Quiz 2

Lewis MAT 17A, Fall 2006

Name: ______________________________
Section:______________________________

You will have 15 minutes to answer the following problems. You may use any calculator you wish, but all other electronic devices are prohibited. Please show all work clearly, and underline your answers. When possible, indicate what rules, properties, or facts you use to draw your conclusions.

1. (*10 points*) The sequence $\{a_n\}$ is defined via the following recursion relation:

$$a_{n+1} = \frac{1}{2}(a_n + 3) \ ; \ a_0 = 1$$

a) Find values of a_1, a_2, and a_3.
$a_1 = \frac{1}{2}(1+3) = 2, \ a_2 = \frac{1}{2}(2+3) = 5/2, \ a_3 = \frac{1}{2}(5/2+3) = 11/4$
b) Find the fixed point of the sequence. Recall that a fixed point is a number a, such that, if $a_0 = a$, then $a_1 = a$. In general, $a_{n+1} = a_n = a$, for all n. Compare to your answers from part a).
$a = \frac{1}{2}(a+3) \Rightarrow 2a = a + 3 \Rightarrow a = 3.$
i.e., $\lim_{n\to\infty} a_n = 3$

2. (*20 points*) Consider the function:

$$f(x) = \begin{cases} \ln(x+1) \ ; & \text{if} x \geq 0 \\ e^{-x} + a \ ; & \text{if} x < 0 \end{cases}$$

a) Graph the function for $a = 0$, and determine whether $f(x)$ continuous for this choice of a.
b) How must you choose a so that $f(x)$ is continuous for all $x \in (-\infty, \infty)$?
First, let:

$$L_+ = \lim_{x\to 0} \ln(x+1) = 0, \text{ and } L_- = \lim_{x\to 0} e^{-x} + a = 1 + a.$$

For continuity at x=0, we need:

$$L_+ = L_-. \text{This implies}: 0 = 1 + a \Rightarrow a = -1.$$

Thus, for a=-1, f is continuous at $x = 0$. for all other x, f is continuous, since logs and exponentials are continuous, and compositions of continuous functions remain continuous.