Confidence Interval Example

The following data ($n = 10$) were drawn from a normal population

-4.26549, -4.50909, 1.26475, 1.42241, 2.73875, 11.954, 3.61592, -9.68883, -2.96558, -3.48133

The sample mean is

$$\mu_{10} = -0.39145$$

and the sample variance is

$$S_{10}^2 = 35.431$$

We use the Student T-statistic to construct a 95% confidence interval. The first thing we must compute is that value of $t_{\alpha/2}$ so that[1]

$$P\left(-t_{\alpha/2} < T_9 < t_{\alpha/2}\right) = 0.95$$

To do this I used Mathematica

```
<<Statistics`ContinuousDistributions`;
f[t_,n_]:=PDF[StudentTDistribution[n],t];
```

to get the density of T_n. (In our example $n = 10$.) I now used `NIntegrate` to find that value of $t_{\alpha/2}$ so that

$$\int_{-t_{\alpha/2}}^{t_{\alpha/2}} f_T(t, 9)\, dt = 0.95$$

By trying different values for the endpoints I found

$$t_{\alpha/2} = 2.2622$$

Thus the confidence interval is $[a, b]$ where

$$\begin{aligned} a &= \mu_{10} - t_{\alpha/2}\frac{S_{10}}{\sqrt{10}} = -4.65 \\ b &= \mu_{10} + t_{\alpha/2}\frac{S_{10}}{\sqrt{10}} = 3.87 \end{aligned}$$

[1] Recall that when the population is of size n we must use a T-statistic variable with $n - 1$ degrees of freedom.

That is we can say with 95% confidence that the theoretical mean μ lies in the interval $[-4.65, 3.87]$.

The same experiment was repeated but this time for $n = 100$. (I won't display the 100 data points.) Now the sample mean is

$$\mu_{100} = 2.1373$$

and the sample variance is

$$S_{100}^2 = 31.855$$

In this case I found, again for 95% confidence, that

$$t_{\alpha/2} = 1.9842$$

and a confidence interval equal to $[1.02, 3.26]$. This is a dramatically smaller interval. It is due to the fact that n is larger (so the $\sqrt{n}$ in the denominator for a and b makes for a smaller interval as well as the $t_{\alpha/2}$ is a bit smaller for $n = 100$.)