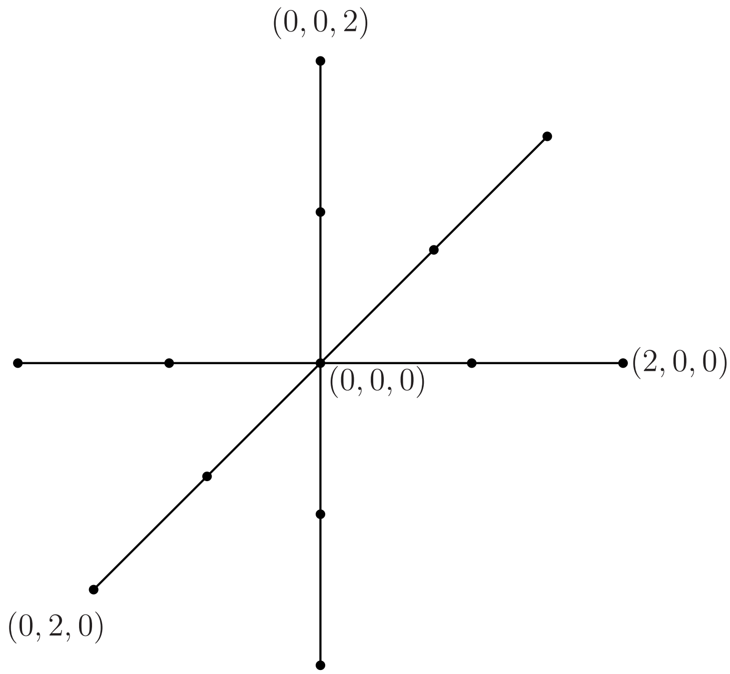
Math 135B: HW #3

#1. Consider random walk on the simple cubic lattice $\mathbb{Z}^3$ where now the probability of transition to one of the six neighboring sites is p and the probability to the next-neighboring sites is q. For example (see picture below), from the site $(0,0,0)$ the walker hops to the sites $(\pm 1,0,0)$, $(0,\pm 1,0)$ and $(0,0,\pm 1)$ with probability p and to the sites $(\pm 2,0,0)$, $(0,\pm 2,0)$ and $(0,0,\pm 2)$ with probability q. Clearly we must have

$$6p + 6q = 1$$

and so $0 \leq p \leq 1/6$.



1. Show that the probability that the walker starting at the origin $(0,0,0)$ will sooner or later return to the origin is given by

$$f(p) = 1 - \frac{1}{g(p)}$$

where

$$g(p) = \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{1 - 2p\sum_{j=1}^{3}\cos(k_j) - 2q\sum_{j=1}^{3}\cos(2k_j)} \, dk_1 dk_2 dk_3 \tag{1}$$

2. Using (1) show that $f(1/6) = f(0)$. Do you expect this? Explain. Hint: What problem corresponds to $p = 1/6$ and what problem corresponds to $p = 0$. Are they really different? Why?

3. As a numerical project, produce a graph of $f = f(p)$ for $0 \leq p \leq 1/6$.