Using *Mathematica* in Linear Algebra

Entering matrices: Suppose we want to enter the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & \frac{1}{2} \\ 1 & 2 & 1 \end{pmatrix}$$

into a *Mathematica* session. We type

```
A={{1,0,3},{-1,2,1/2},{1,2,1}}
```

Determinants and Inverses: The $\det(A)$ is calculated by

```
In[2]:= Det[A]
Out[2]= -11
```

Since $\det(A) \neq 0$ we know A has an inverse. Let us have *Mathematica* calculate it for us:

```
In[3]:= Inverse[A]
Out[3]= {{-(1/11),-(6/11),6/11},{-(3/22),2/11,7/22},{4/11,2/11,-(2/11)}}
```

You can have the result displayed in matrix form if you use the command `MatrixForm`.

Characteristic Polynomial: You can calculate the characteristic polynomial $p(x) = \det(A - xI)$

```
In[13]:= CharacteristicPolynomial[A,x]
Out[13]= -11-x+4 x^2-x^3
```

You can use the `Solve` command to find the three roots of this cubic equation. The result is rather messy so we settle for their numerical values and use the command `NSolve`

```
In[18]:= NSolve[Out[13]==0,x]
Out[18]= {{x->-1.34417},{x->2.67209-1.02149 I},{x->2.67209+1.02149 I}}
```

Note that in my *Mathematica* session the characteristic polynomial was `Out[13]`, so rather than retype the result I used its label. In *Mathematica* the symbol `I` is used for the imaginary number i, $i^2 = -1$.

Eigenvalues and Eigenvectors: Once you have entered a matrix A you can have *Mathematica* try to find the eigenvalues and eigenvectors. The commands are `Eigenvalues[A]` and `Eigenvectors[A]`. Since this cannot always be done in closed form, you may have to settle for their numerical values. For the above matrix A

```
In[19]:= N[Eigenvalues[A]]
Out[19]= {2.67209+1.02149 I,2.67209-1.02149 I,-1.34417}

In[20]:= N[Eigenvectors[A]]
Out[20]= {{1.30655-0.798182 I,0.182767+0.909835 I,1.},
{1.30655+0.798182 I,0.182767-0.909835 I,1.},
{-1.27977,-0.532201,1.}}
```

Matrix Exponential: There is even a command that attempts to compute the exponential of the matrix A. The command is `MatrixExp[A]`. Many times you have to do this numerically: `N[MatrixExp[A]]`.