Title: *A Proposal to Numerically Simulate a Cosmic Shock Wave by Use of a Locally Inertial Glimm Scheme*

Author: Blake Temple, AMS Annual Meeting, Special Session Numerical Relativity, New Orleans, January 2007

Abstract: In this talk I discuss a proposal to numerically simulate a cosmological shock wave using ideas that arose in the author's earlier work on the locally inertial Glimm Scheme. The problem is motivated by prior joint work with J. Smoller, (Proc. Nat. Acad. Sci., USA, Vol. 100, no. 20, pp. 11216-11218), in which we introduced a new exact solution of the Einstein equations in which the explosion of the Big Bang generates an outgoing, spherical, entropy satisfying shock wave. In this model, the Big Bang begins inside a (time reversed) Black Hole–a White Hole in which everything is running backwards, exploding outward instead of collapsing inward. Our recent work indicates that a wave qualitatively similar to this exact solution would emerge from the standard inflationary cosmology if the spacelike slice that emerged co-moving with the perfect fluid at the end of inflation, were a space of "finite total mass". In this talk I discuss a proposal to numerically simulate this exactly using numerical ideas arising from the author's analysis of a locally inertial Glimm Scheme for spherically symmetric spacetimes. We wonder whether the secondary waves might account for the anomalous acceleration of the galaxies that is currently accounted for by the mysterious "Dark Energy". The author's work in cosmology is all joint work with Joel Smoller, while the idea of the locally inertial Glimm Scheme was introduced in the author's joint work with Jeffrey Groah. Involved in the numerical project will be UC-Davis students Brian Wissman and Zeke Vogler. (Articles and commentaries can be found on author's website: http://www.math.ucdavis.edu/ temple/articles/)

Numerical Refinement of a Finite Mass Shock-Wave Cosmology

Blake Temple, UC-Davis

Collaborators: *J. Smoller, Z. Vogler, B. Wissman*

References

- Exact solution incorporating a shock-wave into the standard FRW metric for cosmology...

- Smoller-Temple, *Shock-Wave Cosmology Inside a Black Hole*, PNAS Sept 2003.

- Smoller-Temple, *Cosmology, Black Holes, and Shock Waves Beyond the Hubble Length*, Meth. Appl. Anal., 2004.

References:

- Connecting the shock wave cosmology model with Guth's theory of inflation...

- *How inflationary spacetimes might evolve into spacetimes of finite total mass*, with J. Smoller, Meth. and Appl. of Anal., Vol. **12**, No. 4, pp. 451-464 (2005).

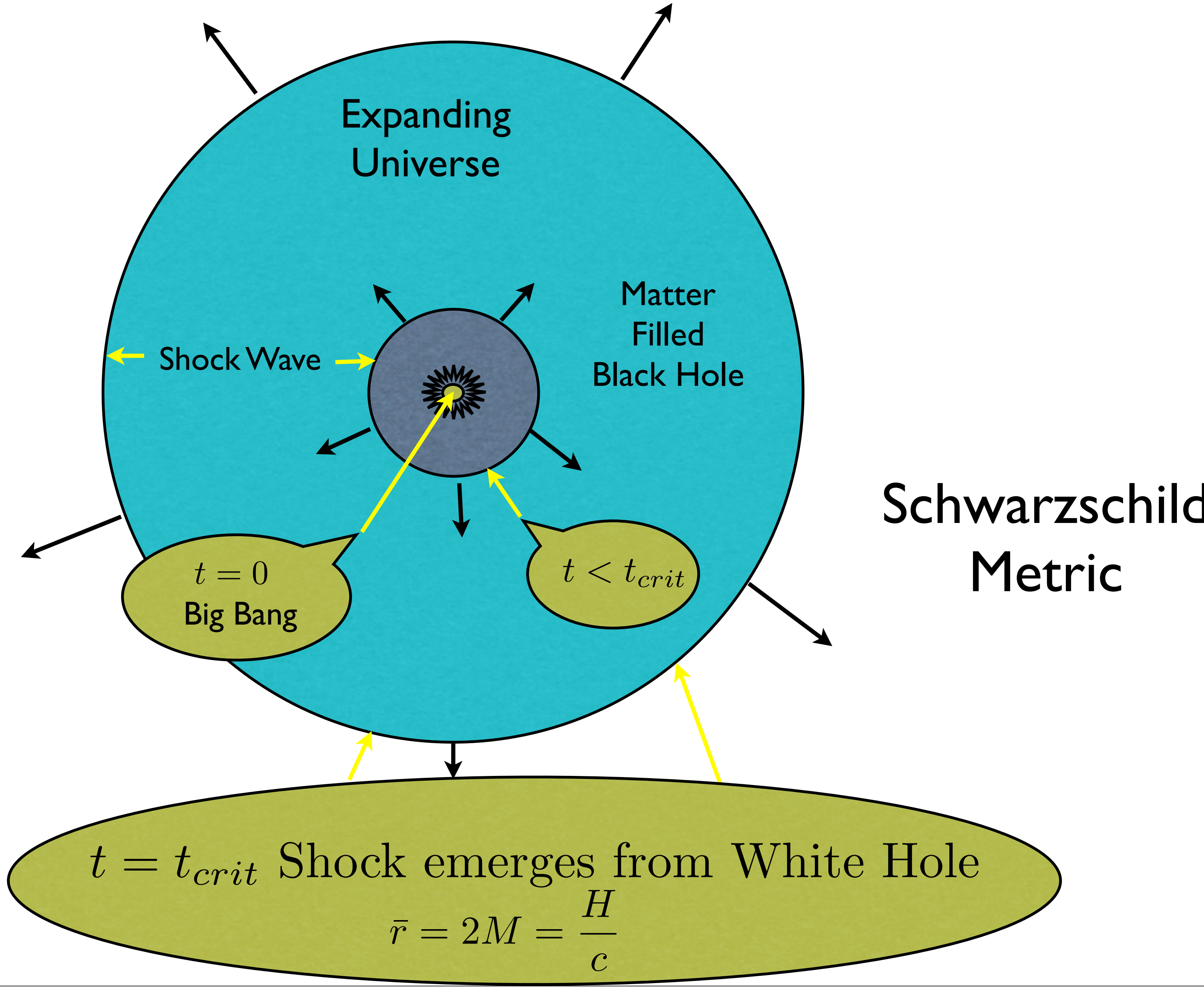
- *How inflation is used to solve the flatness problem*, with J. Smoller, Jour. of Hyp. Diff. Eqns. (JHDE) Vol. 3, no. 2, 375-386 (2006).

References:

- The locally inertial Glimm Scheme...

- *A shock-wave formulation of the Einstein equations*, with J. Groah, Meth. and Appl. of Anal., **7**, No. 4,(2000), pp. 793-812.
- *Shock-wave solutions of the Einstein equations: Existence and consistency by a locally inertial Glimm Scheme*, with J. Groah, Memoirs of the AMS, Vol. 172, No. 813, November 2004.
- *Shock Wave Interactions in General Relativity: A Locally Inertial Glimm Scheme for Spherically Symmetric Spacetimes*, with J. Groah and J. Smoller, Springer Monographs in Mathematics, 2007.

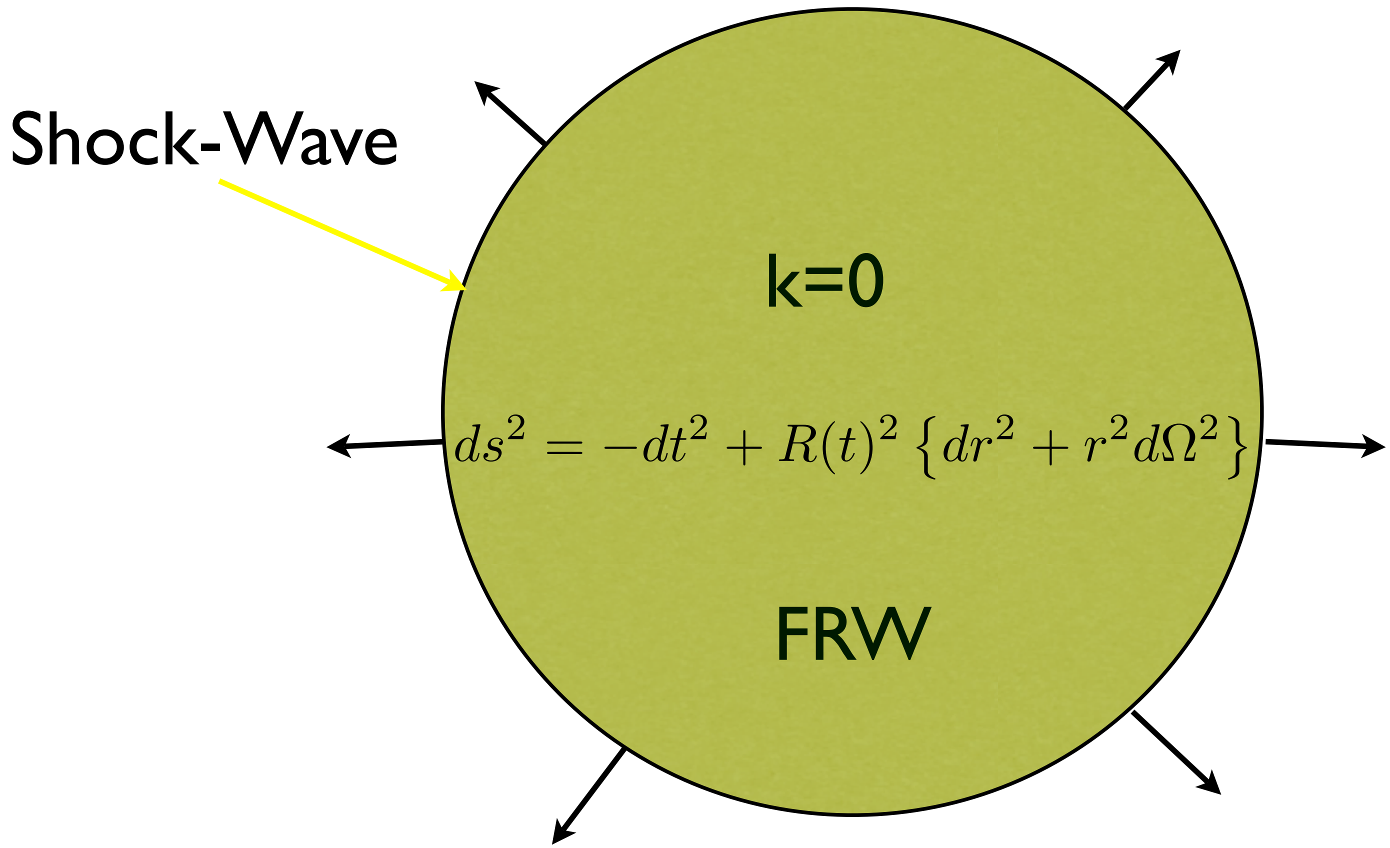
Our Shock Wave Cosmology Solution



- The solution can be viewed as a natural generalization of a $k=0$ Oppenheimer-Snyder solution to the case of non-zero pressure, inside the Black Hole----

$$2M/r>1$$

The Shock Wave Cosmology Solution



TOV: $$ds^2 = -B(\bar{r})d\bar{t}^2 + \frac{1}{1 - \frac{2M(\bar{r})}{\bar{r}}}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$\frac{2M}{\bar{r}} > 1$ ⇒ $\bar{r}$ is timelike

- In [Smoller-Temple, PNAS] we constructed an exact shock wave solution of the Einstein equations by matching a (k=0)-FRW metric to a TOV metric *inside the Black Hole* across a subluminal, entropy-satisfying shock-wave, out beyond one Hubble length

- To obtain a large region of uniform expansion at the center consistent with observations we needed $\frac{2M}{\bar{r}} > 1$

- $$\frac{2M}{\bar{r}} = 1 \Longleftrightarrow \bar{r} = Hubble\ Length \equiv \frac{H}{c}$$

1

- The TOV metric inside the Black Hole is the simplest metric that cuts off the FRW at a finite total mass.
- Approximately like a classical explosion of finite mass with a shock-wave at the leading edge of the expansion.
- The solution decays time asymptotically to Oppenheimer-Snyder----a finite ball of mass expanding into empty space outside the black hole, something like a gigantic supernova.

- Limitation of the model: TOV density and pressure are determined by the equations that describe the matching of the metrics

- $p = \frac{c^2}{3}\rho$ can only be imposed on the FRW side

- Imposing $p = \frac{c^2}{3}\rho$ on the TOV side introduces secondary waves which can not be modeled in an exact solution

- Question: How to model the secondary waves?

- OUR QUESTION: How to refine the model to incorporate the correct TOV equation of state, and thereby model the secondary waves in the problem?

- OUR PROPOSAL: Get the initial condition at the end of inflation

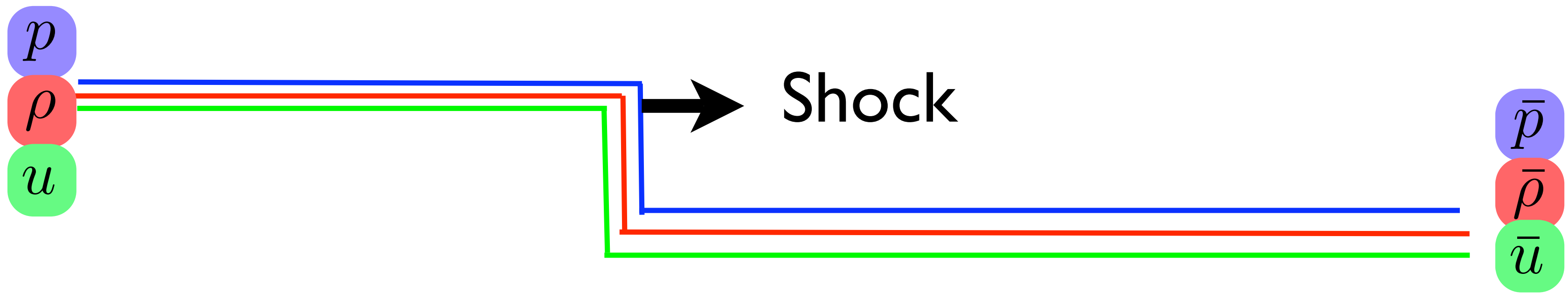
- Use the Locally Inertial Glimm Scheme to simulate the region of interaction between the FRW and TOV metrics

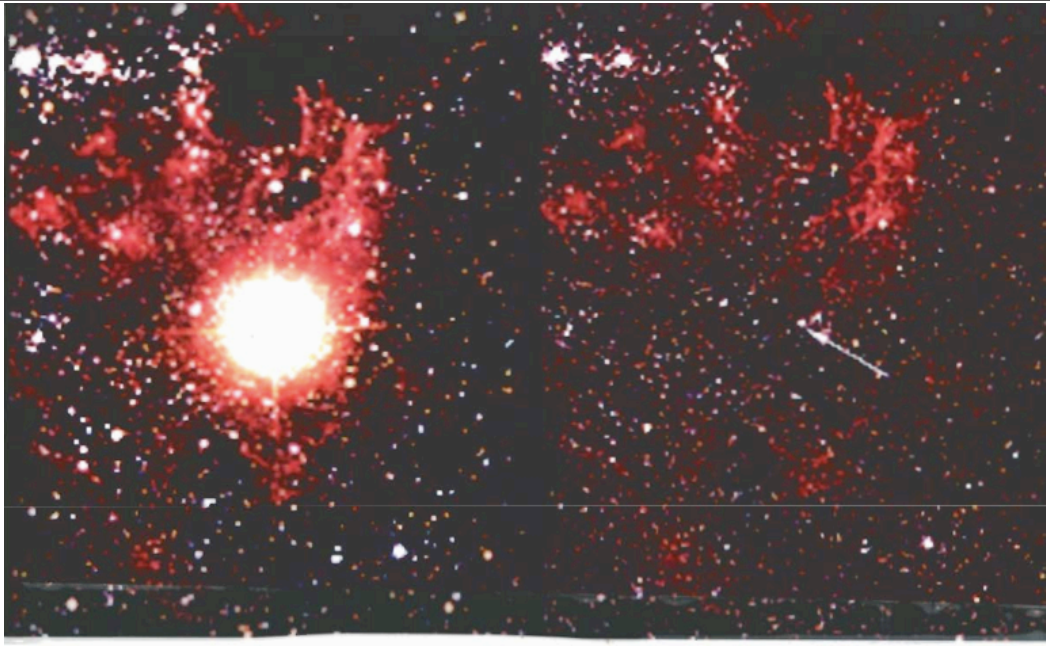
Incorporating a Shock Wave into the Standard Model of Cosmology

- A *blast-wave/shock-wave* marks the leading edge of a classical explosion

- *Shock-wave* $\approx$ *discontinuity* in density and pressure between the explosion and the material beyond the explosion

- Any explosion with a finite mass/energy behind it must generate such a *blast-wave*



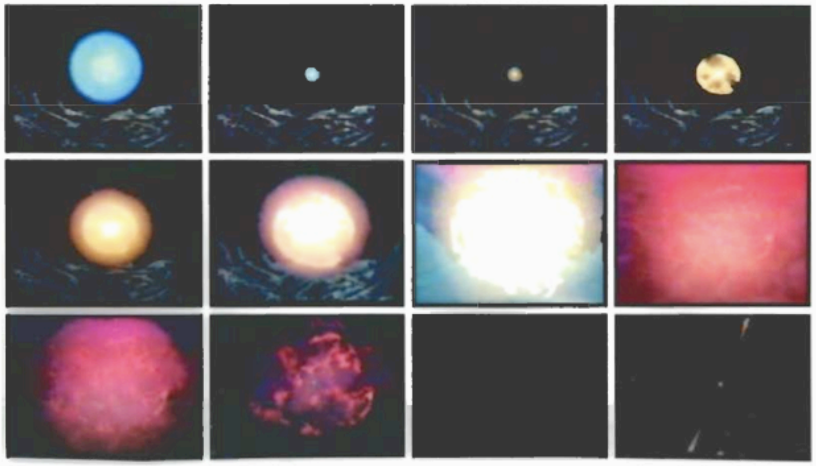


The above two photographs are of the same part of the sky. The photo on the left was taken in 1987 during the supernova explosion of SN 1987A, while the right hand photo was taken beforehand. Supernovae are one of the most energetic explosions in nature, making them like a 1028 megaton bomb (i.e., a few octillion nuclear warheads).

SN1987A
(NASA)

Supernova Images

This is the set of images used to create the supernova inline animation.



One of the most energetic explosive events known is a supernova. These occur at the end of a star's lifetime, when its nuclear fuel is exhausted and it is no longer supported by the release of nuclear energy. If the star is particularly massive, then its core will collapse and in so doing will release a huge amount of energy. This will cause a blast wave that ejects the star's envelope into interstellar space. The result of the collapse may be, in some cases, a rapidly rotating neutron star that can be observed many years later as a radio pulsar.

Title #1

Shock-Wave Cosmology
Inside
A Black Hole

Joel Smoller and Blake Temple

http://www.math.ucdavis.edu/~temple

Title #2

What Would Happen If The Big Bang Were an Explosion of Finite Total Mass

Joel Smoller and Blake Temple

http://www.math.ucdavis.edu/~temple

Title #3

The Big Bang
or
Have You Ever Seen an Explosion Without a Shock-Wave Before?

Joel Smoller and Blake Temple
http://www.math.ucdavis.edu/~temple

We have been working on a theory of shock wave propagation in general relativity-----

Intriguing Question: ``Could there be a shock-wave at the leading edge of the expansion of the galaxies?"

- If there is a shock wave, then the Big Bang was an explosion of Finite Total Mass

- In the standard model of cosmology, the Big Bang is an explosion of

Infinite Mass

and

Infinite Extent

- There is nothing beyond the galaxies in the Standard Model

In the shock wave model---an exact solution of the Einstein equations

- $t = 0:$ Shock-wave emerges from the origin

  $\bar{r} = 0$ ⟺ Big Bang

  ----Inside a Black Hole----

- $t = t_{crit}:$ Shock-wave emerges from the event horizon of a Black Hole in a Schwarzschild Metric

- Shock-wave can be arbitrarily far out.

INTRODUCTION TO COSMOLOGY

Edwin Hubble (1889-1953)

- Hubble's Law (1929):

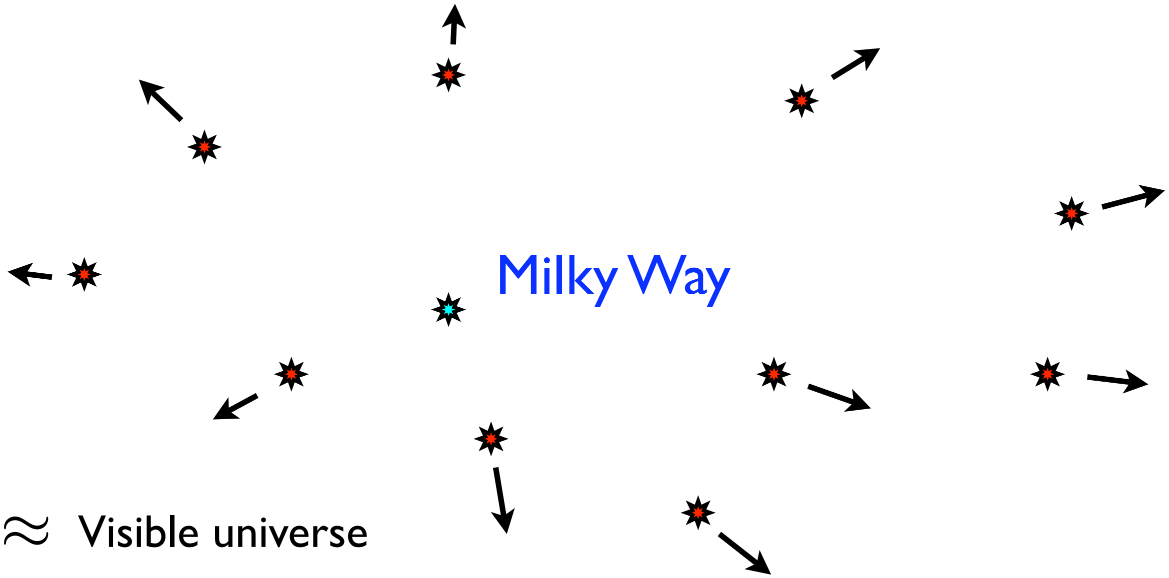
``The galaxies are receding from us at a velocity proportional to distance"

↓

Universe is Expanding

- Based on Redshift vs Luminosity

Cosmic Length Scales



- 10 billion lightyears ≈ Visible universe

  1 billion lightyears ≈ Uniform density

- 50 million lightyears ≈ Separation between clusters of galaxies

  10 million lightyears ≈ Diameter of a cluster

- 1 million lightyears ≈ Separation between galaxies in a cluster

  100 thousand lightyears ≈ Distance across Milky Way

- 28 thousand lightyears ≈ Distance to galactic center

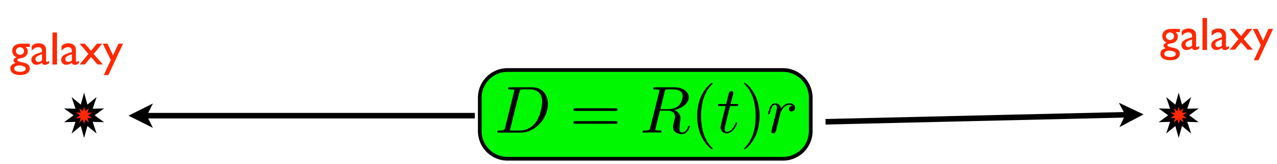
  4 lightyears ≈ Distance to the nearest star

Standard Model of Cosmology

- Assume FRW metric, *k=0:*

$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

- $D = Rr$ Measures distance between galaxies at each fixed t



- Conclude: $$\dot{D} = \dot{R}r = \frac{\dot{R}}{R} Rr = HD$$

$$\dot{D} = HD$$ ← Hubble's Law

- $H = \frac{\dot{R}}{R}$ = Hubble's Constant

- Standard Model of Cosmology

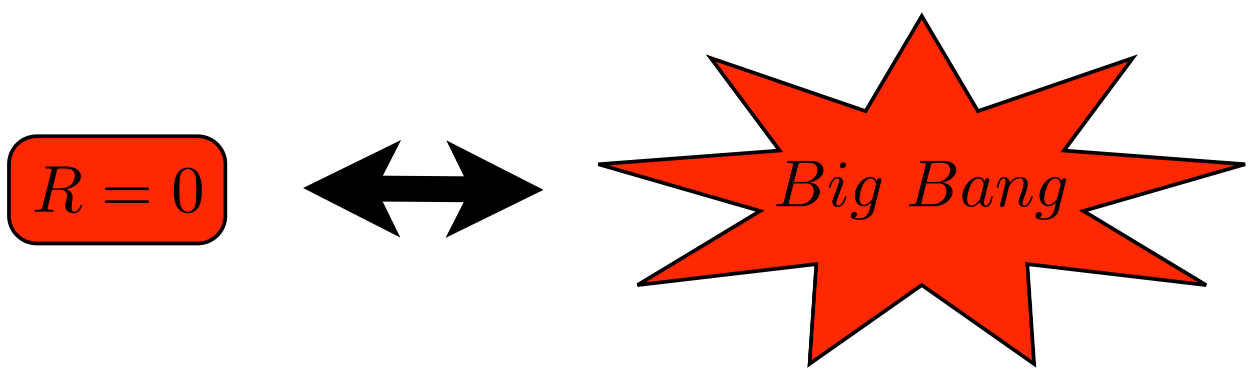
$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

- Hubble's Law:

$$\dot{D} = H D$$

- Conclude--

``The universe is expanding like a balloon"



The Hubble ``Constant''

$$H = \frac{\dot{R}}{R} \approx h_0 \frac{100\ km}{s\ mpc}$$

- A galaxy at 1 mpc $\approx$ 3.26 million lightyears

  recedes at $h_0 \dfrac{100\ km}{sec}$ $.5 \leq h_0 \leq .8$

$$\frac{1}{H_0} \approx 10^{10}\ years \approx age\ of\ universe$$

- $\dfrac{c}{H_0} \approx$ Hubble Length $\approx 10^{10}$ lightyears

  $\approx$ farthest we can see across the universe

- FRW metric: $ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$

- Any point can be taken as $r = 0$

↓

Homogeneous and Isotropic about every point

↓

Copernican Principle:
Earth is not in a special place in the universe

- $lim_{t \to \infty} R(t) r = \infty$

↓

Universe is infinite at each t=const surface

Conclude: In the Standard Model----

Even though we can only See ≈ One Hubble Length, the Standard Model presumes that uniformity extends to infinity in all directions....

- Our Question: ``What if the expansion of the galaxies is only a finite expansion of bounded extend?''

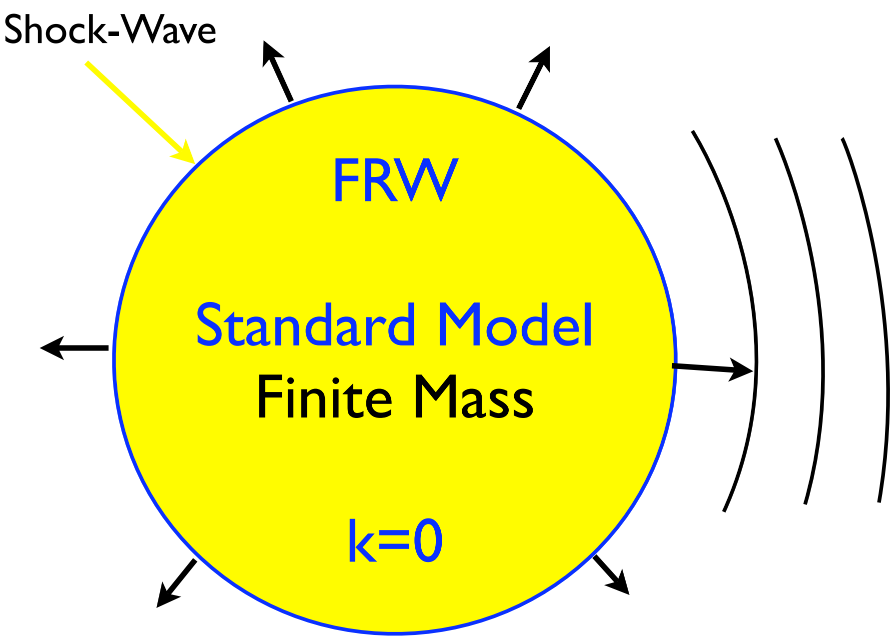
↓

There exists a wave at the leading edge of the expansion of the galaxies

↓

- Math Question: Can you incorporate a shock-wave at the leading edge of the galaxies in the FRW metric in a rigorous mathematical model?

- Answer: Yes (Te/Sm PNAS Sept. 2003)



TOV metric
``inside the Black Hole''

$$\frac{2M}{\bar{r}} > 1$$

- In order to account for the large region of uniform expansion that we observe in the universe today, the explosion must occur within a

``time-reversed Black Hole'' = White Hole

- Ans: Yes

- There exists a free parameter in the model

  $\sim$ Total Mass of the explosion

  $\sim$ Distance to the Shock-Wave

  $\sim$ Extent and Duration of the region of uniformity

- We expect every sufficiently large explosion of finite total Mass to look qualitatively like this

- If close enough, the wave is observable

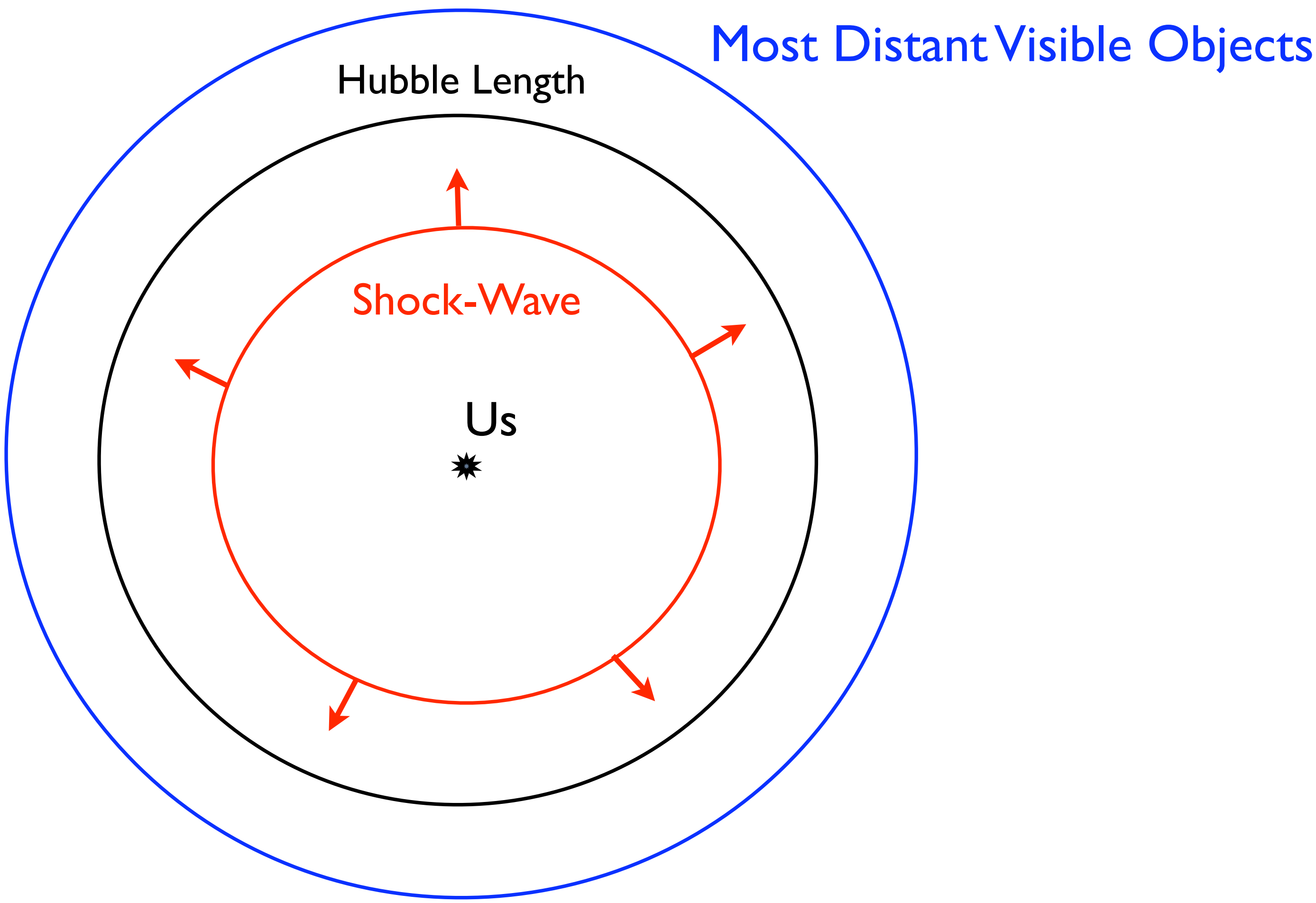
- The Main Point:

To be consistent with observations, the shock-wave must lie out beyond one Hubble length, which places it inside a Black Hole in the sense that

$$\frac{2M}{\bar{r}} > 1$$

For Example: We can see out to about 1.5 Hubble Lengths...

Picture:



The Schwarzschild Radius of the Universe

- Schwarzschild Metric: $$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{(1 - \frac{2M}{r})} dr^2 + r^2 d\Omega^2$$

- Schwarzschild Radius: $$r_S = 2M \equiv \frac{2\mathcal{G}}{c^2} M$$

- The radius at which *M* forms a Black Hole:
  - $r_S(\text{Earth}) \approx 3\ cm$
  - $r_S(\text{Sun}) \approx 2\ km$
  - $r_S(\text{Galaxy}) \approx 10^{12} \times 2\ km$
  - $r_S(\text{Visible Universe}) \approx \frac{c}{H} = \text{One Hubble Length}$

- **Conclude:** ``A Shock-wave beyond one Hubble Length must lie inside a Black Hole"

- I.e. In order for *M* to be continuous at the shock, the spacetime beyond the shock-wave must satisfy

$$\frac{2M}{\bar{r}} > 1$$

To construct a shock-wave at the leading edge of the galaxies beyond one Hubble length, we need to show:

(1) The FRW metric continues to a spacetime metric

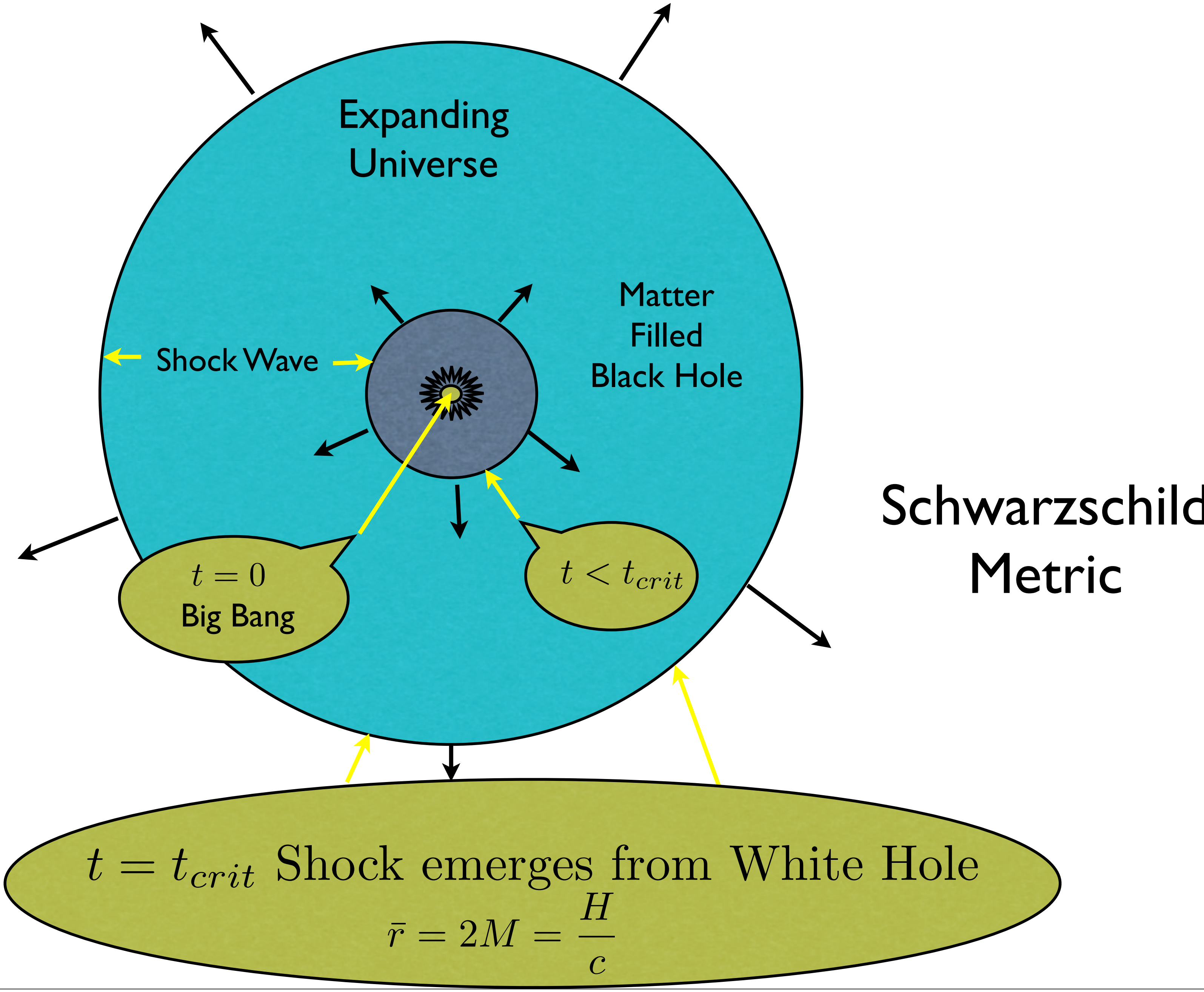
with $\frac{2M}{\bar{r}} > 1$ at radius $\bar{r} > \frac{c}{H}$

$\bar{r} = Rr$

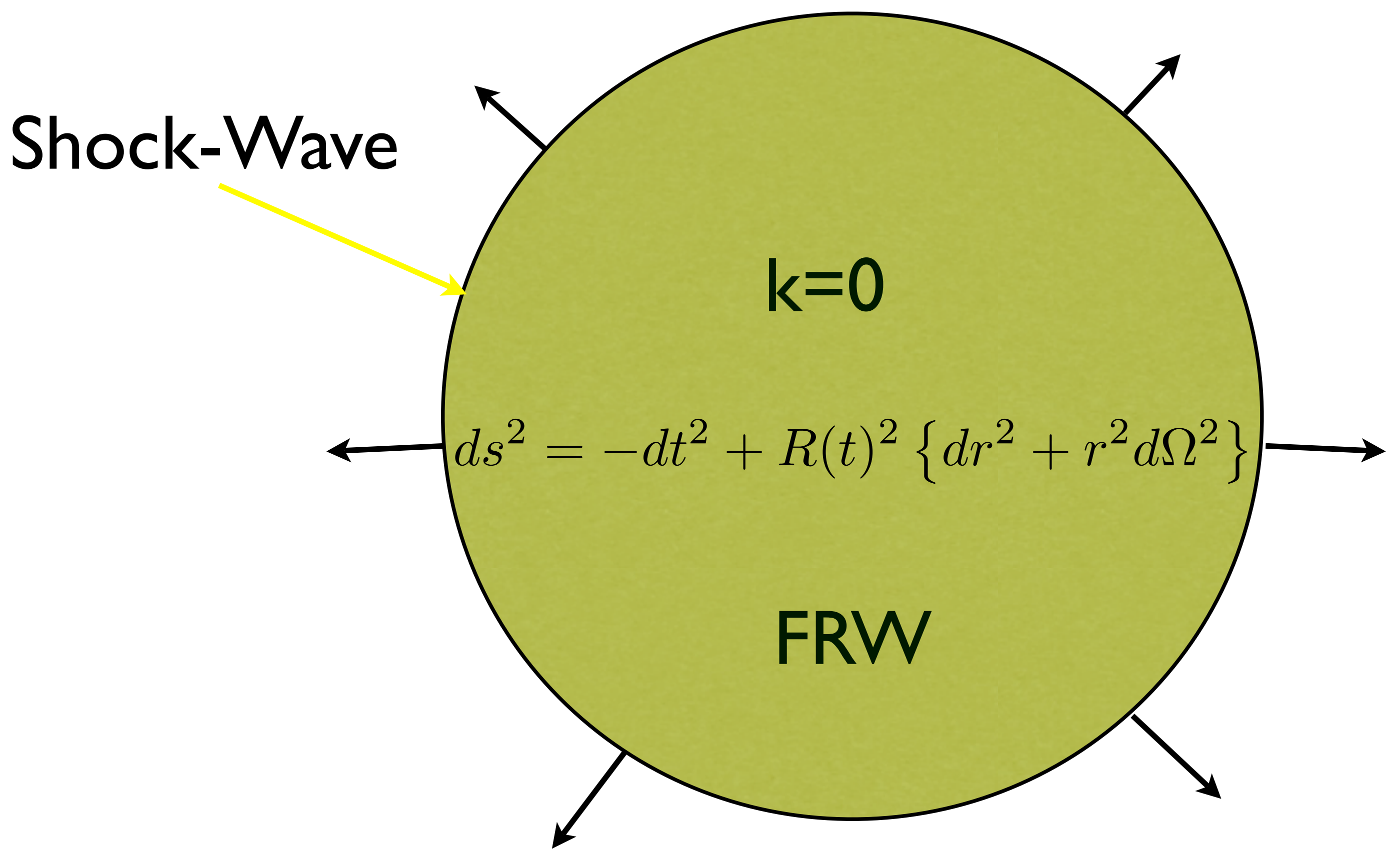
(2) The interface describes a Shock-Wave

DETAILS
OF THE
EXACT SOLUTION

Our Shock Wave Cosmology Solution



The Shock Wave Cosmology Solution



TOV: $$ds^2 = -B(\bar{r})d\bar{t}^2 + \frac{1}{1 - \frac{2M(\bar{r})}{\bar{r}}} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$\frac{2M}{\bar{r}} > 1$ $\Rightarrow$ $\bar{r}$ is timelike

- Limitation of the model: TOV density and pressure are determined by the equations that describe the matching of the metrics

- $p = \frac{c^2}{3}\rho$ can only be imposed on the FRW side

- Imposing $p = \frac{c^2}{3}\rho$ on the TOV side introduces secondary waves which can not be modeled in an exact solution

- Question: How to model the secondary waves?

- OUR QUESTION: How to refine the model to incorporate the correct TOV equation of state, and thereby model the secondary waves in the problem?

- OUR PROPOSAL: Get the initial condition at the end of inflation

- Use the Locally Inertial Glimm Scheme to simulate the region of interaction between the FRW and TOV metrics

To get the equations that describe the shock wave:

- Start with an FRW spacetime with $p = \sigma\rho$
- Impose Einstein equations on the TOV metric
- Impose shock-matching conditions on shock position
- Make the change of variables $u = \frac{\bar{p}}{\rho}, \quad S = \frac{1}{\sqrt{N}}$

  where $N =$ distance from FRW center to the shock

- Assume $\sigma = constant$ ⇒ Final Equations

The Final Equations

$p = \sigma\rho$

$\sigma = const.$

Autonomous System

$$S' = 2S(1+3u)\left\{(\sigma - u) + (1+u)S\right\} \equiv F(S,u)$$

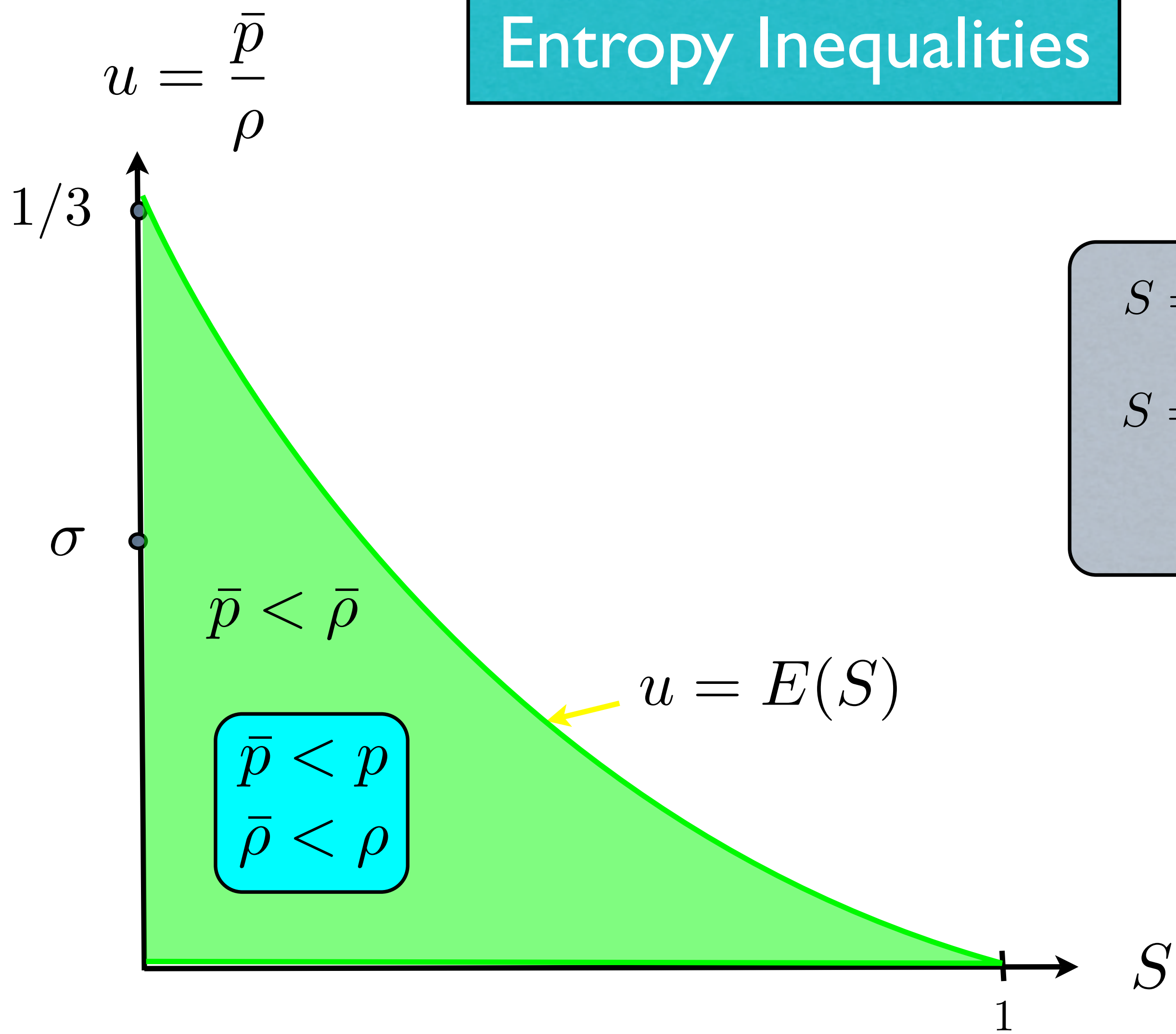
$$u' = (1+u)\left\{-(1-3u)(\sigma - u) + 6u(1+u)S\right\} \equiv G(S,u)$$

Big Bang $\equiv 0 \leq S \leq 1 \equiv$ Emerges From White Hole

Entropy Condition: $\bar{p} < p, \ \bar{\rho} < \rho, \ \bar{p} < \bar{\rho}$

$$S < \left(\frac{1-u}{1+u}\right)\left(\frac{\sigma - u}{\sigma + u}\right) \equiv E(u)$$

Entropy Inequalities



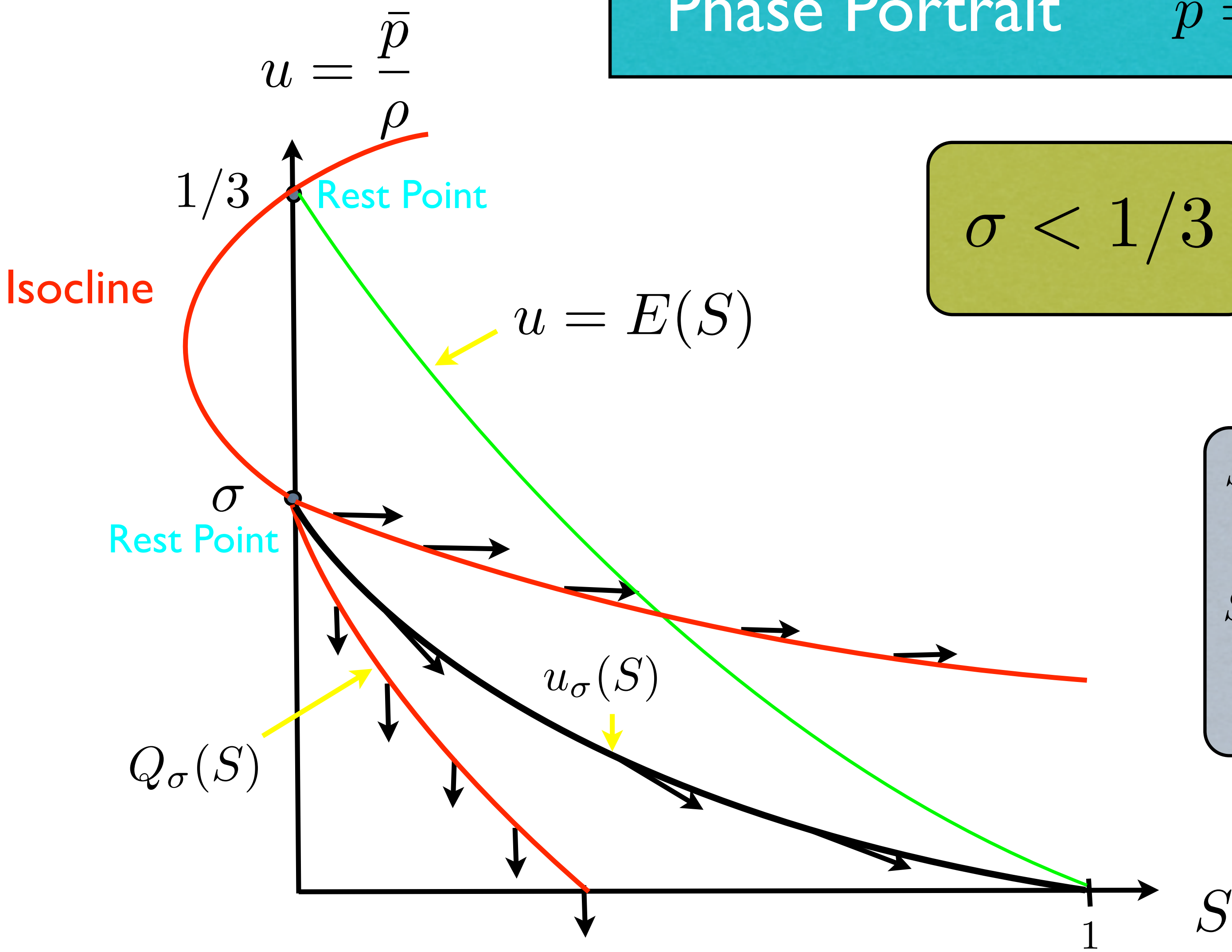
$S = 0$ *Big Bang*

$S = 1$ *Solution emerges from White Hole*

The entropy conditions for an outgoing shock hold when $u_\sigma(S) < E(S)$

Phase Portrait $p = \sigma\rho$

$\sigma < 1/3$



$S = 0$ *Big Bang*

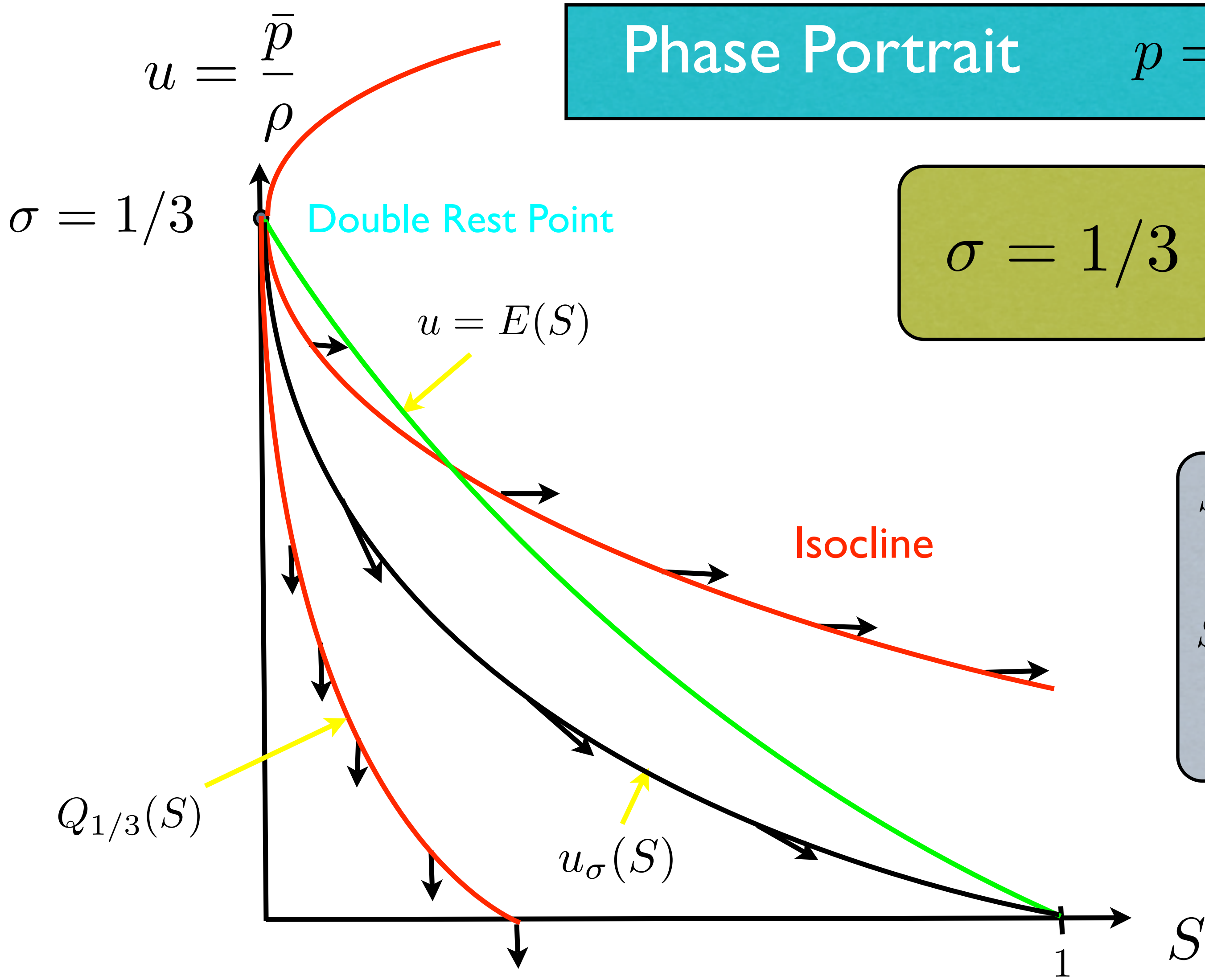
$S = 1$ *Solution emerges from White Hole*

$s_\sigma(S) \equiv$ *shock speed* $< c$ for $0 < S < 1$

$s_\sigma(S) \to 0$ as $S \to 0$

Phase Portrait $p = \sigma\rho$

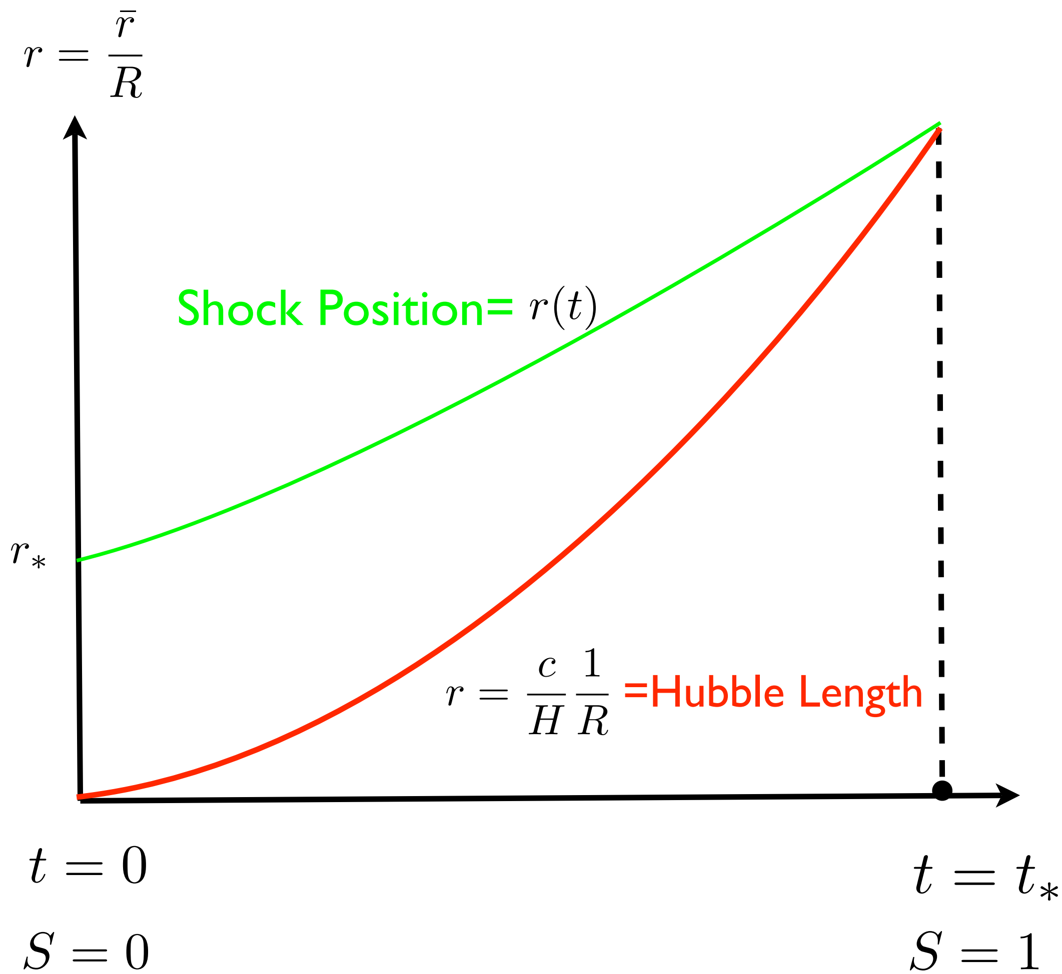
$\sigma = 1/3$



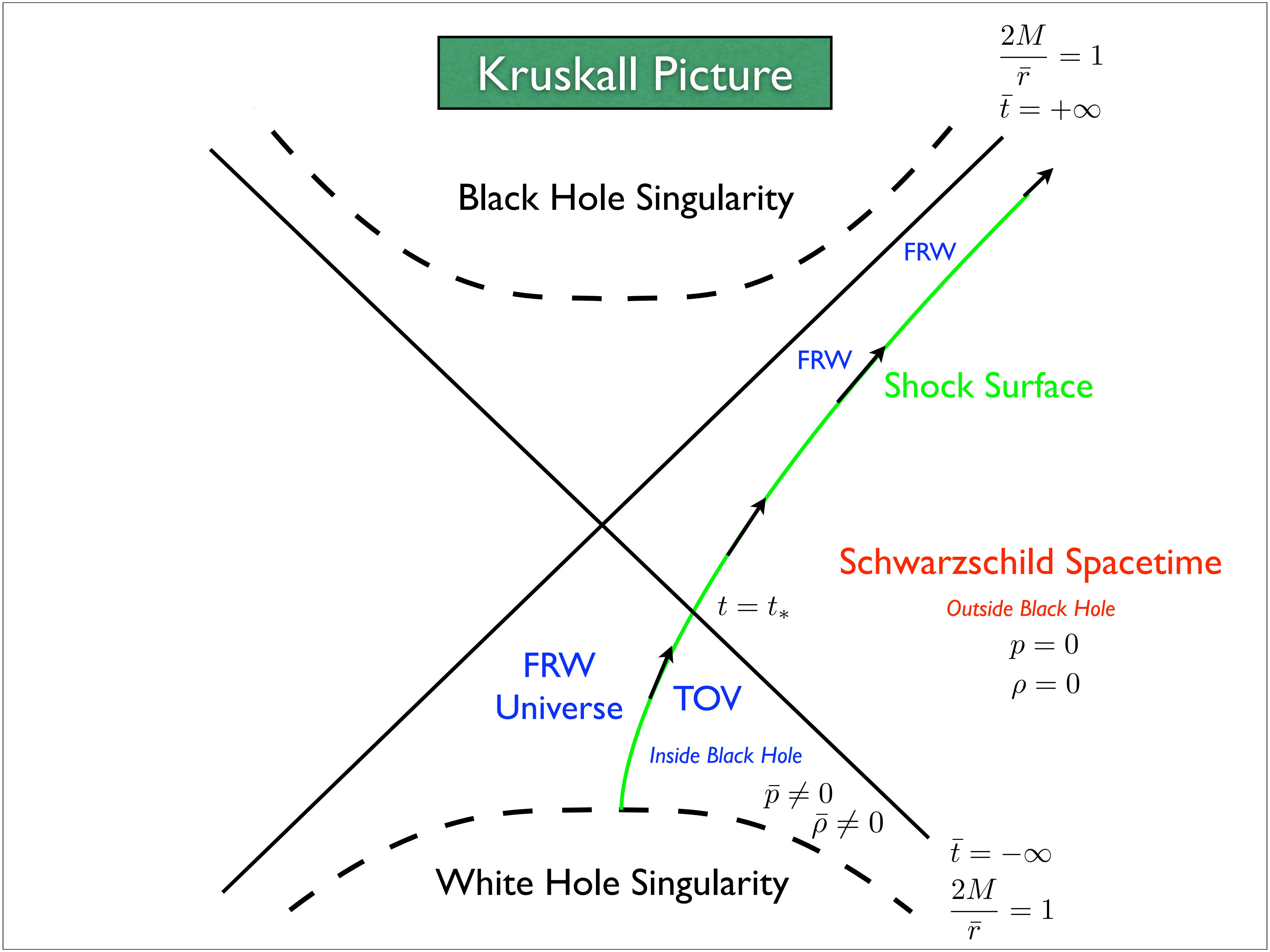
$S = 0$ *Big Bang*

$S = 1$ *Solution emerges from White Hole*

$s_\sigma(S) \to c$ as $S \to 0$



- The Hubble length catches up to the shock-wave at S=1, the time when the entire solution emerges from the White Hole


Kruskall Picture
$\frac{2M}{\bar{r}} = 1$
$\bar{t} = +\infty$
Black Hole Singularity
FRW
FRW
Shock Surface
Schwarzschild Spacetime
Outside Black Hole
$p = 0$
$\rho = 0$
$t = t_*$
FRW
Universe
TOV
Inside Black Hole
$\bar{p} \neq 0$
$\bar{\rho} \neq 0$
$\bar{t} = -\infty$
$\frac{2M}{\bar{r}} = 1$
White Hole Singularity

- OUR QUESTION: How to refine the model to incorporate the correct TOV equation of state, and thereby model the secondary waves in the problem?
- OUR PROPOSAL: Get the initial condition at the end of inflation
- Use the Locally Inertial Glimm Scheme to simulate the region of interaction between the FRW and TOV metrics

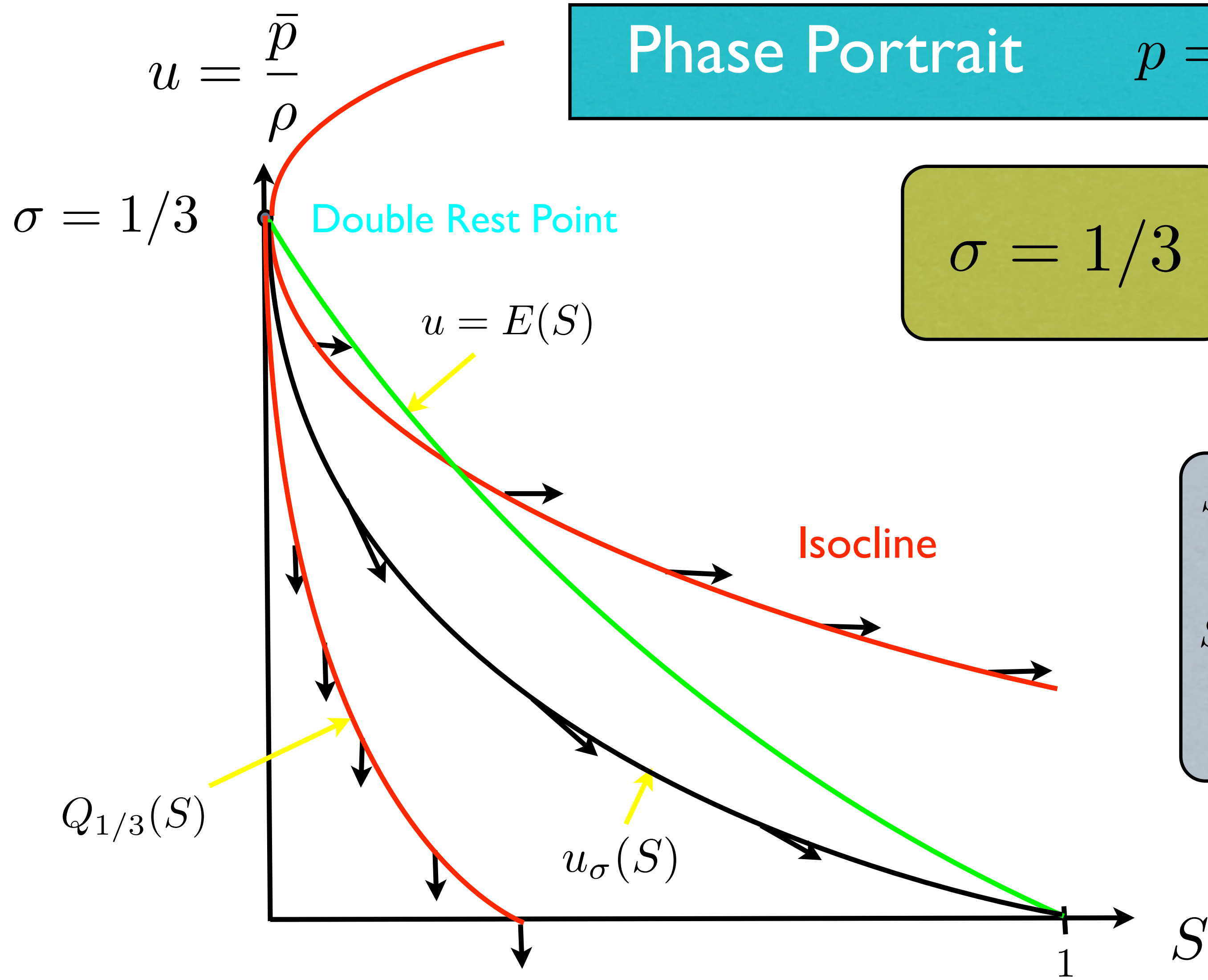
- We are interested in the case $\sigma = 1/3$
  $\approx$ correct for $t = Big\ Bang$ to $t = 10^5 yr$

- ρ and p on the FRW and TOV side tend to the same values as $t \rightarrow 0$

- It is as though the solution is emerging from a spacetime of constant density and pressure at the Big Bang $\approx$ Inflation

Phase Portrait $p = \sigma\rho$

$\sigma = 1/3$



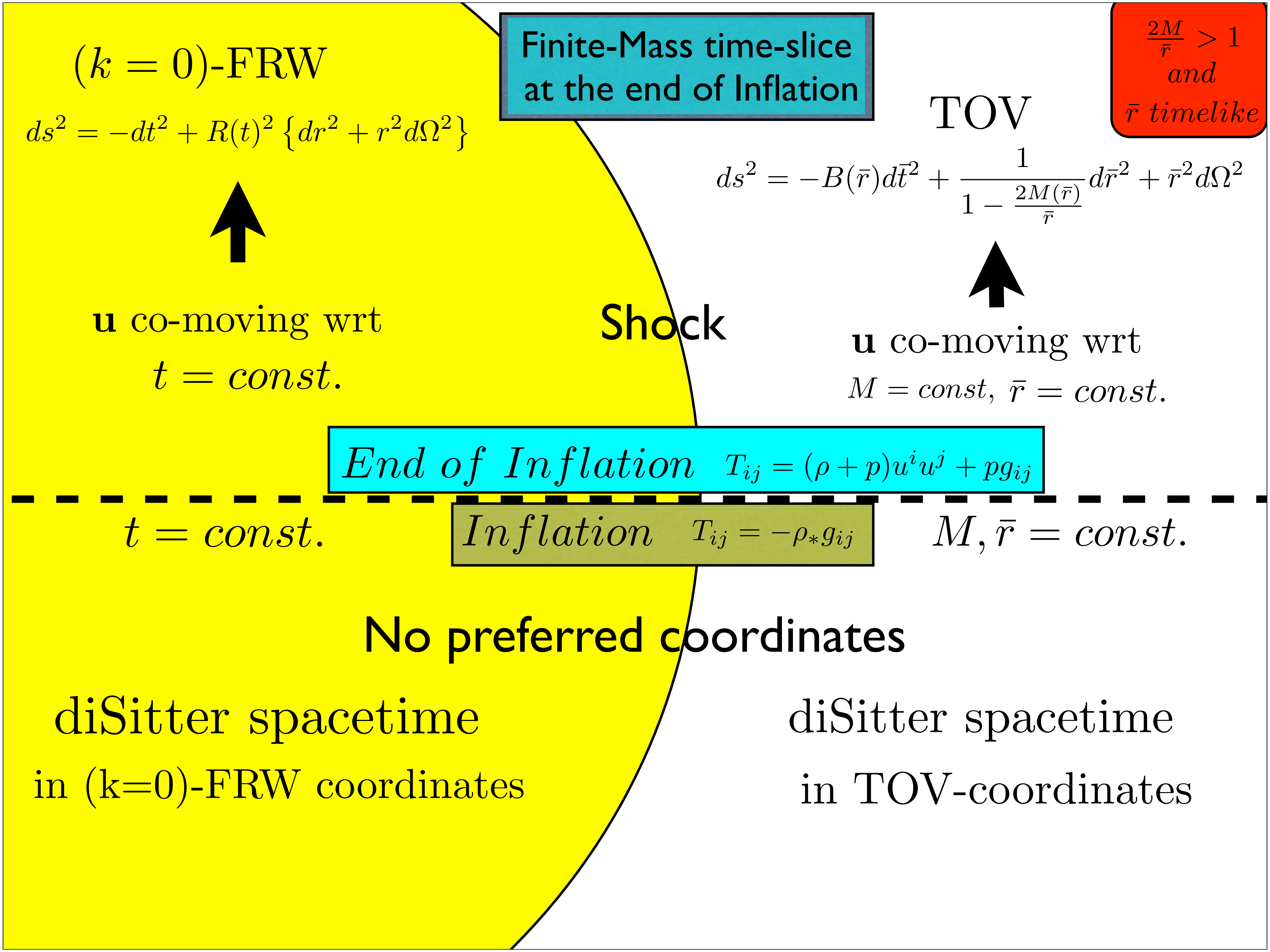
$S = 0$ *Big Bang*

$S = 1$ *Solution emerges from White Hole*

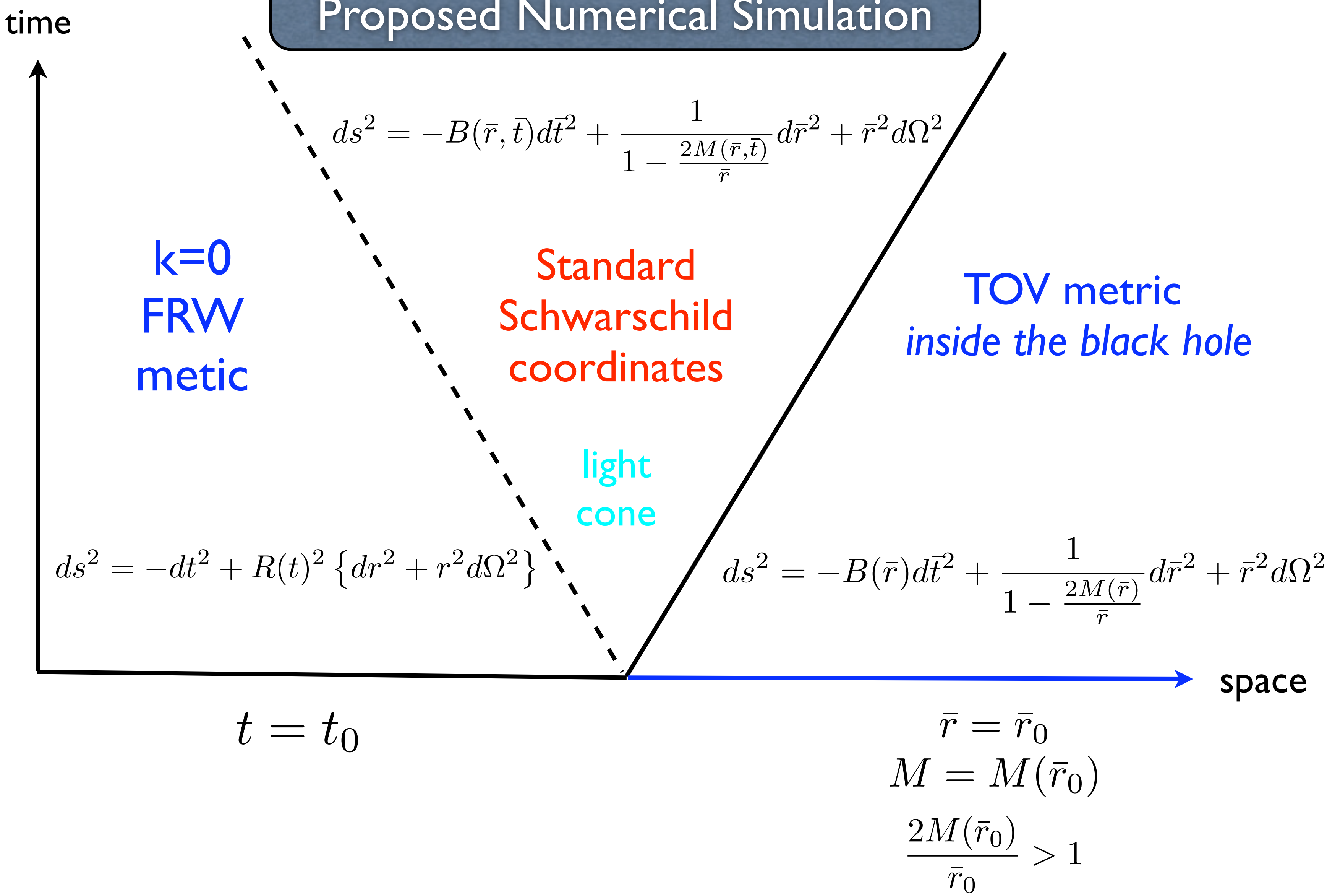
$s_\sigma(S) \to c \quad \text{as} \quad S \to 0$

Conclude: A solution like this would emerge at the end of inflation if the fluid at the end of inflation became co-moving wrt a $(k = 0)$ FRW metric for $\bar{r} < \bar{r}_0$, and co-moving wrt the simplest spacetime of finite total mass for $\bar{r} > \bar{r}_0$.

The inflationary diSitter spacetime has all of the symmetries of a vaccuum, and so there is no preferred frame at the end of inflation


Finite-Mass time-slice at the end of Inflation
$\frac{2M}{\bar{r}} > 1$ and $\bar{r}$ timelike
$(k=0)$-FRW
$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$
TOV
$ds^2 = -B(\bar{r})d\bar{t}^2 + \frac{1}{1-\frac{2M(\bar{r})}{\bar{r}}} d\bar{r}^2 + \bar{r}^2 d\Omega^2$
$\mathbf{u}$ co-moving wrt $t = const.$
Shock
$\mathbf{u}$ co-moving wrt $M = const,\ \bar{r} = const.$
$End\ of\ Inflation$ $T_{ij} = (\rho + p)u^i u^j + p g_{ij}$
$t = const.$
$Inflation$ $T_{ij} = -\rho_* g_{ij}$
$M, \bar{r} = const.$
No preferred coordinates
diSitter spacetime in (k=0)-FRW coordinates
diSitter spacetime in TOV-coordinates

Proposed Numerical Simulation



$$t = \text{the end of inflation} \approx 10^{-30}s = t_0$$

A Locally Inertial Method for Computing Shocks

Einstein equations-Spherical Symmetry

$$ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + r^2\left(d\theta^2 + \sin^2\theta\; d\phi^2\right)$$

$$G = 8\pi T$$

$$\begin{aligned}
\frac{A}{r^2 B}\left\{r\frac{B'}{B} + B - 1\right\} &= \kappa A^2 T^{00} && (1)\\
-\frac{B_t}{rB} &= \kappa A B T^{01} && (2)\\
\frac{1}{r^2}\left\{r\frac{A'}{A} - (B-1)\right\} &= \kappa B^2 T^{11} && (3)\\
-\frac{1}{rAB^2}\left\{B_{tt} - A'' + \Phi\right\} &= \frac{2\kappa r}{B} T^{22}, && (4)
\end{aligned}$$

$$B = \frac{1}{1 - \frac{2M}{r}}$$

$$\begin{aligned}
\Phi = & -\frac{BA_tB_t}{2AB} - \frac{B}{2}\left(\frac{B_t}{B}\right)^2 - \frac{A'}{r} + \frac{AB'}{rB}\\
& + \frac{A}{2}\left(\frac{A'}{A}\right)^2 + \frac{A}{2}\frac{A'}{A}\frac{B'}{B}.
\end{aligned}$$

(1)+(2)+(3)+(4) ⟺ (1)+(3)+div T=0

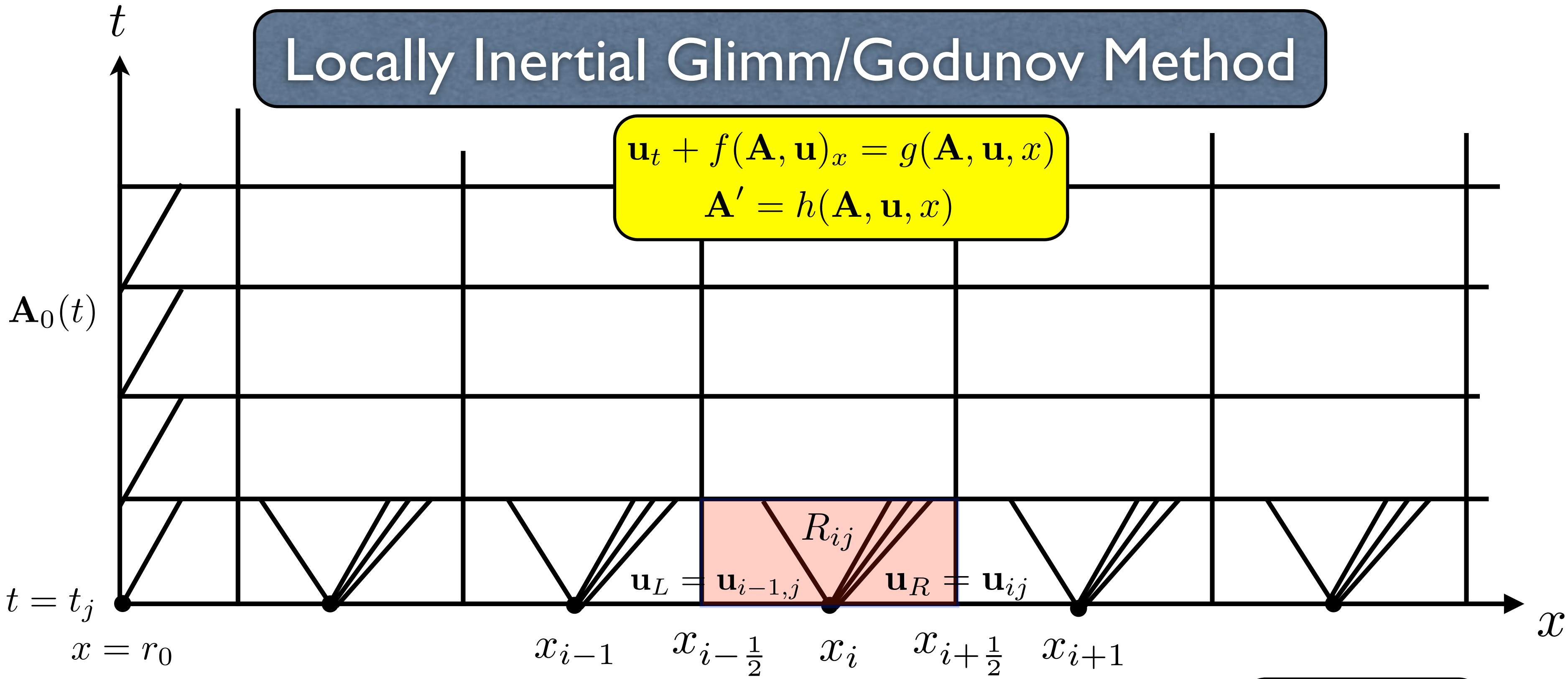
(weakly)

References:

- The locally inertial Glimm Scheme...

- *A shock-wave formulation of the Einstein equations*, with J. Groah, Meth. and Appl. of Anal., **7**, No. 4,(2000), pp. 793-812.
- *Shock-wave solutions of the Einstein equations: Existence and consistency by a locally inertial Glimm Scheme*, with J. Groah, Memoirs of the AMS, Vol. 172, No. 813, November 2004.
- *Shock Wave Interactions in General Relativity: A Locally Inertial Glimm Scheme for Spherically Symmetric Spacetimes*, with J. Groah and J. Smoller, Springer Monographs in Mathematics, 2007.

Locally Inertial Glimm/Godunov Method



$R_{ij}: \quad \mathbf{A} = \mathbf{A}_{ij}$

$\mathbf{A} = (A, B)$

$\mathbf{u} = (u^1, u^2)$

- $\mathbf{A}_{ij} = const. \implies$ Locally Flat in each Grid Cell
- Fractional Step Method: — Solve RP for 1/2 timestep, Solve ODE for 1/2 time step → Sample or Average → $\mathbf{A}' = h(\mathbf{A},\mathbf{u},x)$, $\mathbf{A}(r_0, t_{j+1}) = \mathbf{A}_0(t_{j+1})$
- $p = \frac{c^2}{3}\rho \implies$ Nishida System $\implies$ Global Exact Soln of RP, [Smol,Te]

Remarkable Change of Variables

- Equations close under change to Local Minkowski variables: T $\longrightarrow$ $\mathbf{u}=T_M$

- I.e., Div T=0 reads:

$$0=T^{00}_{,0}+T^{01}_{,1}+\frac{1}{2}\left(\frac{2A_t}{A}+\frac{B_t}{B}\right)T^{00}+\frac{1}{2}\left(\frac{3A'}{A}+\frac{B'}{B}+\frac{4}{r}\right)+\frac{B_t}{2A}T^{11}$$

$$0=T^{01}_{,0}+T^{11}_{,1}+\frac{1}{2}\left(\frac{A_t}{A}+\frac{3B_t}{B}\right)T^{01}+\frac{1}{2}\left(\frac{A'}{A}+\frac{2B'}{B}+\frac{4}{r}\right)T^{11}+\frac{A'}{2B}T^{00}-2\frac{r}{B}T^{22}$$

- Time derivatives A_t and B_t cancel out under change $T\rightarrow\mathbf{u}$

- Good choice because o.w. there is no A_t equation to close $Div\ T=0$!

$$\frac{A}{r^2B}\left\{r\frac{B'}{B}+B-1\right\} = \kappa A^2T^{00} \quad (1)$$
$$-\frac{B_t}{rB} = \kappa ABT^{01} \quad (2)$$
$$\frac{1}{r^2}\left\{r\frac{A'}{A}-(B-1)\right\} = \kappa B^2T^{11} \quad (3)$$
$$-\frac{1}{rAB^2}\left\{B_{tt}-A''+\Phi\right\} = \frac{2\kappa r}{B}T^{22}, \quad (4)$$

Locally Inertial Formulation

$$\{T_M^{00}\}_{,0} + \left\{\sqrt{\frac{A}{B}}T_M^{01}\right\}_{,1} = -\frac{2}{x}\sqrt{\frac{A}{B}}T_M^{01}, \quad (1)$$

$$\{T_M^{01}\}_{,0} + \left\{\sqrt{\frac{A}{B}}T_M^{11}\right\}_{,1} = -\frac{1}{2}\sqrt{\frac{A}{B}}\left\{\frac{4}{x}T_M^{11} + \frac{(B-1)}{x}(T_M^{00} - T_M^{11}) + 2\kappa x B(T_M^{00}T_M^{11} - (T_M^{01})^2) - 4xT^{22}\right\}, \quad (2)$$

$$\frac{B'}{B} = -\frac{(B-1)}{x} + \kappa x B T_M^{00}, \quad (3)$$

$$\frac{A'}{A} = \frac{(B-1)}{x} + \kappa x B T_M^{11}. \quad (4)$$

$$\mathbf{u} = (T_M^{00}, T_M^{01})$$

$$\mathbf{A} = (A, B)$$

$$T_M^{00} = \frac{c^4 + \sigma^2 v^2}{c^2 - v^2}\rho$$

$$T_M^{01} = \frac{c^2 + \sigma^2}{c^2 - v^2}cv\rho$$

$$T_M^{11} = \frac{v^2 + \sigma^2}{c^2 - v^2}\rho c^2$$

Flat Space Relativistic Euler

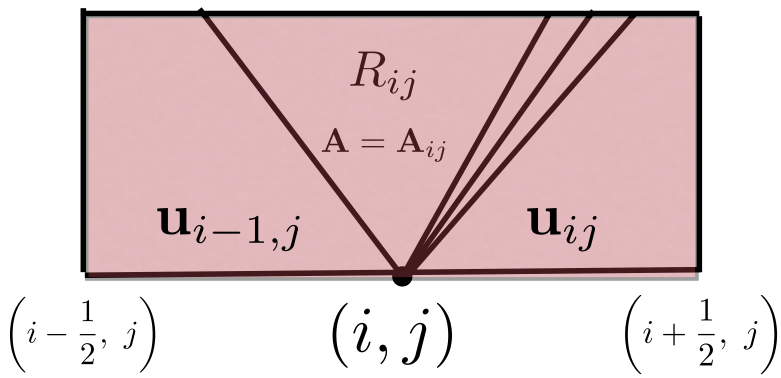
$$\{T_M^{00}\}_{,0} + \{T_M^{10}\}_{,1} = 0$$

$$\{T_M^{01}\}_{,0} + \{T_M^{11}\}_{,1} = 0$$

$$\mathbf{u}_t + f(\mathbf{A}, \mathbf{u})_x = g(\mathbf{A}, \mathbf{u}, x)$$

$$\mathbf{A}' = h(\mathbf{A}, \mathbf{u}, x)$$

Grid Rectangle



- Solve RP for $\frac{1}{2}$-timestep

$$\mathbf{u}_t + f(\mathbf{A}_{ij}, \mathbf{u})_x = 0$$

$$\mathbf{u} = \begin{cases} \mathbf{u}_{i-1,j} & x \le x_i \\ \mathbf{u}_{ij} & x > x_i \end{cases} \quad \rightarrow \quad u_{ij}^{RP}$$

- Solve ODE for $\frac{1}{2}$-timestep

$$\mathbf{u}_t = g(\mathbf{A}_{ij}, \mathbf{u}, x) - \nabla_{\mathbf{A}} f \cdot \mathbf{A}'$$

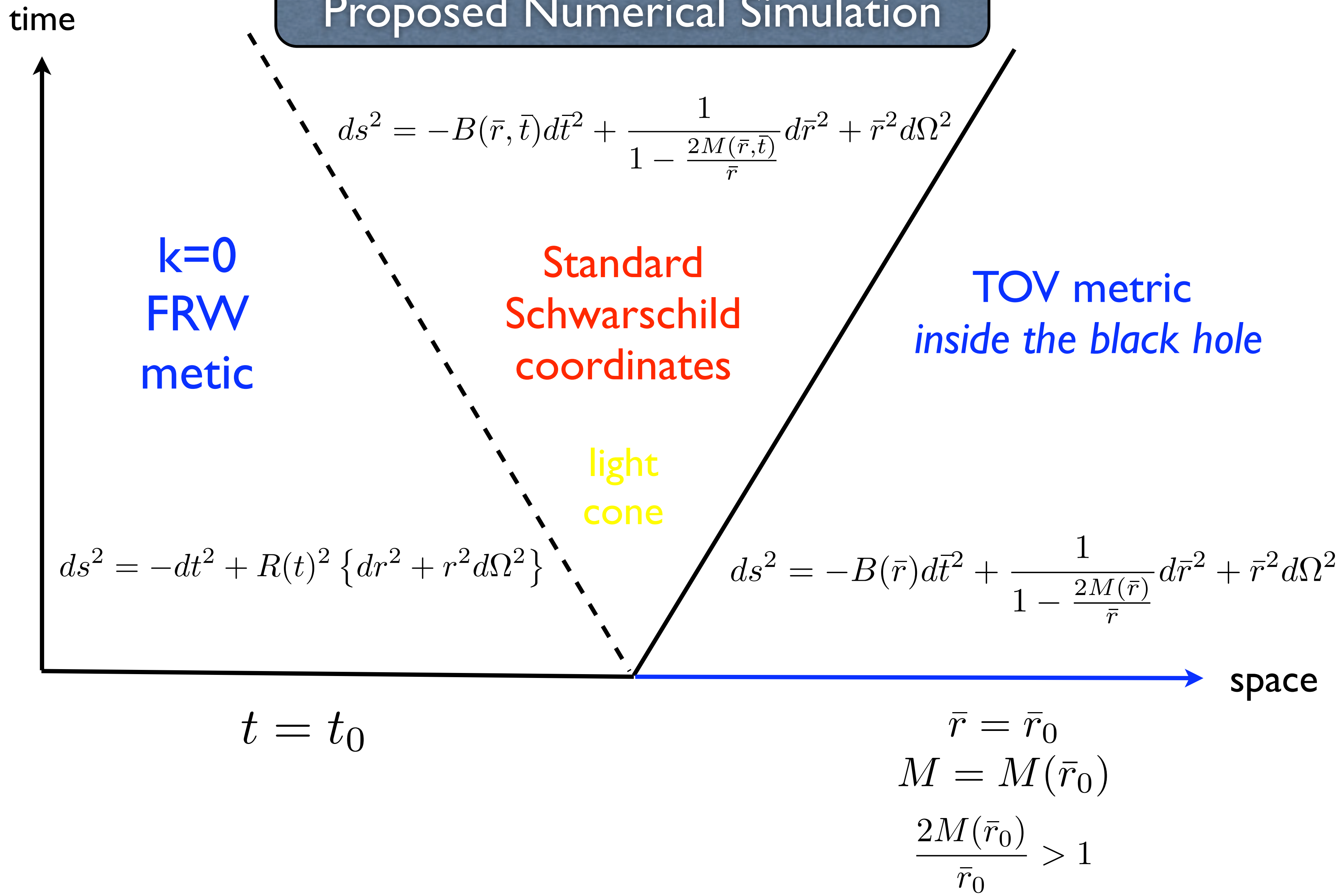
$$\mathbf{u}(0) = \mathbf{u}_{ij}^{RP}$$

- Sample/Average then update $\mathbf{A}$ to time t_{j+1}

$$\mathbf{A}' = h(\mathbf{A}, \mathbf{u}, x)$$

$$\mathbf{A}(r_0, t_{j+1}) = \mathbf{A}_0(t_{j+1})$$

Proposed Numerical Simulation



$$t = \text{the end of inflation} \approx 10^{-30}s = t_0$$

Speculative Question: Could the anomolous acceleration of the Galaxies and Dark Energy be explained within Classical GR as the effect of looking out into a wave?

This model represents the simplest candidate for a simulation of such a wave

Standard Model for Dark Energy

- Assume Einstein equations with a cosmological constant:

$$G_{ij} = 8\pi T_{ij} + \Lambda g_{ij}$$

- Assume $k = 0$ FRW: $$ds^2 = -dt^2 + R(t)^2 \left\{dr^2 + r^2 d\Omega^2\right\}$$

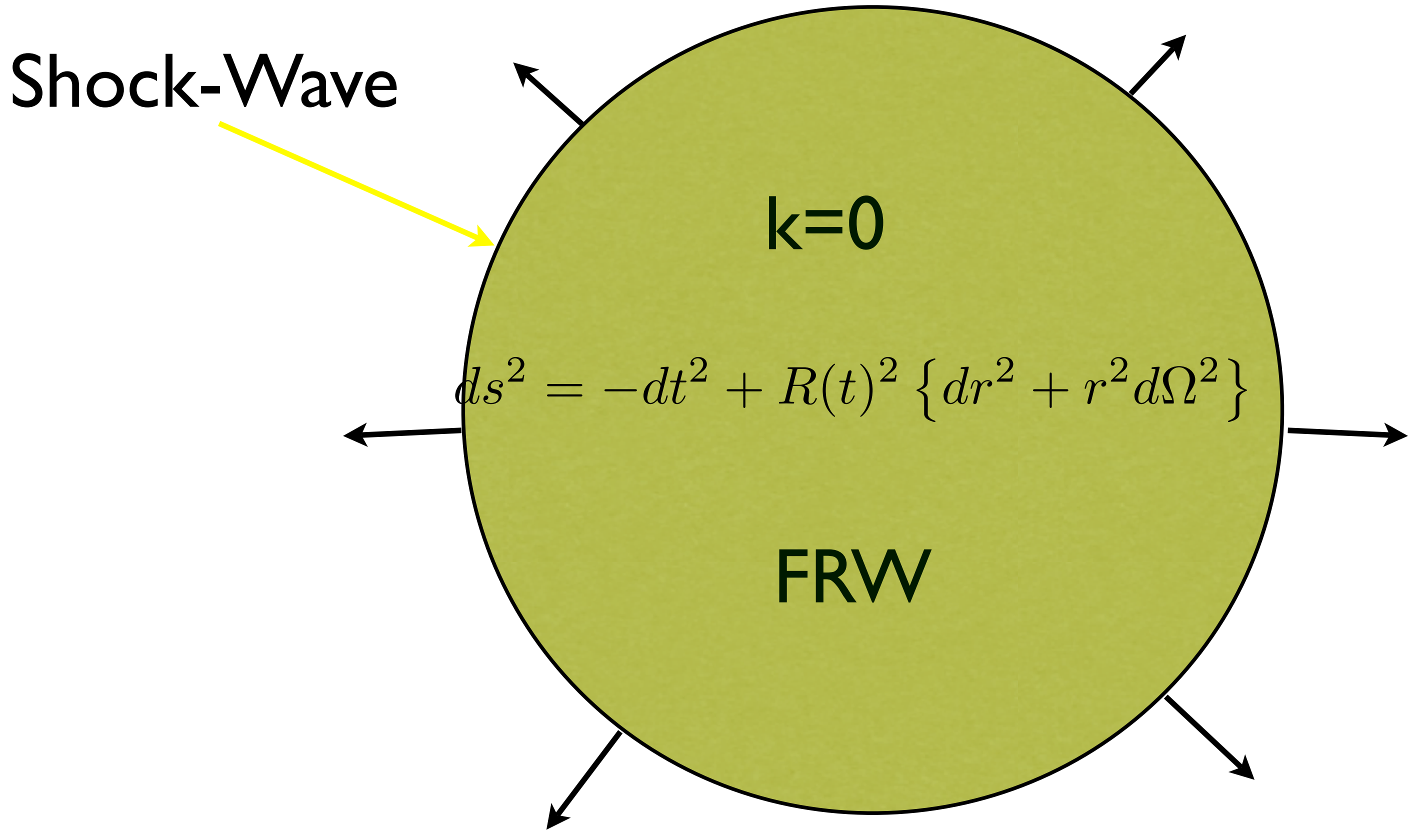
- Leads to: $$H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda$$

- Divide by $H^2 = \rho_{crit}$: $$1 = \Omega_M + \Omega_\Lambda$$

- Best data fit leads to $\Omega_\Lambda \approx .73$ and $\Omega_M \approx .27$

- Implies: The universe is 73 percent dark energy

Could the Anomalous acceleration be accounted for by an expansion behind the Shock Wave?



TOV: $$ds^2 = -B(\bar{r})d\bar{t}^2 + \frac{1}{1 - \frac{2M(\bar{r})}{\bar{r}}} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$\frac{2M}{\bar{r}} > 1$ $\Rightarrow$ $\bar{r}$ is timelike

Conclusion

- We think this numerical proposal represents a natural mathematical starting point for numerically resolving the secondary waves neglected in the exact solution.

- Also a possible starting point for investigating whether the anomalous acceleration ``Dark Energy" could be accounted for within classical GR with classical sources ???