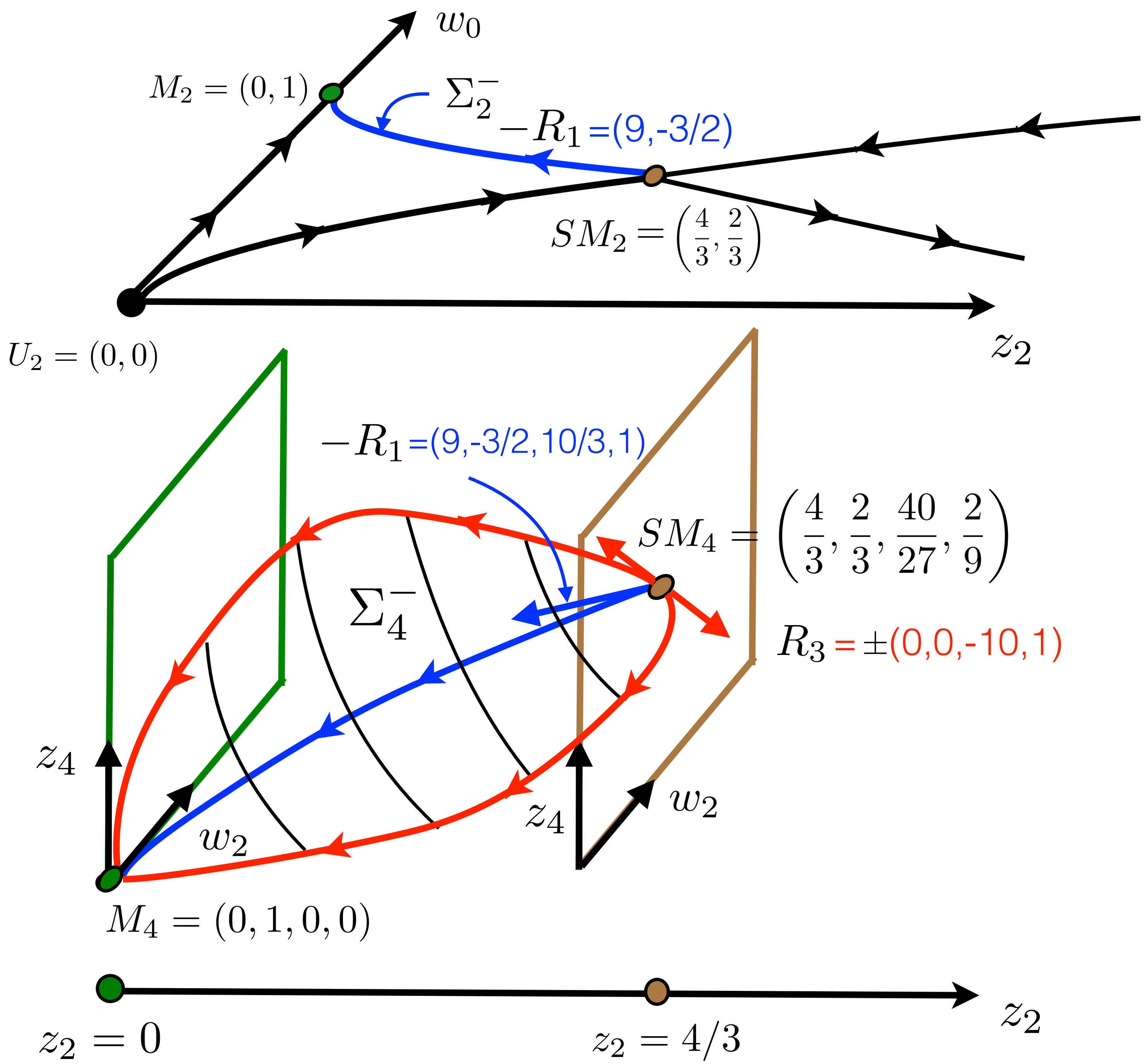
Conclude: The $k \neq 0$ Friedmann spacetimes correspond to one trajectory in the 2-dimensional unstable manifold of SM...

Phase Portrait: STV-ODE (n=2)



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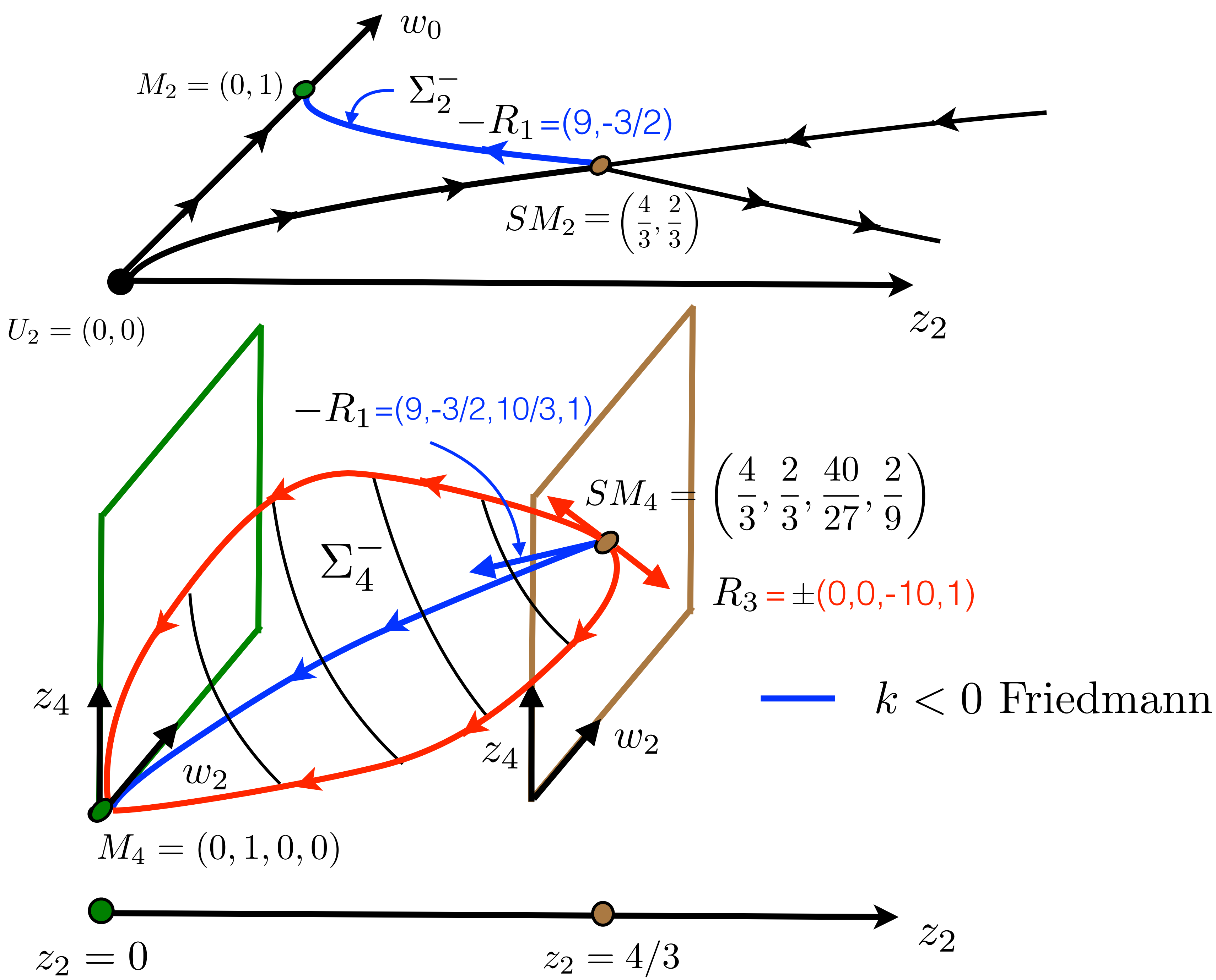

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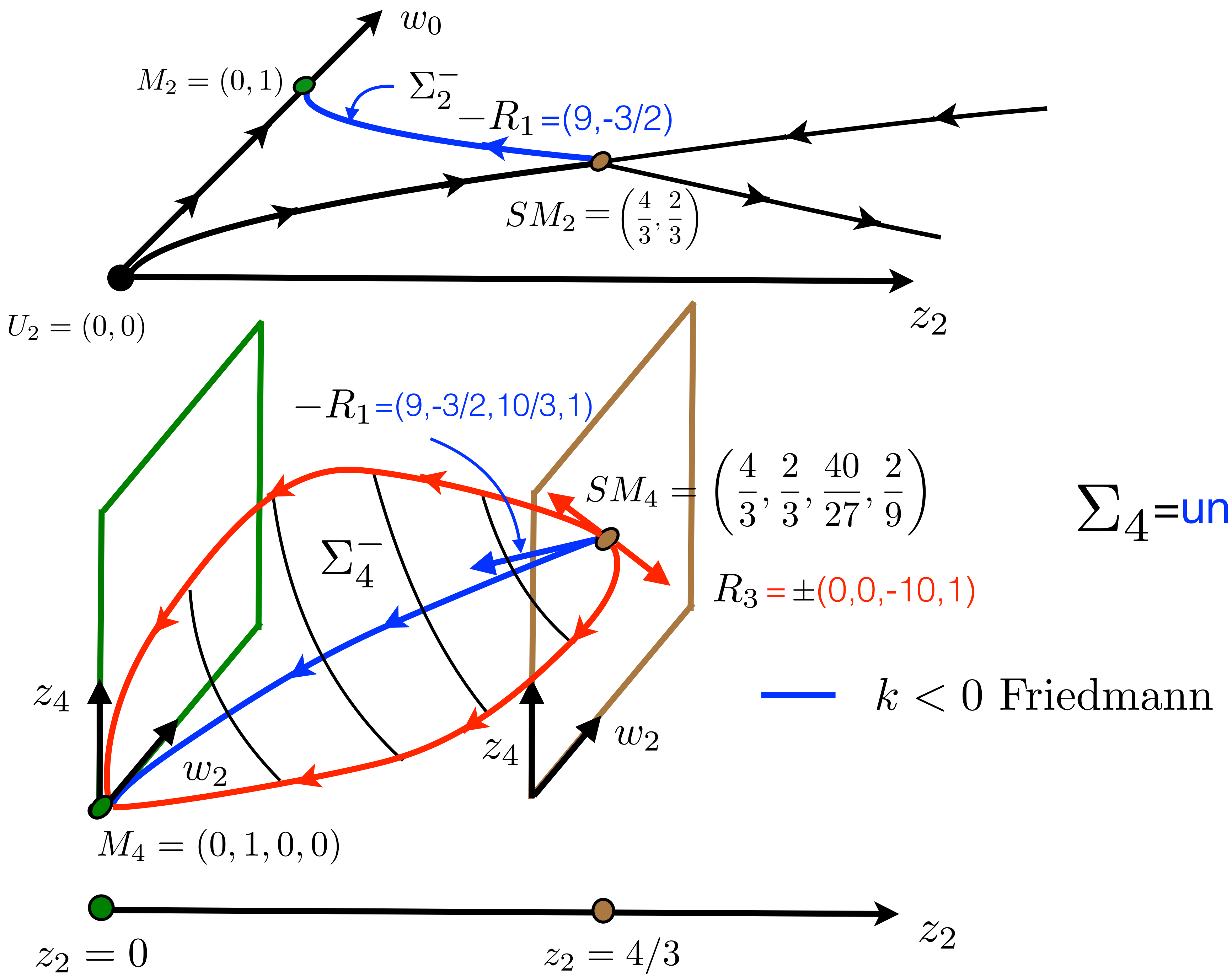
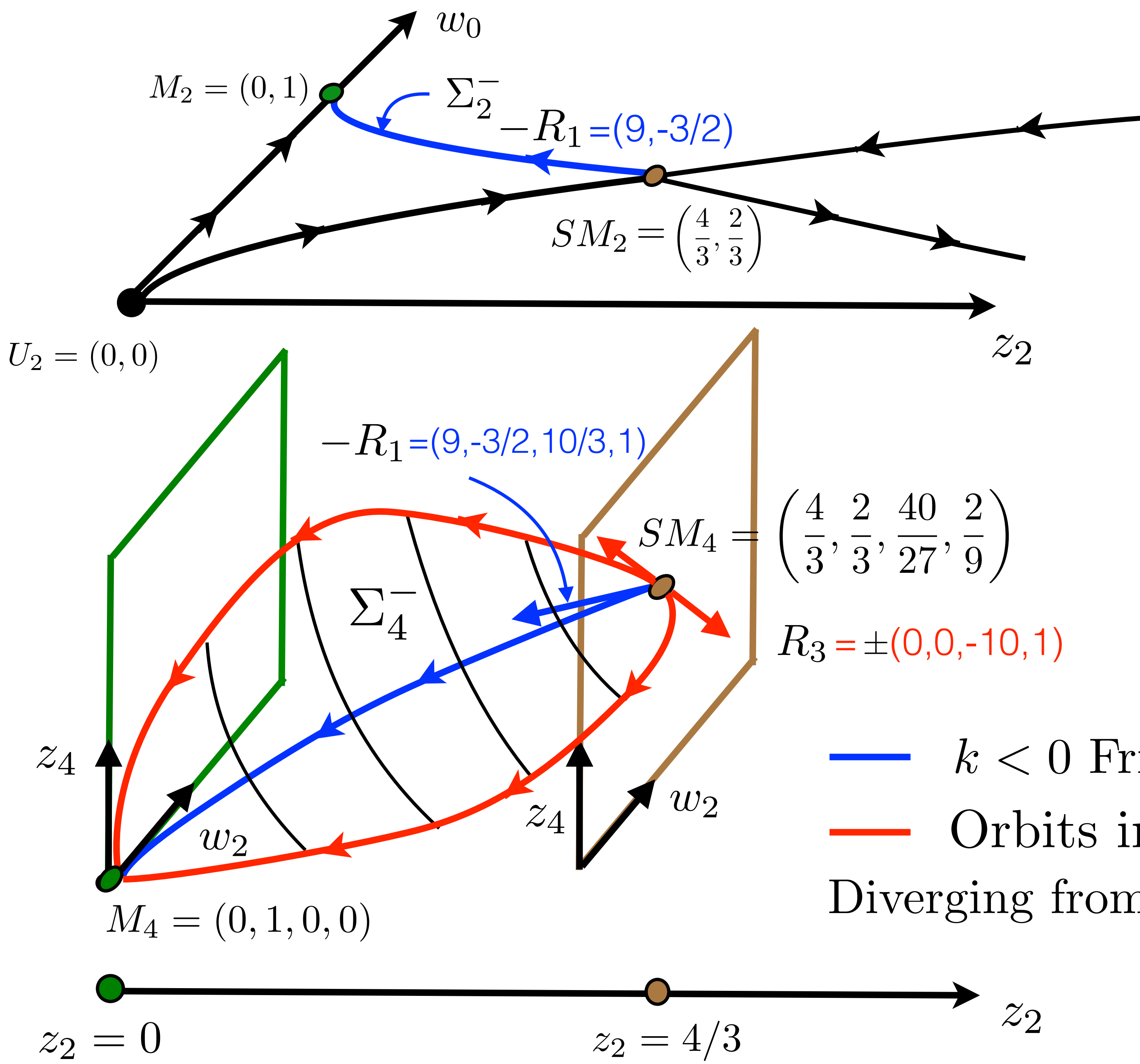


Σ_4=unstable manifold of SM

Phase Portrait: STV-ODE (n=2)



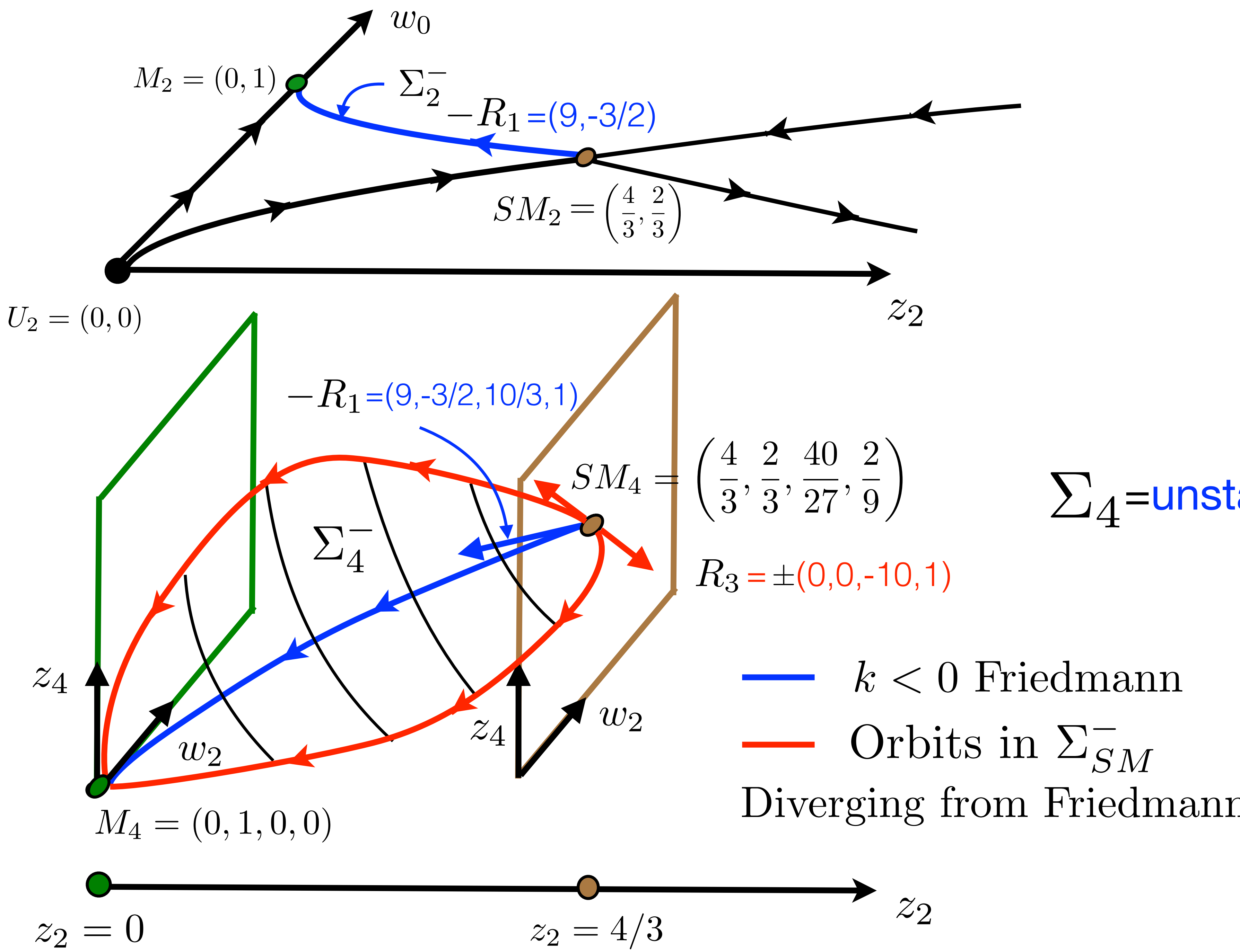
Σ_4=unstable manifold of SM

$k < 0$ Friedmann

Orbits in Σ_{SM}^-

Diverging from Friedmann

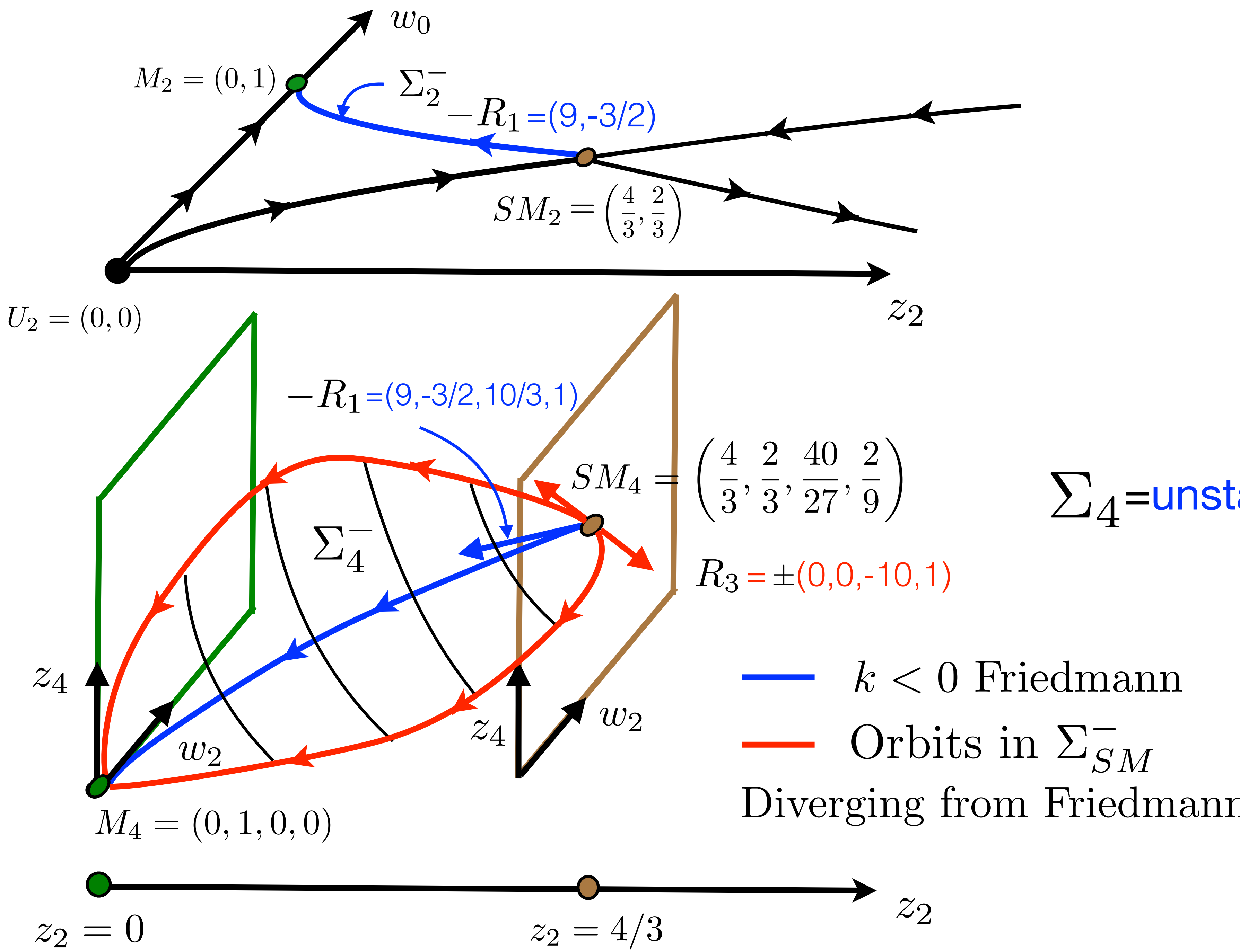
Phase Portrait: STV-ODE (n=2)



Σ_4=unstable manifold of SM

Generically: Solutions agree with Friedman at early times, accelerate away, and then decay back to Friedmann…

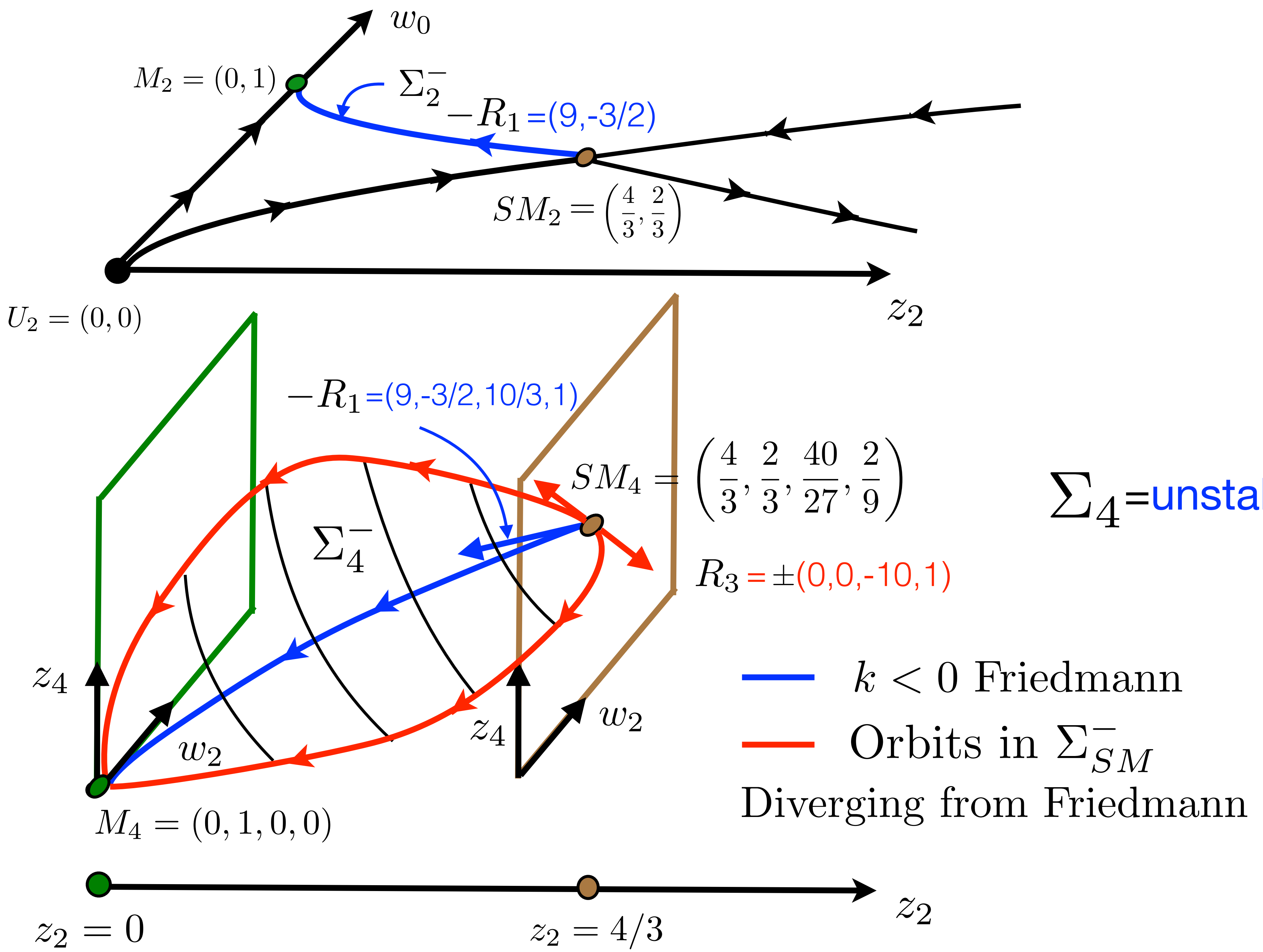
Phase Portrait: STV-ODE (n=2)



Σ_4=unstable manifold of SM

(Theorem: If a solution tends to M at order n=1, then it tends to M at every order n>1 as well.)

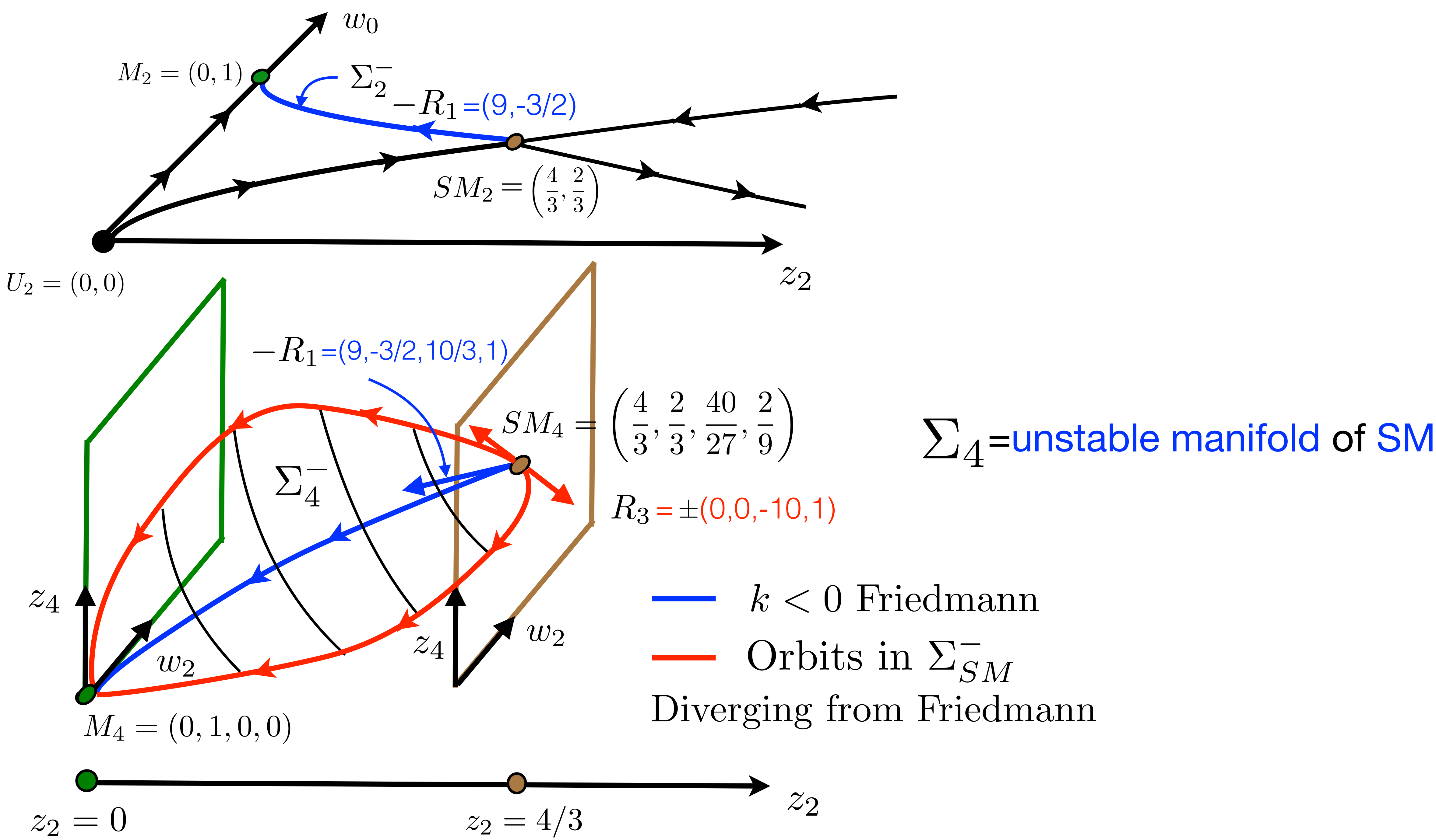
Phase Portrait: STV-ODE (n=2)



Σ_4=unstable manifold of SM

The accelerations fundamentally arise from an instability at Big Bang, not from fluctuations on some length scale…

Phase Portrait: STV-ODE (n=2)



Under-dense solutions all decay back to k<0 Friedmann faster than they decay to Minkowski at M as $t \to \infty$.

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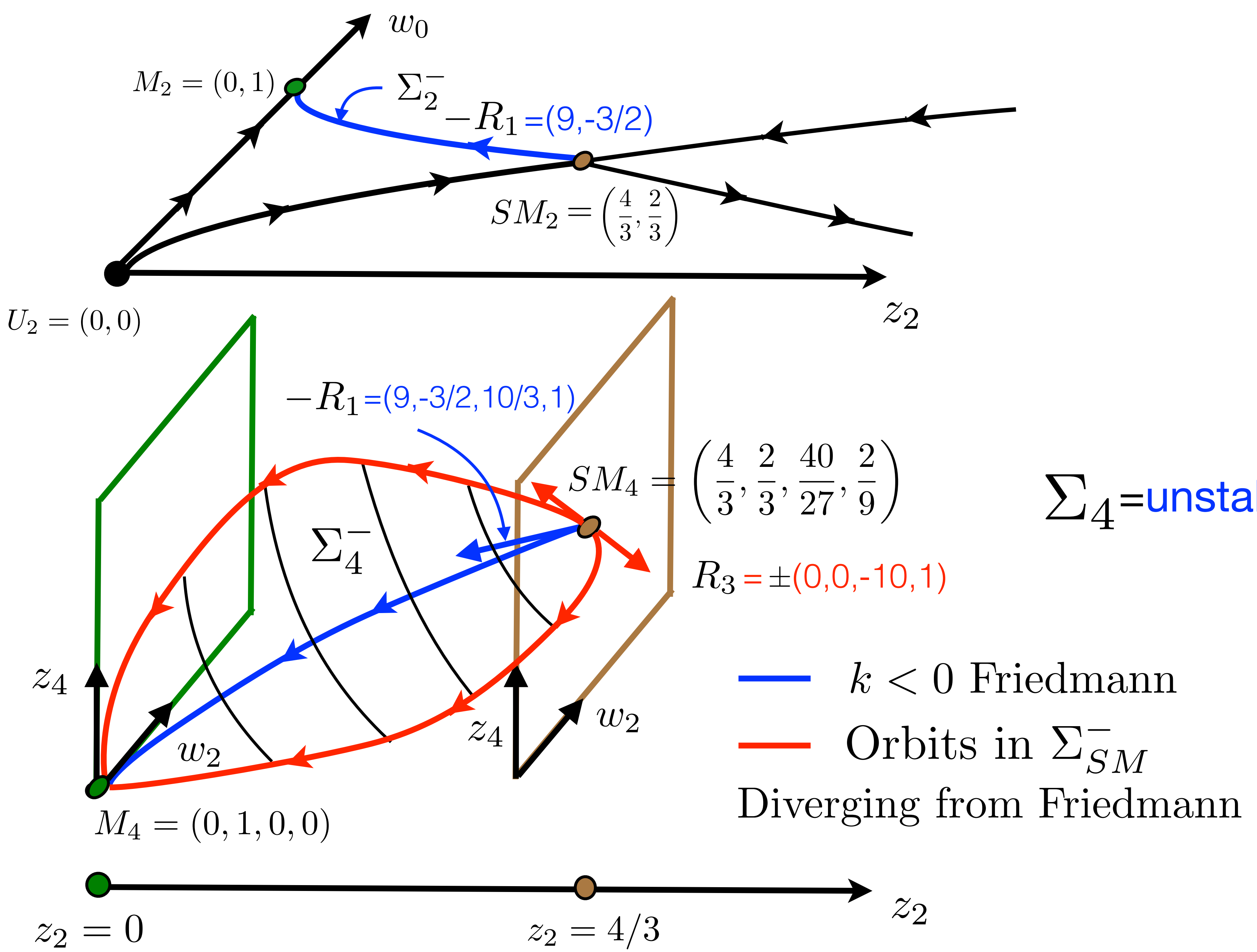
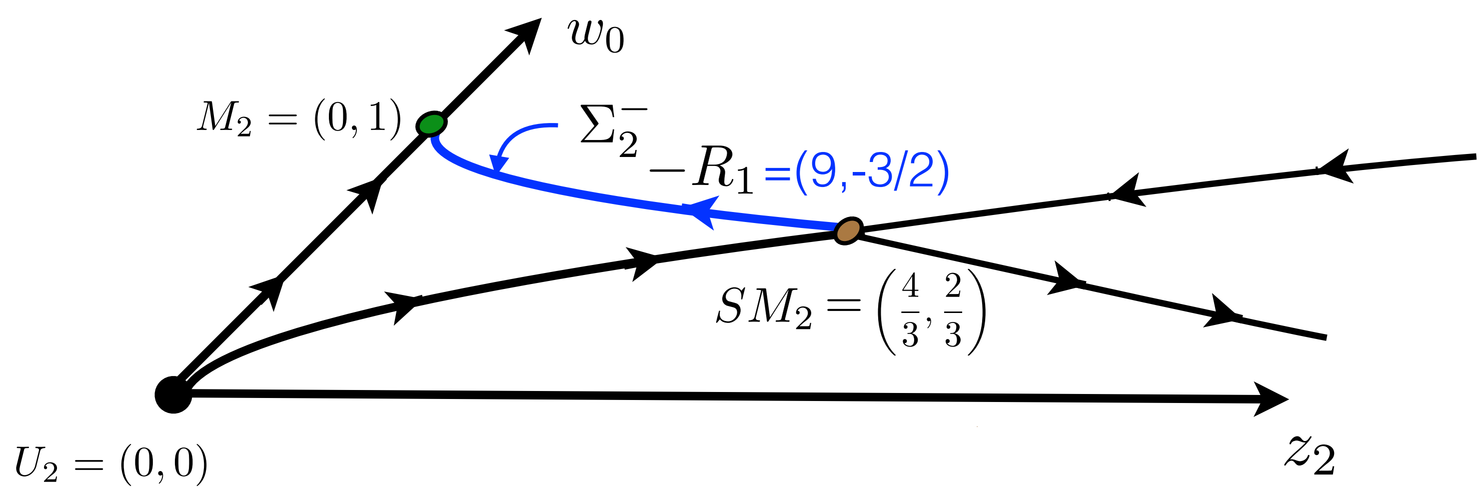
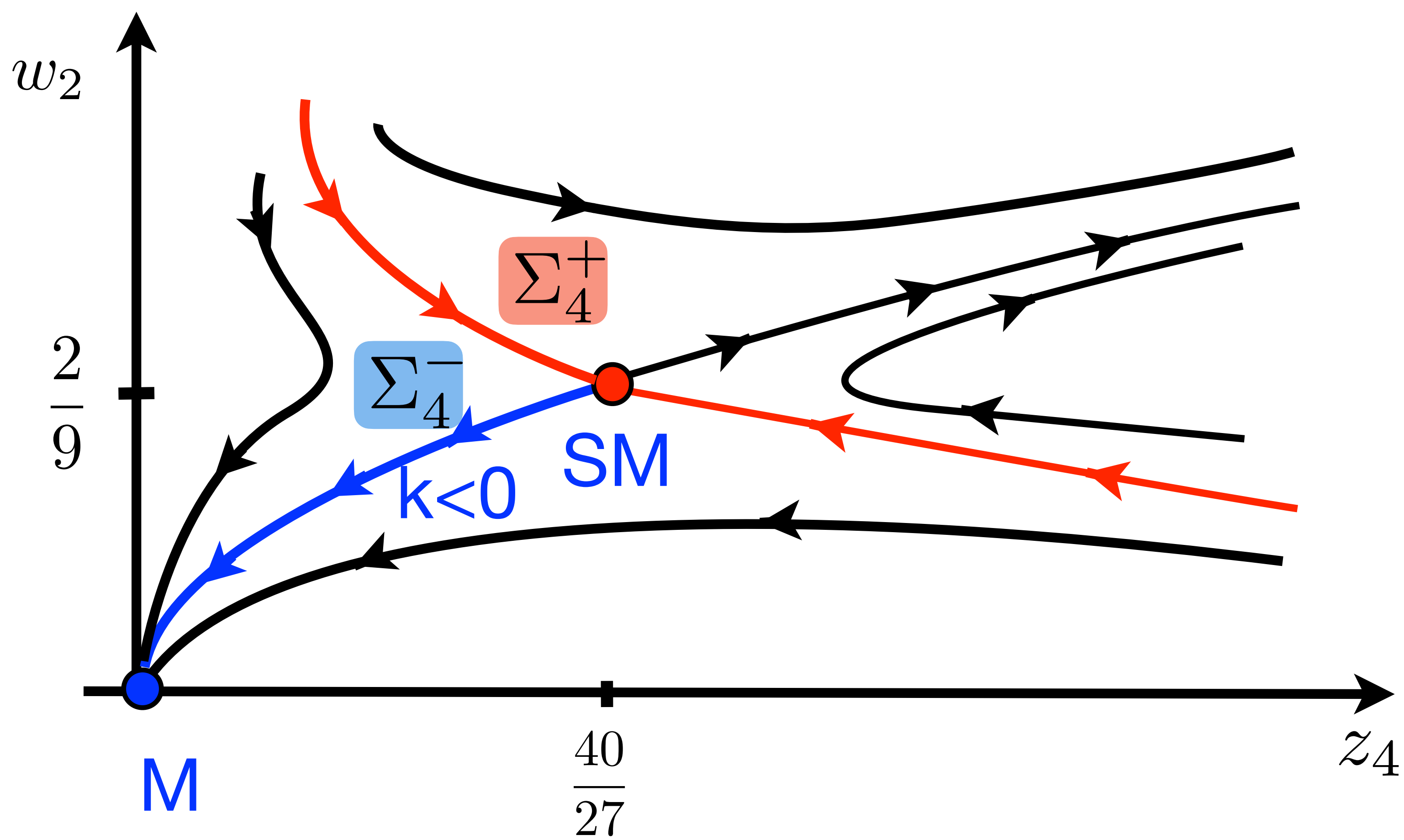


Σ_4=unstable manifold of SM

The negative eigenvalue at SM implies Big Bang generically self-similar like SM only at leading order n=1…

Phase Portrait: STV-ODE (n=2)





Generically…

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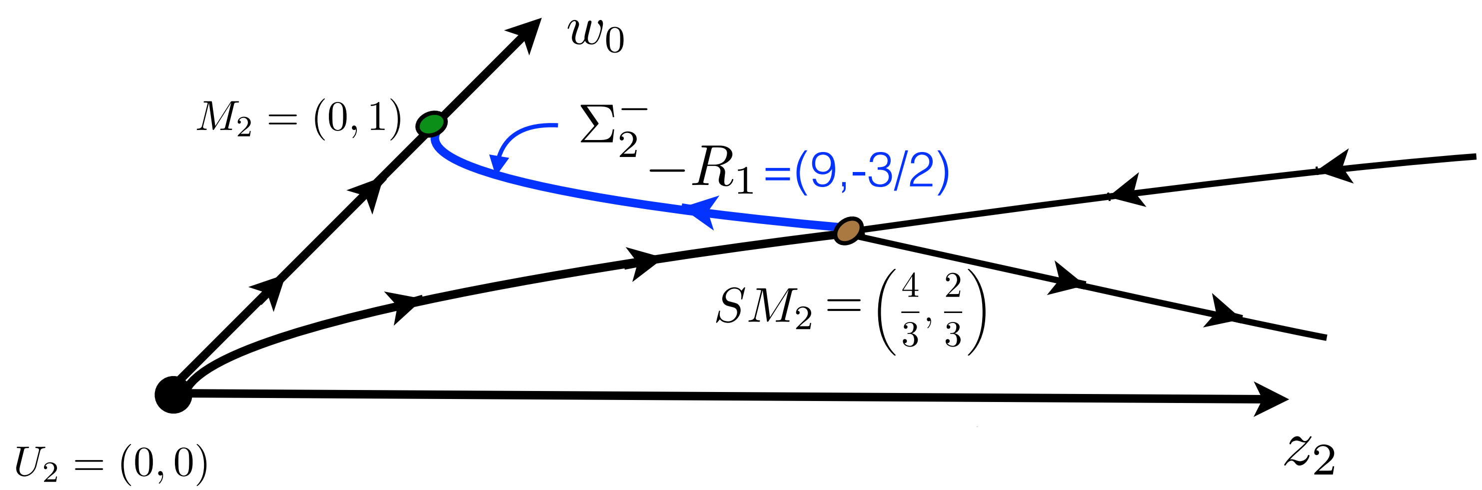
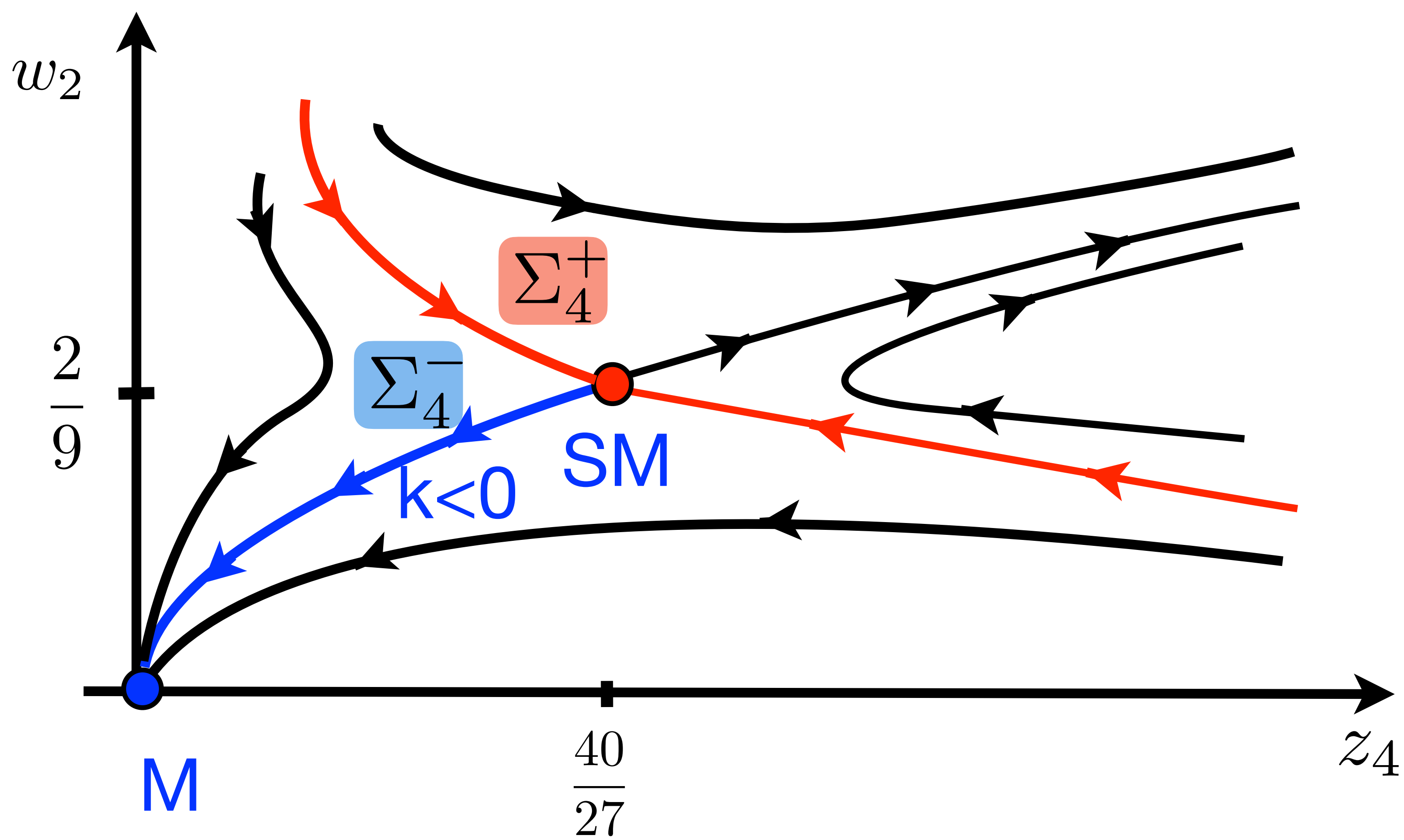
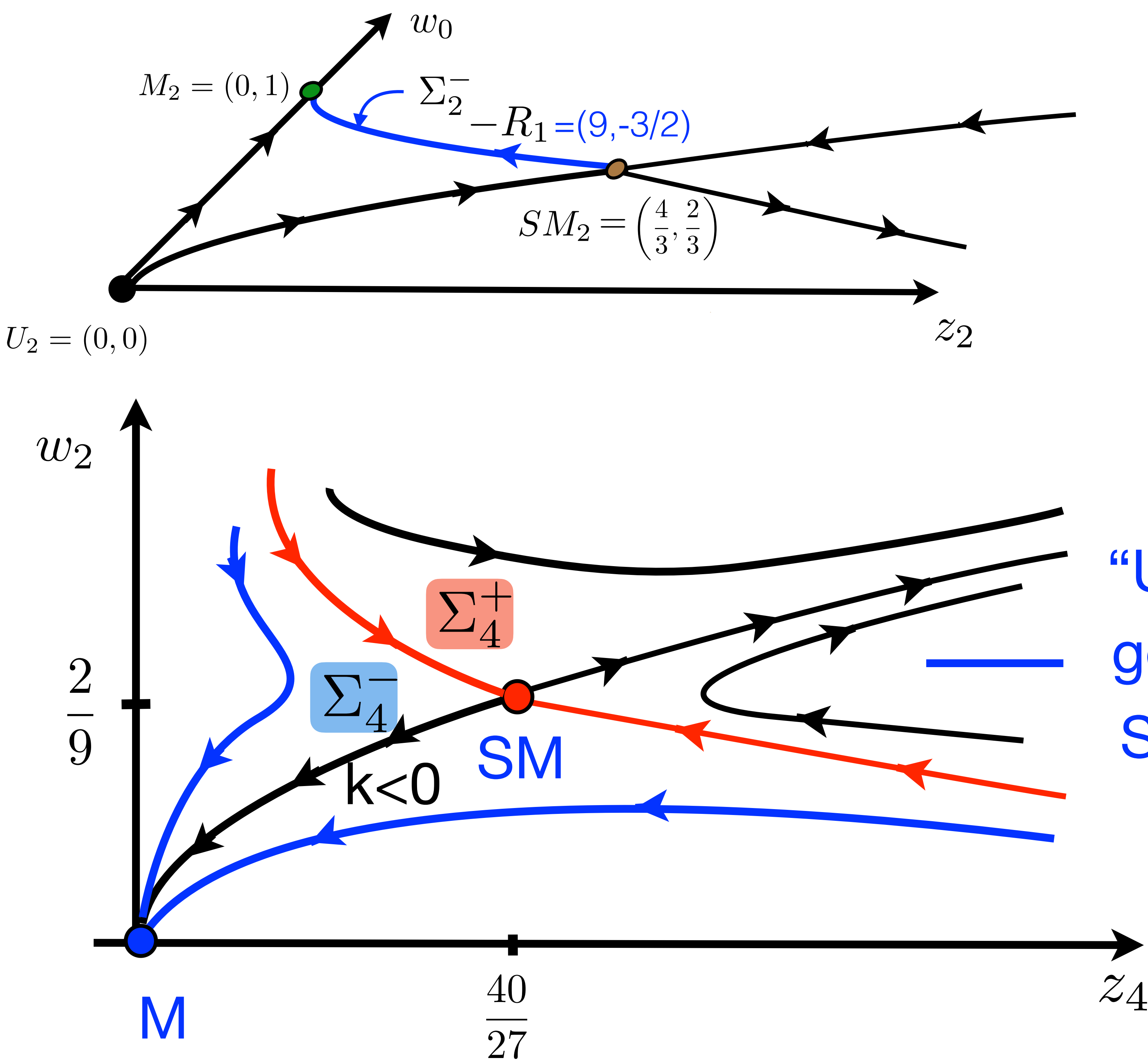




Generically…perturbations of k<0 Friedmann do not hit SM in backward time, but go off to infinity instead…

Phase Portrait: STV-ODE (n=2)



"Unlike k<0 Friedmann generic solns do not hit SM in backward time…"

…so Generically…

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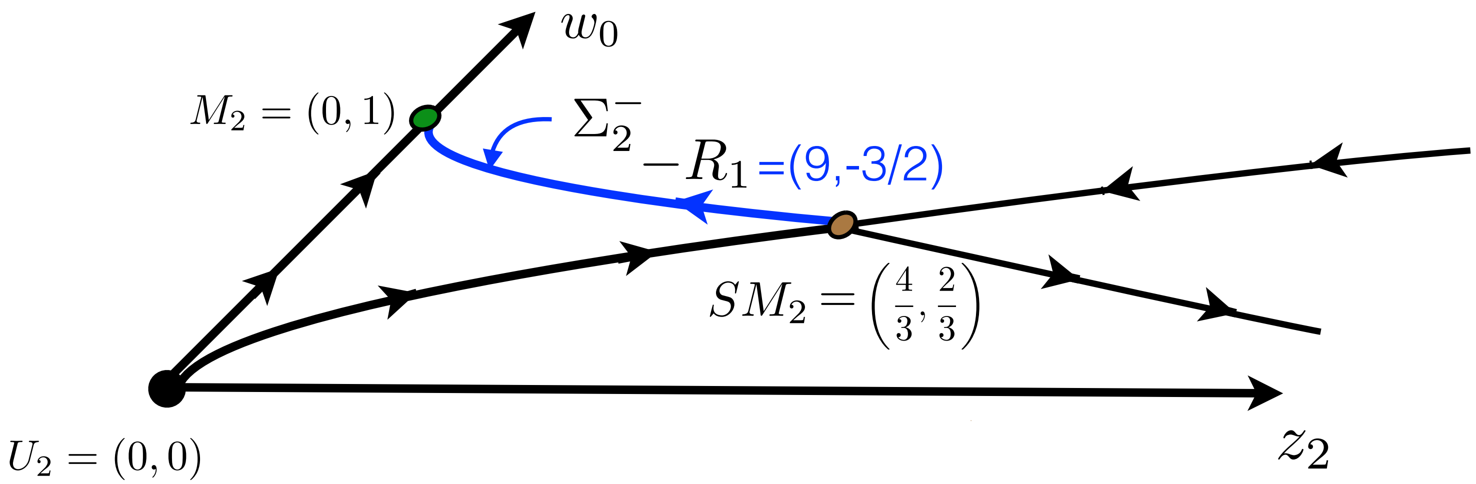

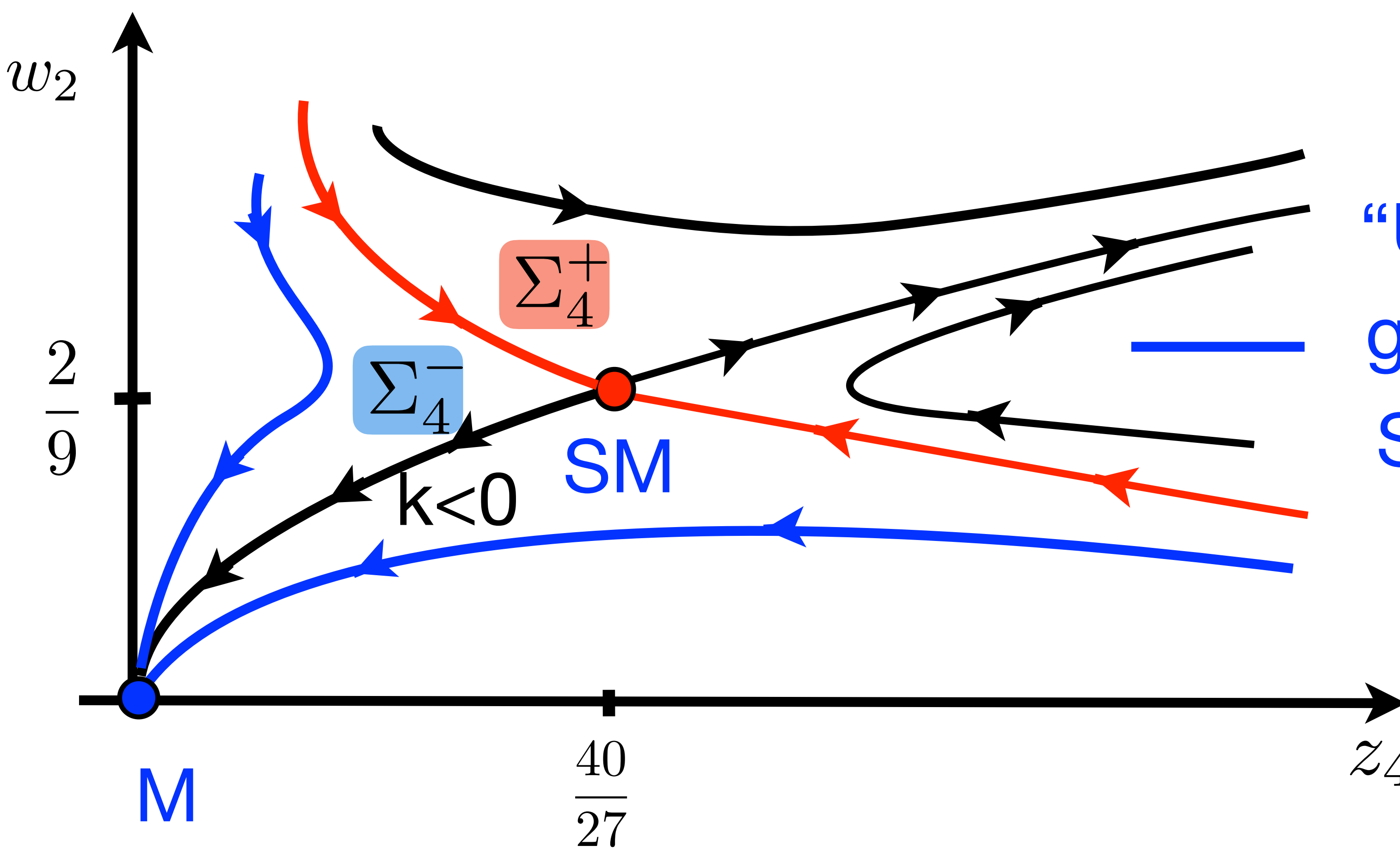

"Unlike k<0 Friedmann generic solns do not hit SM in backward time…"

…so Generically… the Big Bang is self-similar like Friedmann spacetimes only to leading order $n = 1$!

STV-ODE (n=3)

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(Complicated…)

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$$t\dot{z}_6 = 6z_6 - 7(z_6 w_0 + z_2 w_4 + z_2 w_0 D_4 + z_2 w_2 D_2 + z_4 w_0 D_2 + z_4 w_2)$$

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$$-w_0 w_2 A_2 + \frac{1}{2} A_2 D_4 + \frac{1}{2}(A_4 - A_2^2) D_2 - A_2 A_4 + \frac{1}{2} A_2^3$$

$$-w_0^2 D_4 - 4w_0 w_2 D_2 - 3w_2^2$$

$$A_2 = -\frac{1}{3} z_2, \qquad A_4 = -\frac{1}{5} z_4$$

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But phase portrait is determined qualitatively at n=2…

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$$\frac{d}{d\tau}\boldsymbol{v}_n = \begin{pmatrix} (2n+1)(1-w_0)-1 & -(2n+1)z_2 \\ -\frac{1}{2(2n+1)} & 2n(1-w_0)-1 \end{pmatrix} \boldsymbol{v}_n + \boldsymbol{q}_n$$

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…nested ODE's…linear at leading order.

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…minimum positive eigenvalue $\Lambda_{B3} = \frac{1}{3}$ at n=3.

Results

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Definition: Let $\mathcal{F}$ denote the family of smooth solutions of the STV-PDE which agree with a $k<0$ Friedmann spacetime at leading order $n=1$…

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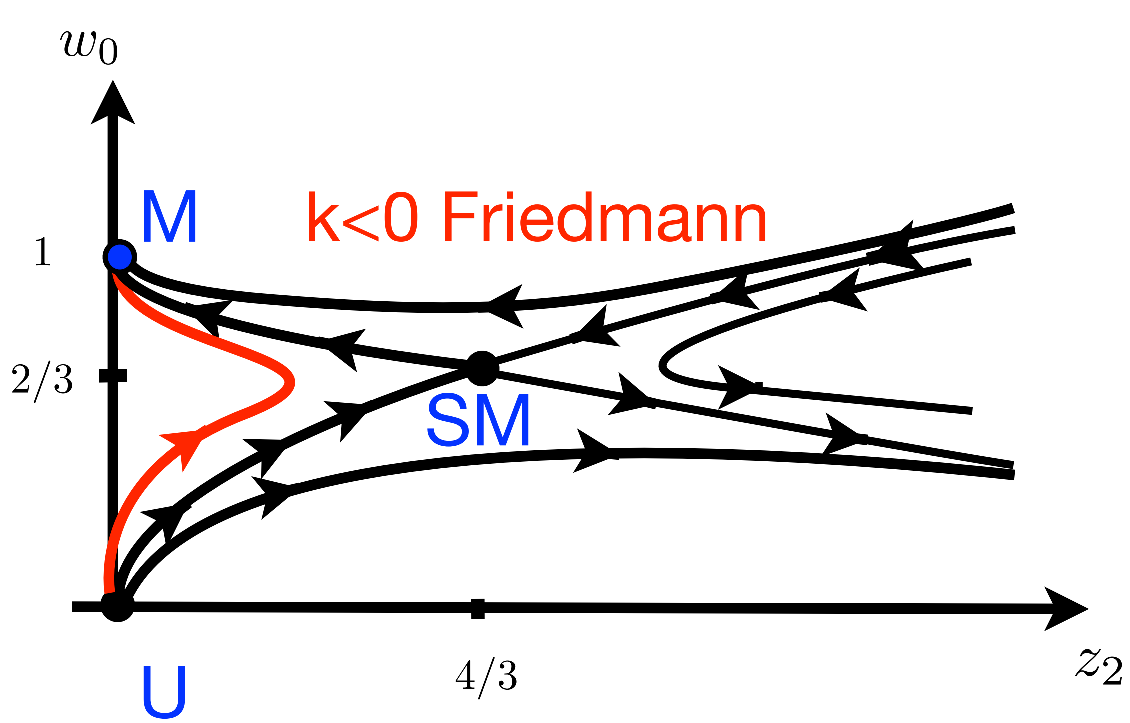

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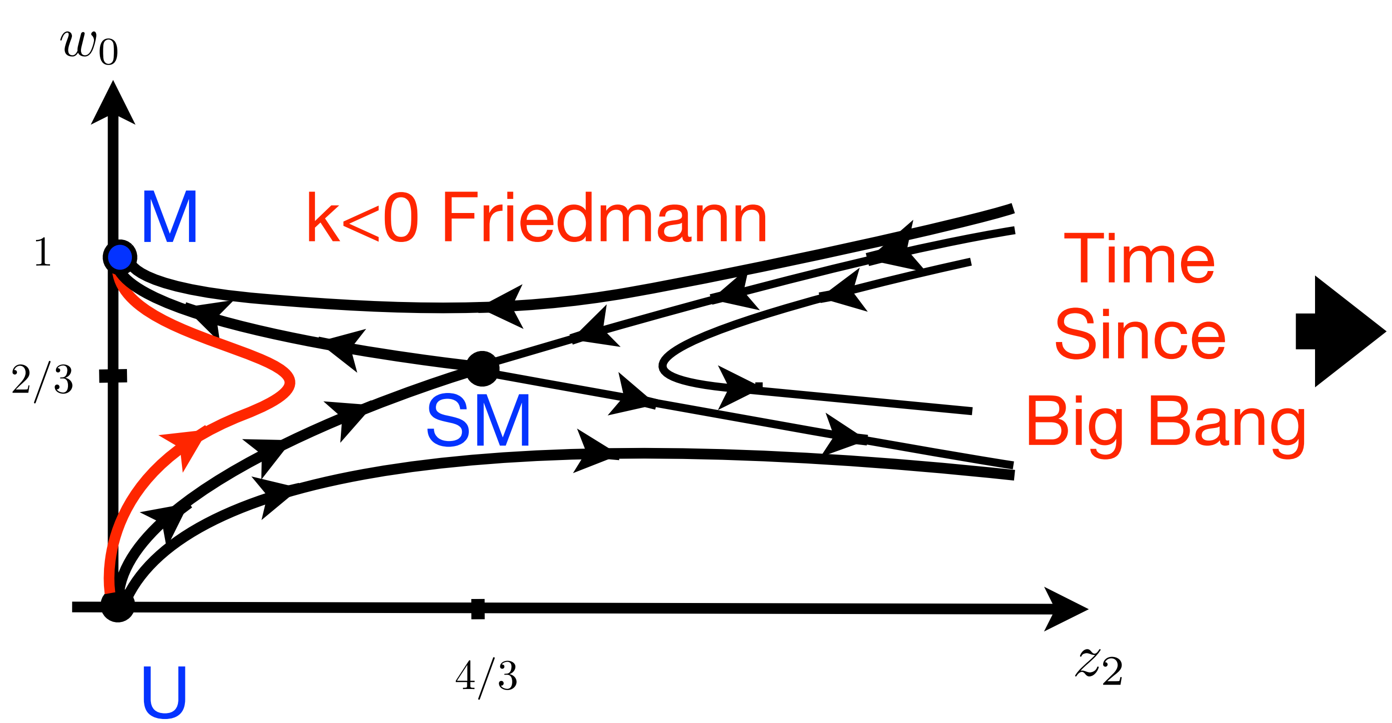

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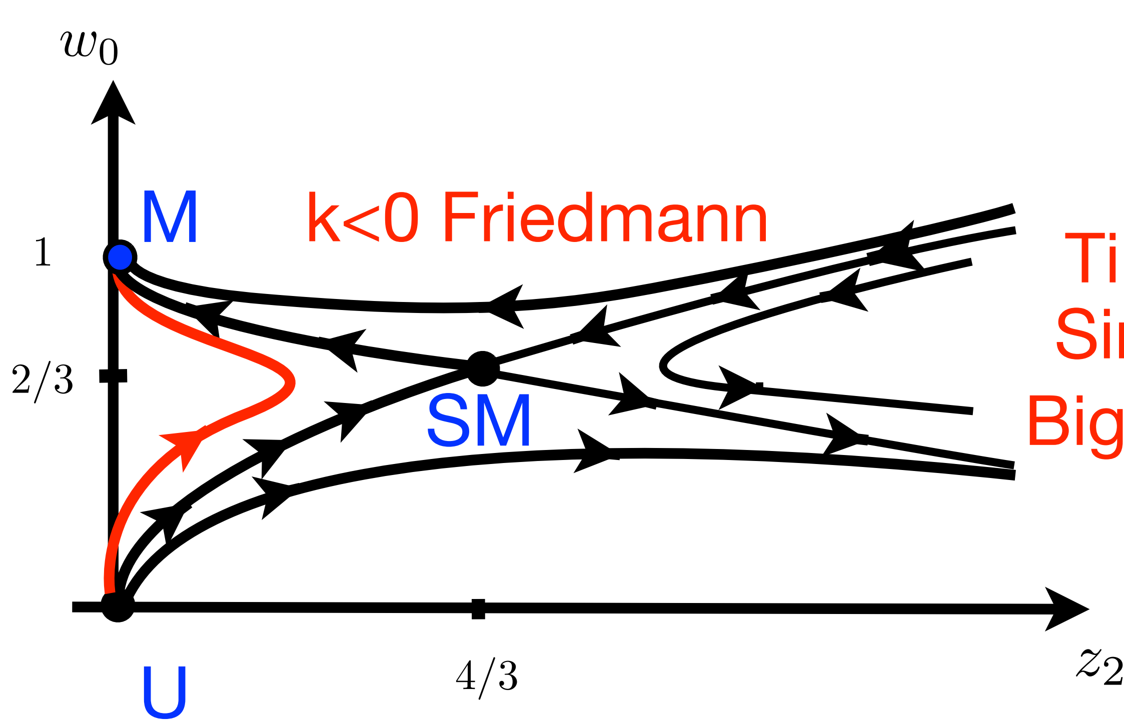

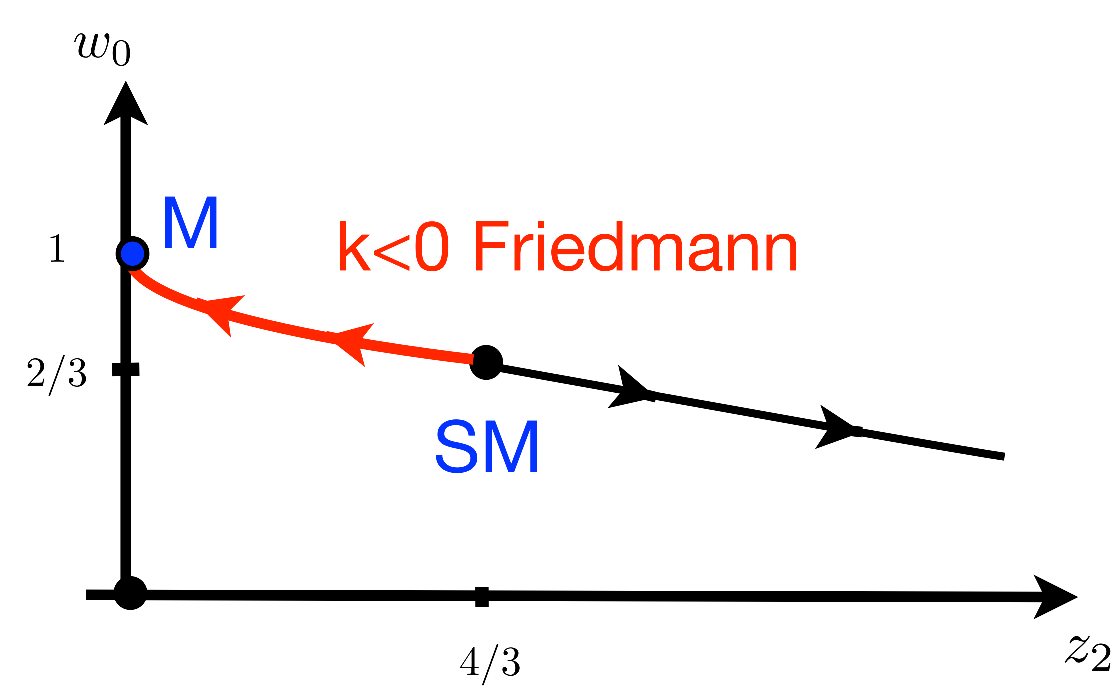

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Solutions in $\mathcal{F}$ which agree at n=1, differ by initial conditions for STV-ODE at orders $n \geq 2$.

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- All eigenvalues of SM are positive for $n \geq 3$!

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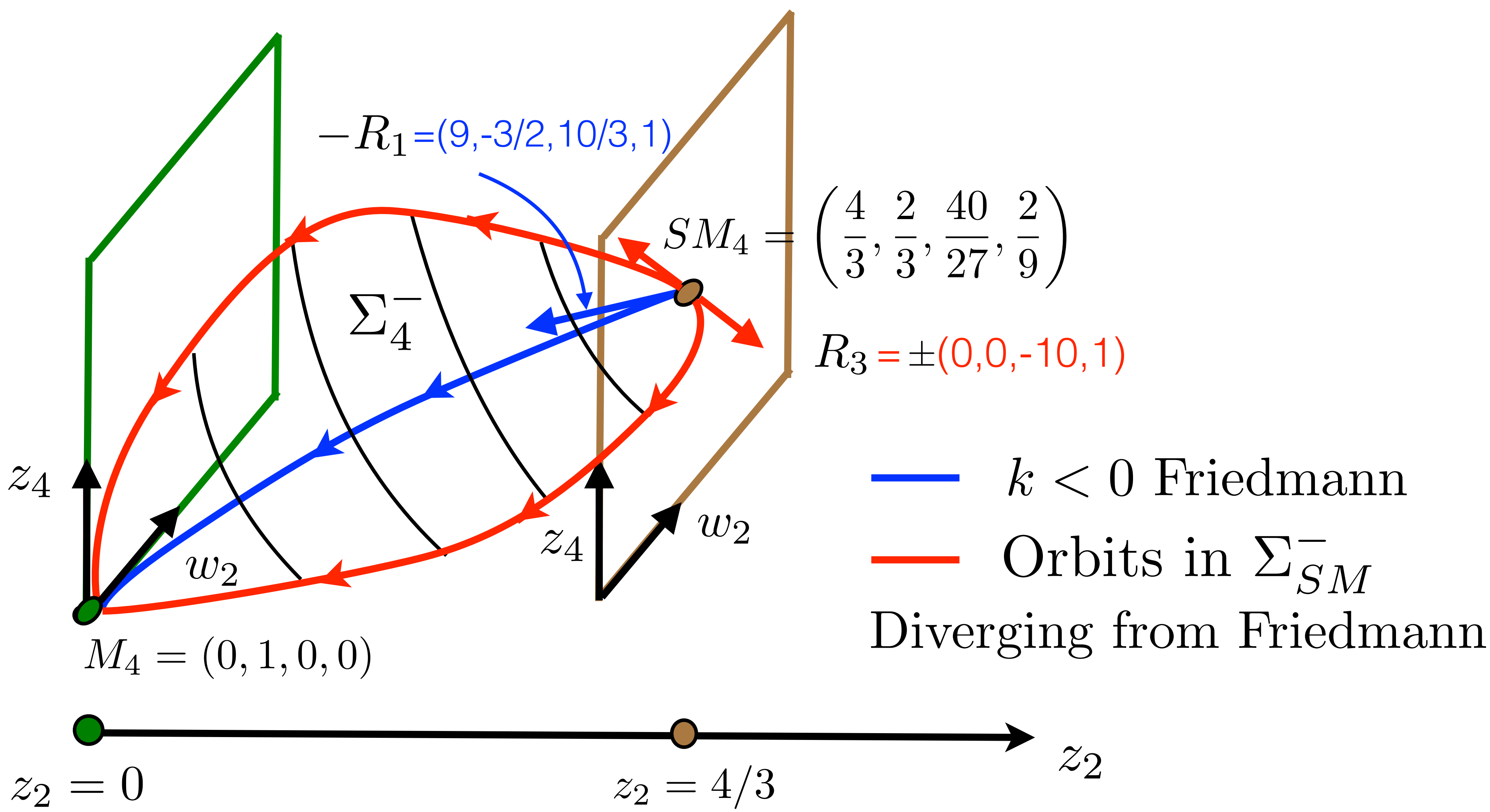

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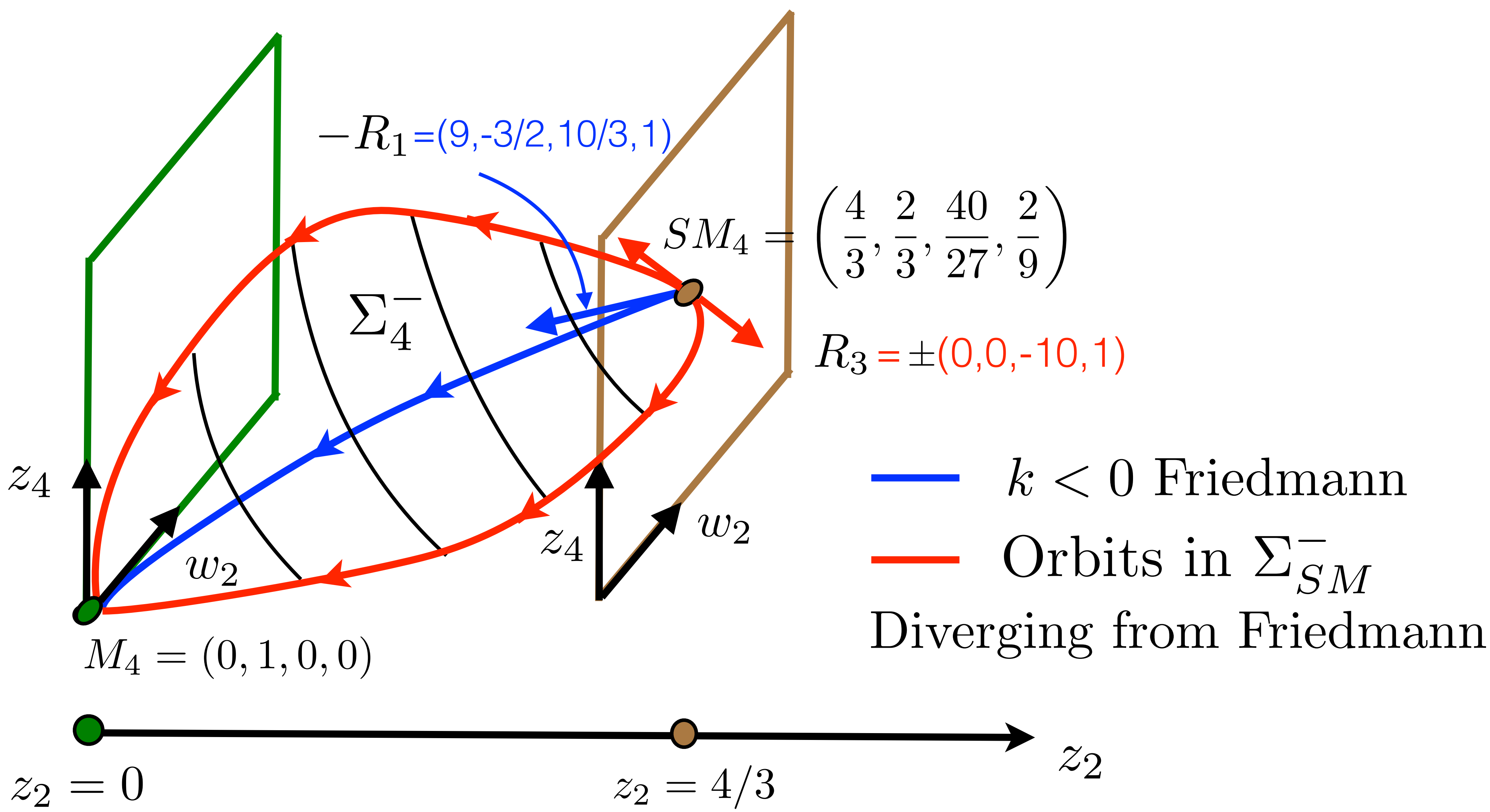
- At $n \geq 2$, k<0 Friedmann is the eigen-trajectory of the leading order eigenvalue of SM in Σ^-.



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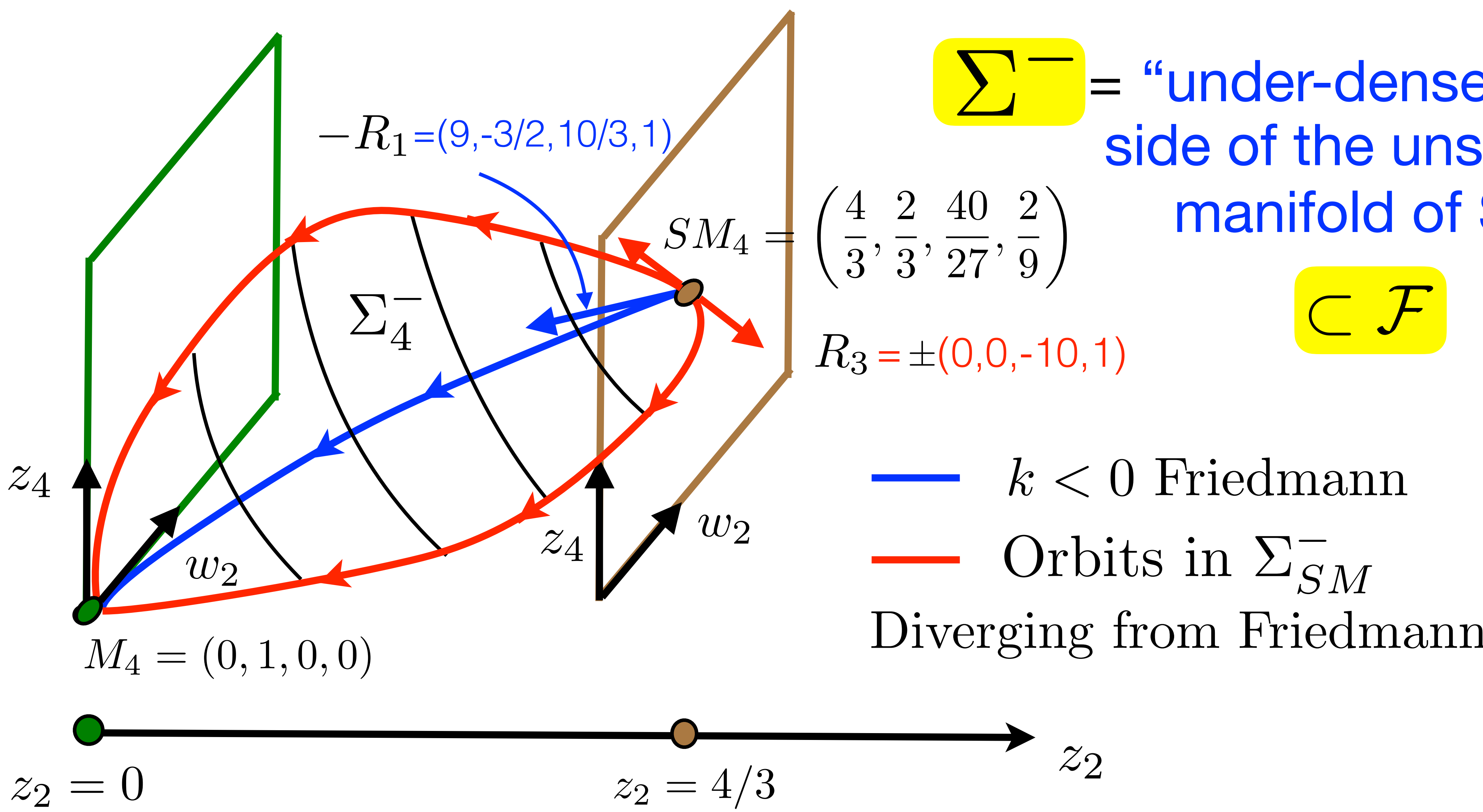

Σ^- = "under-dense" side of the unstable manifold of SM $\subset \mathcal{F}$

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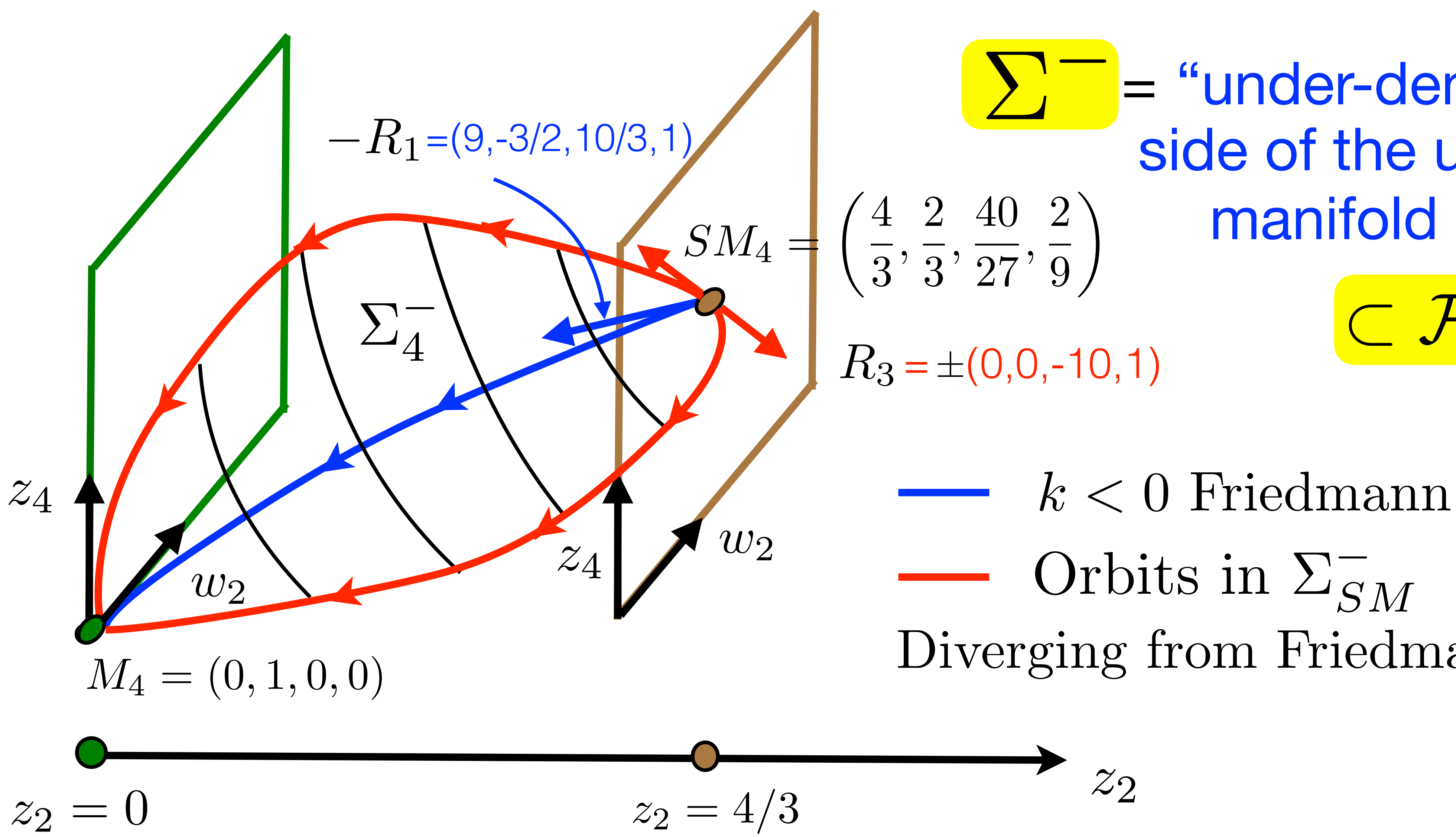
- $\exists$ a second Eigen-direction in Σ^- at order n=2…



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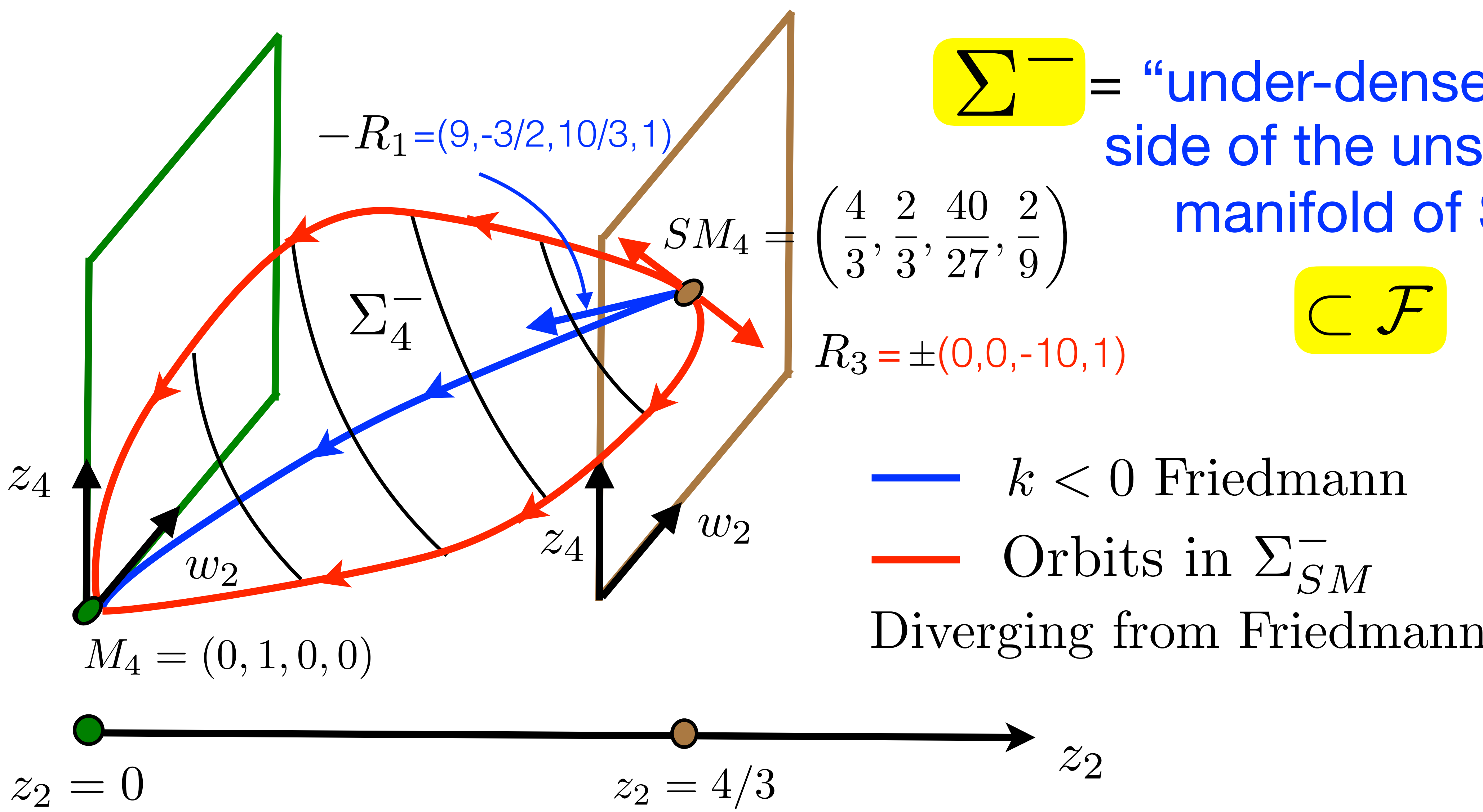
- $\exists$ +2 new Eigen-directions in Σ^- at each $n \geq 2$.



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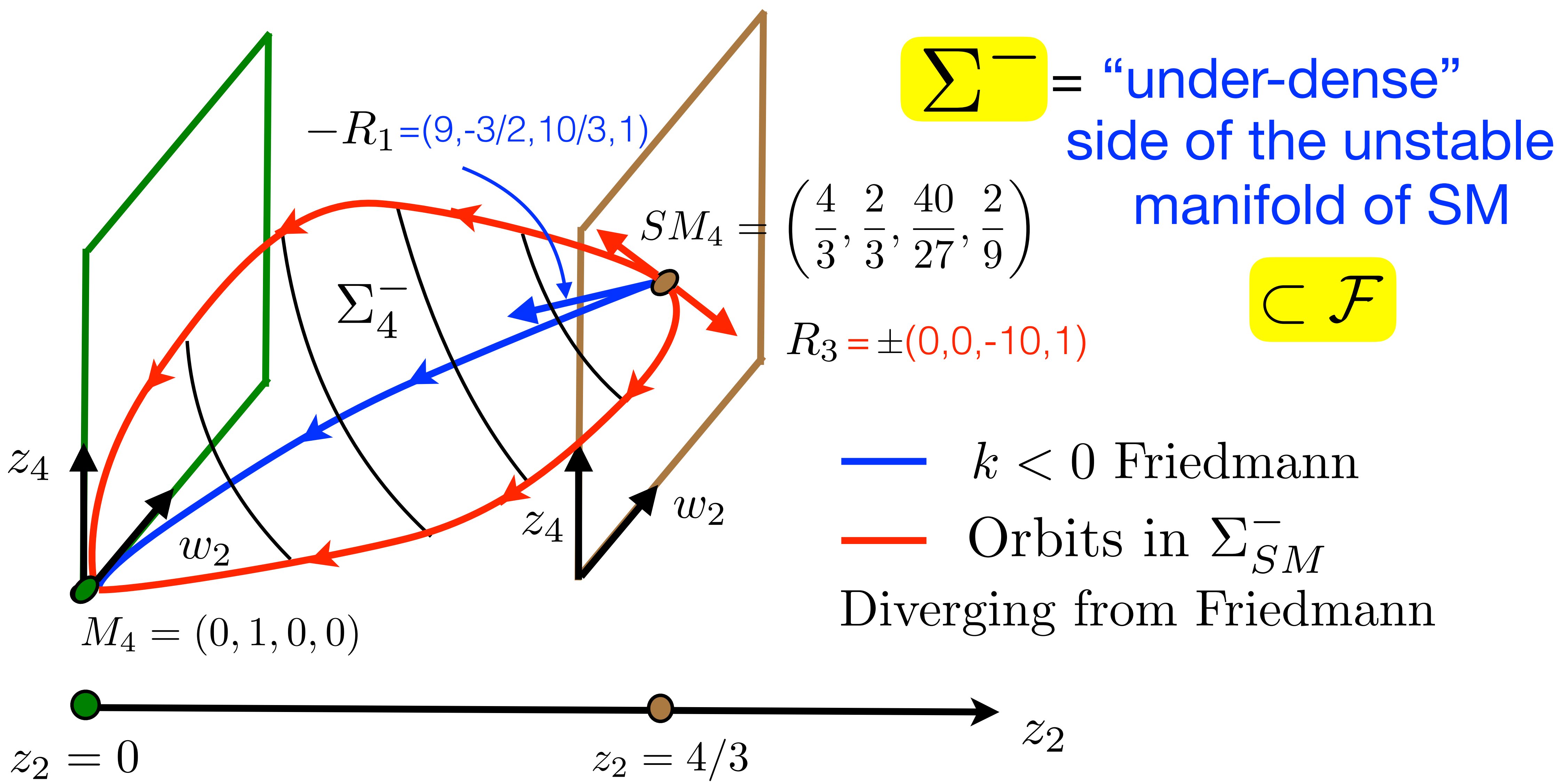
- Solutions in the unstable manifold of SM at order n=2, are in the unstable manifold of SM for all n>2.



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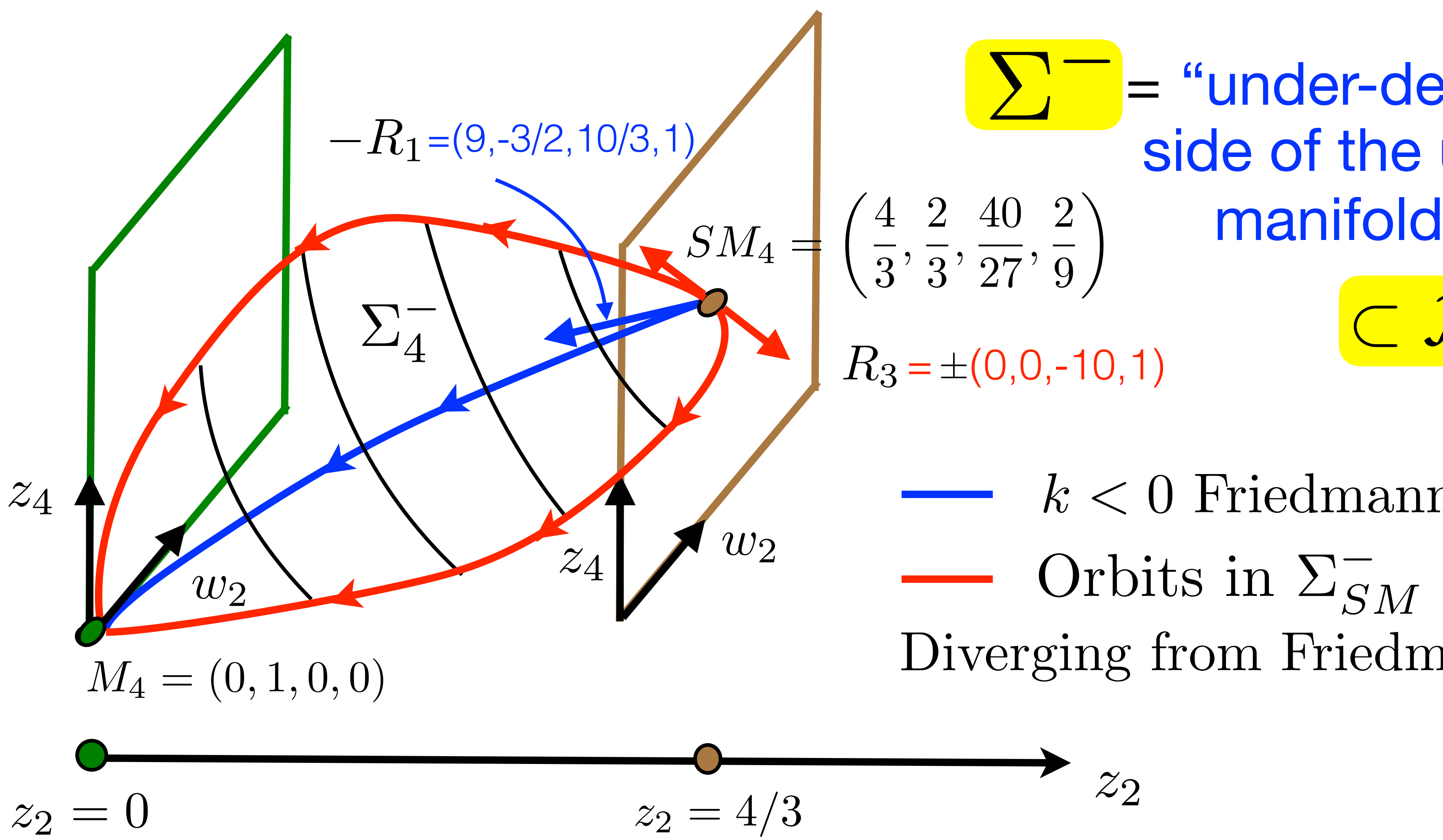
- $n \geq 2$: "Same picture" except more directions to perturb k<0 Friedmann within Σ^-.



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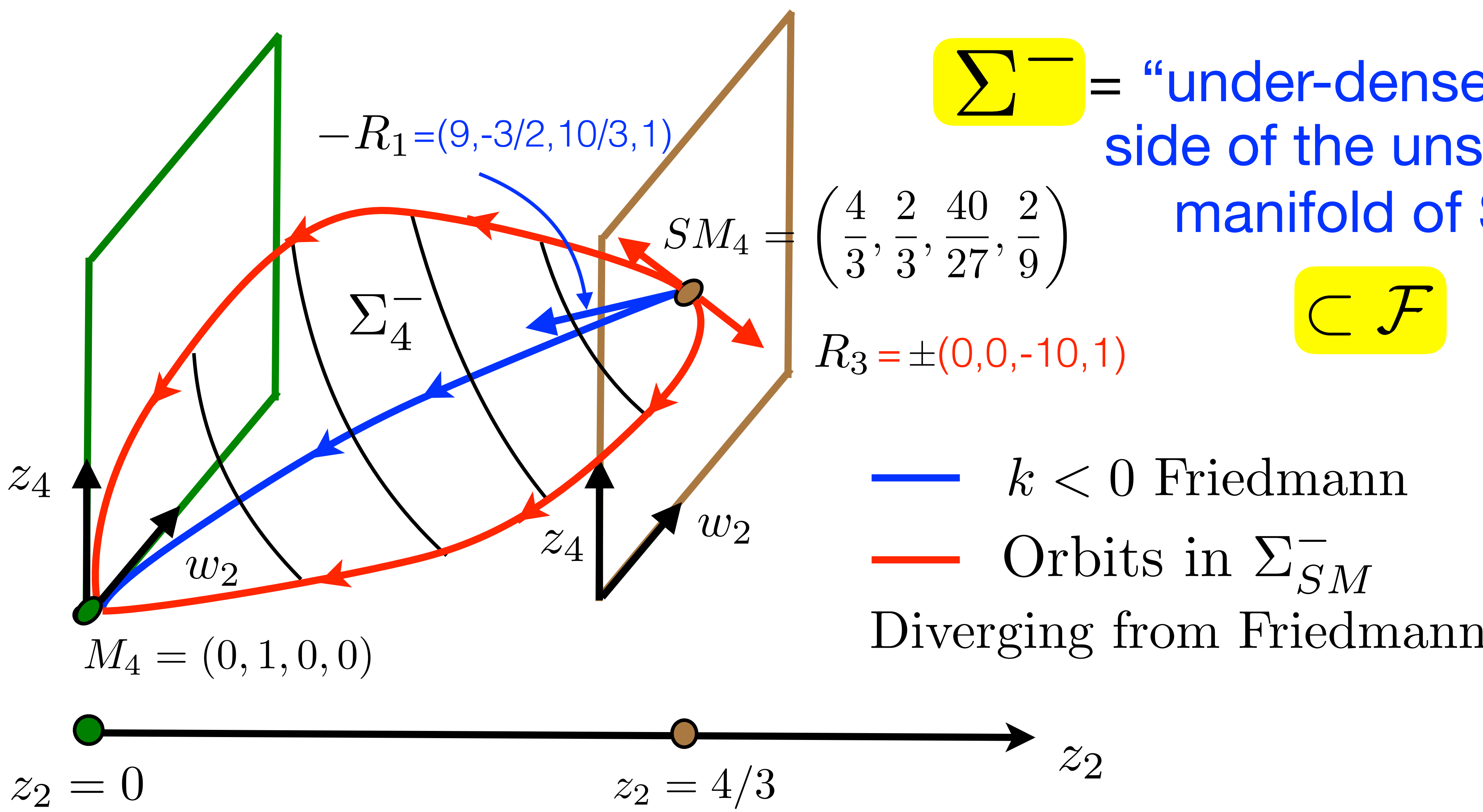
Theorem (ATV): Every solution in $\mathcal{F}$ tends to M from below as $t \to \infty$ at every order $n \geq 1$.

- Conclude: k<0 Friedmann is unstable to perturbation in Σ^- at every order $n \geq 2$!



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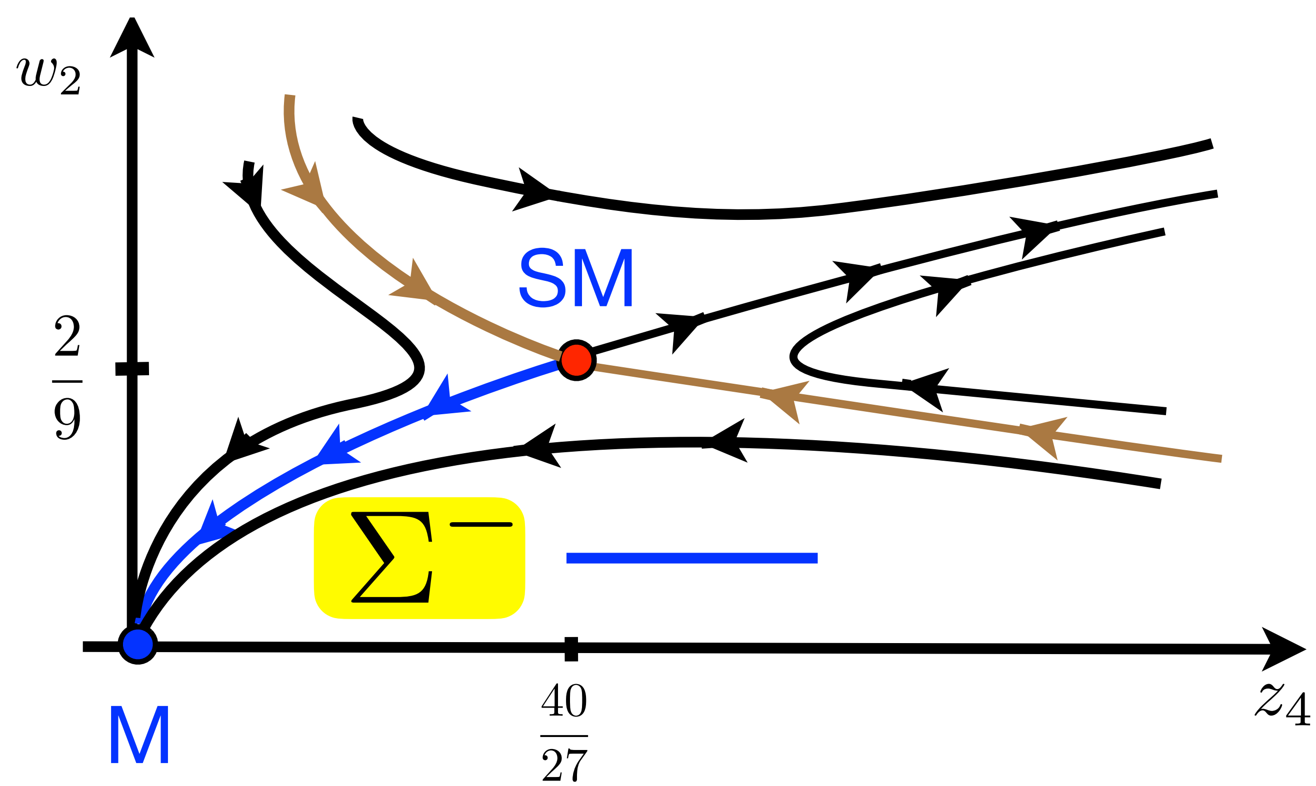

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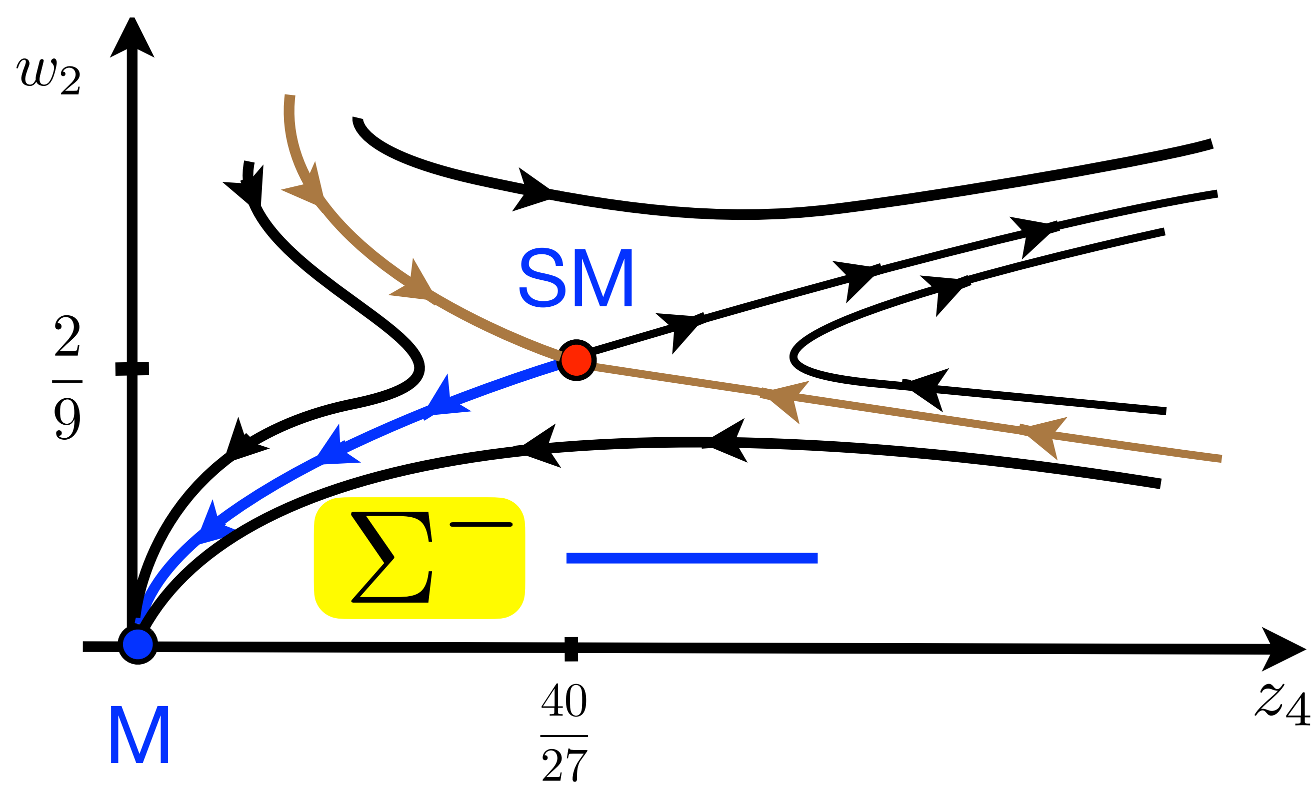
- So it suffices to characterize the instability of k<0 Friedmann within the unstable manifold Σ^- of SM.



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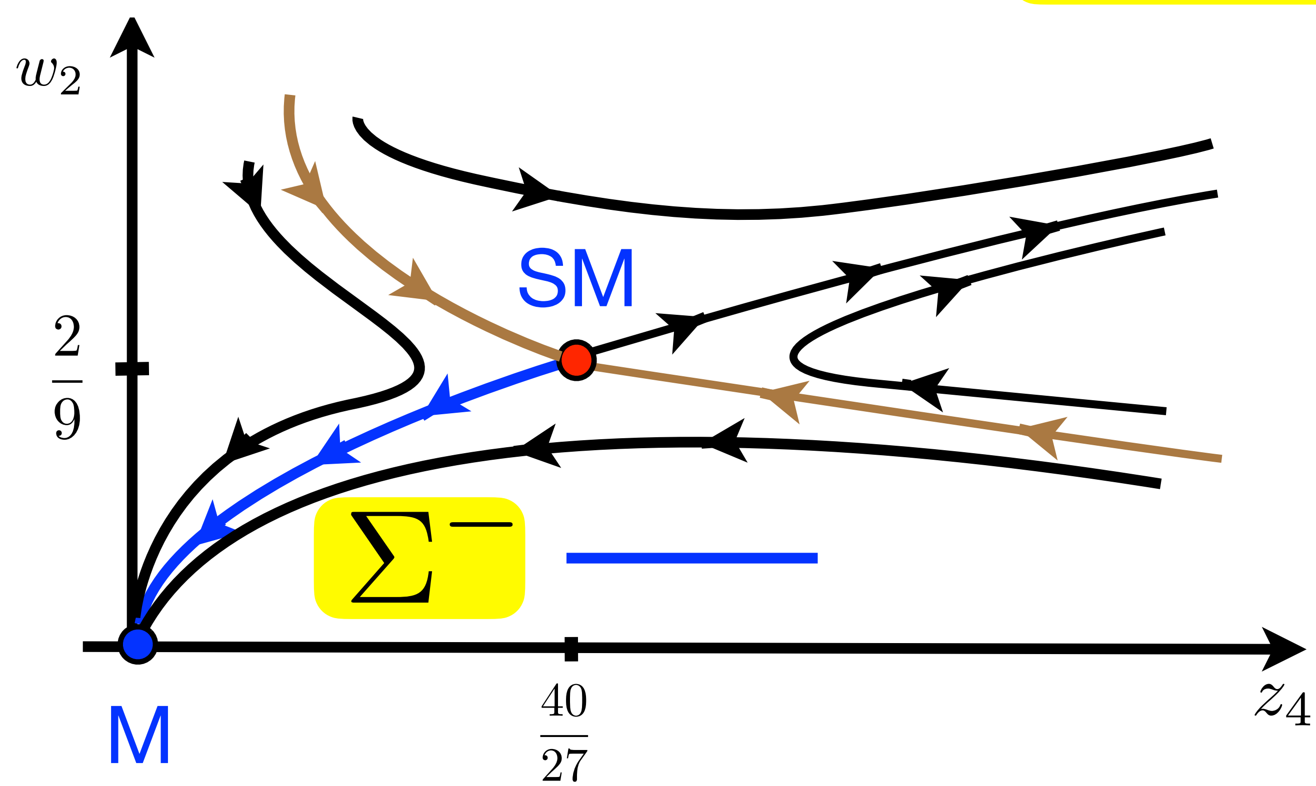
Theorem: Solutions in $\mathcal{F}$ do NOT generically tend to SM in backward time $t \to 0$.



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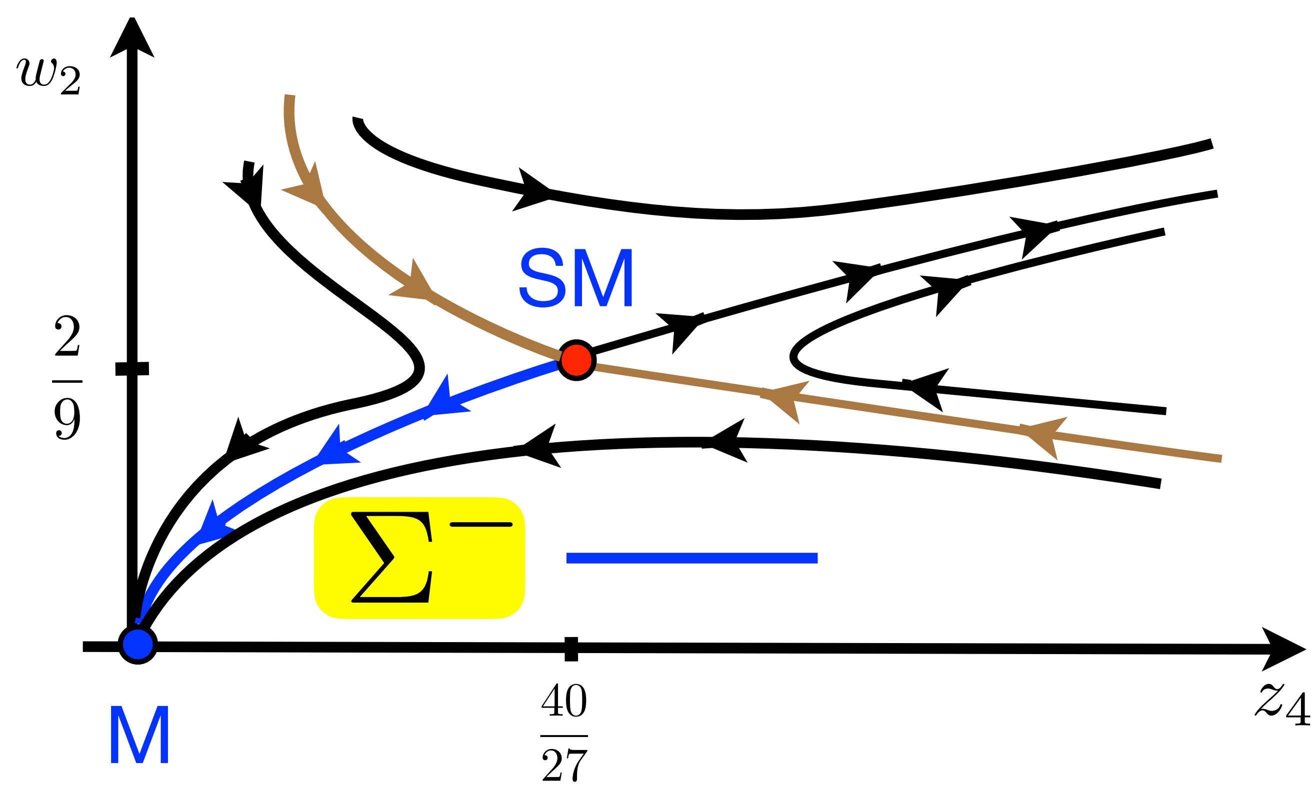
- The Friedmann Big Bang is NOT generic!



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(Same for small perturbations of SM in $\mathcal{F}$.)

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Proof: Taylor's Theorem with decay to M at n=2.

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Theorem: The spatially homogeneous self-similar Big Bang of Friedmann spacetimes is universal at leading order in ξ, but is generically not self-similar beyond leading order.

-In fact, generically: $z_{2n}(t) \to \infty, \quad w_{2n-2}(t) \to \infty$

as $t \to 0$ for all $n \geq 2$

Concluding Remarks:

- Note that trajectories of STV-ODE of order n>1 which tend to M, are bounded on $t \geq t_0 > 0$.

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Problem: Find sharp bounds on i.c.'s which imply convergence of power series to solutions in $\mathcal{F}$:

$$z(t) = \sum_{n=1}^{\infty} z_{2n}(t)\xi^{2n}, \quad w(t) = \sum_{n=1}^{\infty} w_{2n-2}(t)\xi^{2n} \quad \in \mathcal{F}$$

Concluding Remarks:

Define: $\mathcal{F}_n \equiv$ space of approximate solutions

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$\mathcal{F}$ described by convergence/existence Thms

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…we argued this is the hallmark of a "fudge factor"…what you expect to see when you try to add a parameter to the equations in an attempt to fix what is actually an incorrect solution of the original equations…?)

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…just like $k<0$ Friedmann comes from BB?

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- Could this imply a larger region of thermalization?

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- With the understanding that solutions generically accelerate away from Friedmann spacetimes before they decay back, couldn't one almost predict the anomalous acceleration before it was observed?

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So $p = \frac{c^2}{3}\rho$ Big Bangs produce $p = 0$ solutions…

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- We conjectured: Enormous pressure and modulus of genuine nonlinearity might imply decay to non-interacting wave pattern by end of radiation…

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Coincidence: The range of possible values of Q is precisely the same as in theory of Dark Energy!

$$.25 \leq Q \leq .5$$

Results: 2017 RSPA

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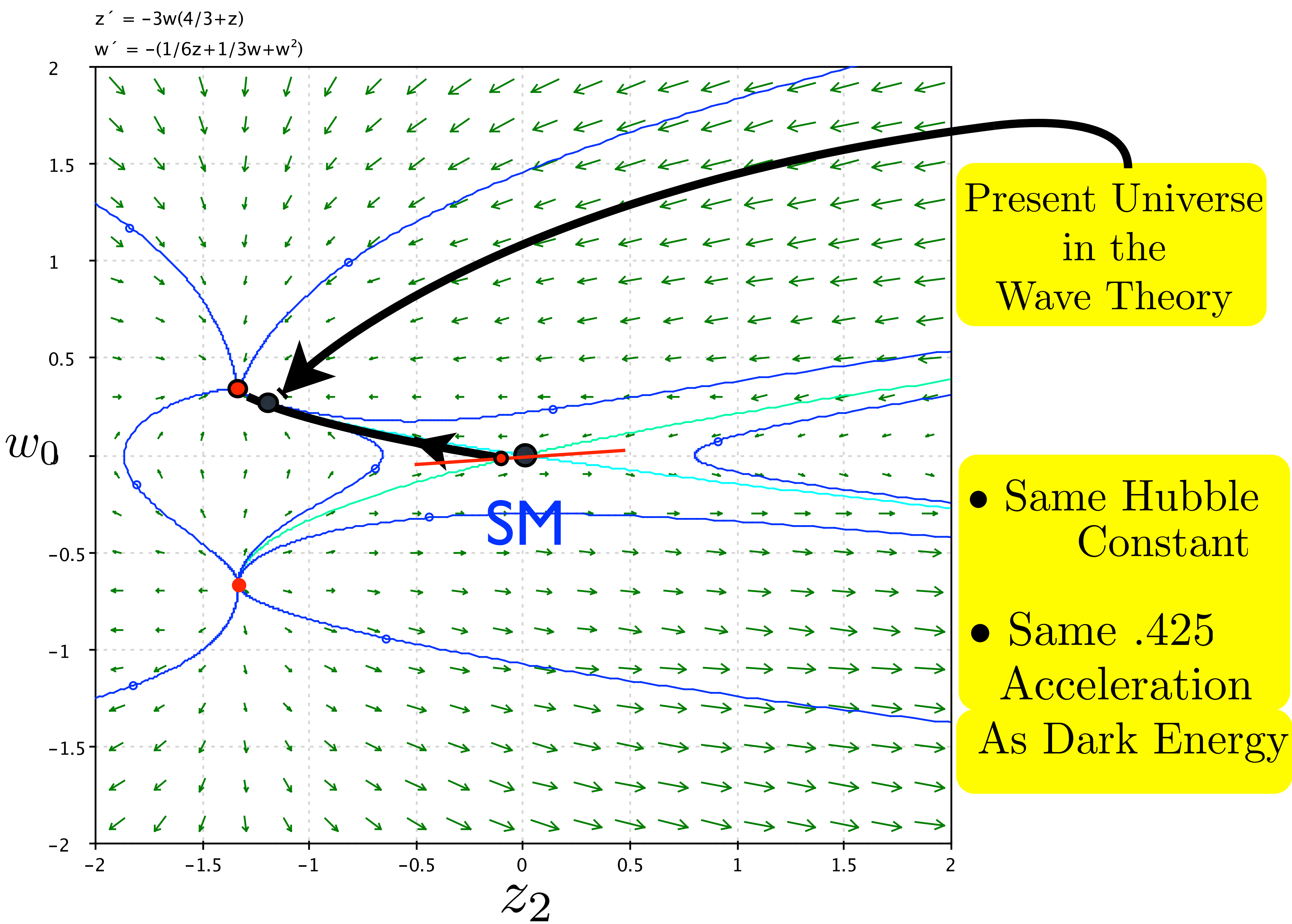
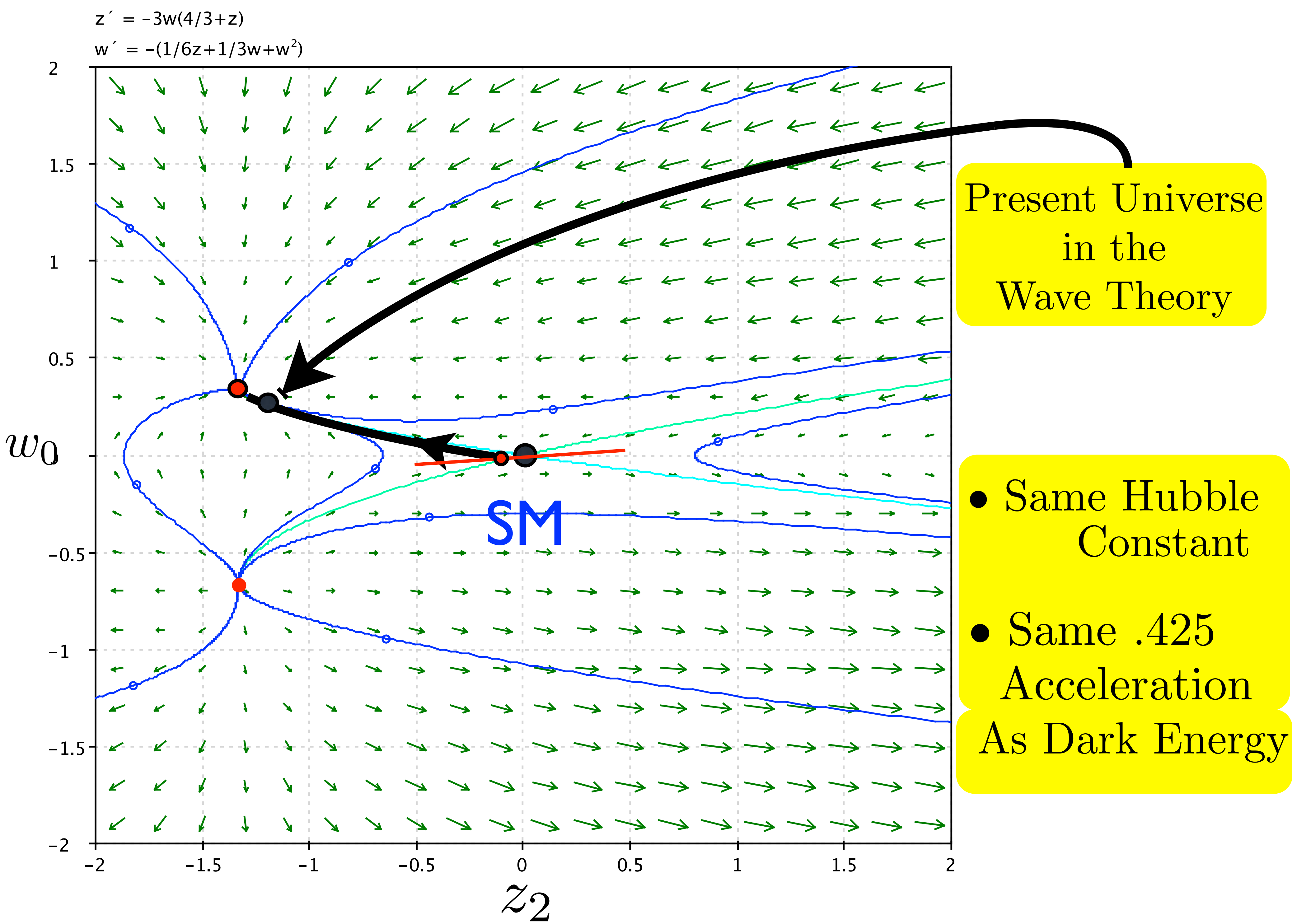


Results: 2017 RSPA

H_0 and Q determine present time at n=1…as well as the acceleration parameter of wave at end of radiation…



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Dark Energy

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Wave Theory

Results: 2017 RSPA

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Conclude: You need redshift vs luminosity to order n=4 to see where you are in the n=2 phase portrait…

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* Theorem: C_4 determines whether the Big Bang is self-similar like Friedmann spacetimes or not…

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- Final Question: Could the anomalous acceleration of the universe be due to an acceleration away from Friedmann arising directly from the Big Bang, and not from a scale of fluctuation in Friedmann created after the Big Bang?

End

Thank you!!