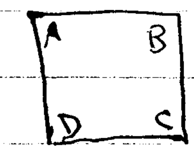
HW #0

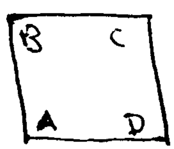
Try to complete this by Monday.

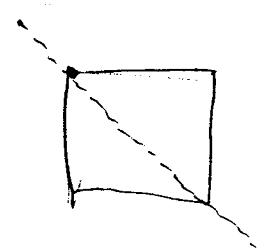
1. (We will do most of this together in class)

a.) With a sheet of paper, make a square, labelled (front + back) compatibly

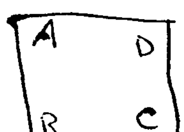


b.) You can move the square however you like so you still see a square in front of you.
For instance, rotating 90 degrees counterclockwise you get



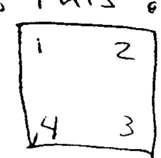


or flipping over the main diagonal you get

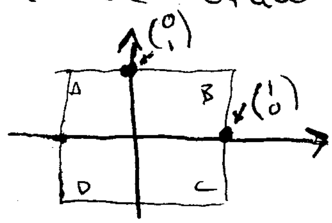


* How many different ways can you do this?

c.) Listing the vertices in order



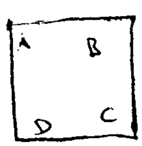
make a list of all possibilities, ie. ABCD, BCDA, ADCB .... etc d.) Notice in fact all of these movements can be seen as linear operators on $\mathbb{R}^2$ if we draw in x- and y-axes



Recall, given a linear operator T, we write its matrix as $\begin{pmatrix} x & z \\ y & w \end{pmatrix}$ if

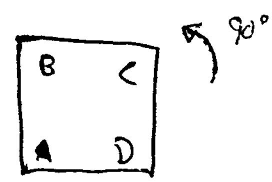
$$T\binom{1}{0} = \binom{x}{y} \quad \text{and} \quad T\binom{0}{1} = \binom{z}{w}.$$

So for instance starting from



ABCD

and moving to



BCDA

the corresponding T sends $\rightarrow \binom{1}{0}$ to $\uparrow \binom{0}{1}$ and $\uparrow \binom{0}{1}$ to $\binom{-1}{0} \leftarrow$

so has matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

For your list from part c.) write beneath each "word" its corresponding matrix

i.e. $\underset{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}{\text{A B C D}}$, $\underset{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}{\text{B C D A}}$, ...

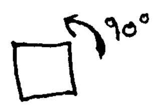
It will help you to have a third row for this list that reminds you what the "movement" is that gets you there from ABCD.

For instance

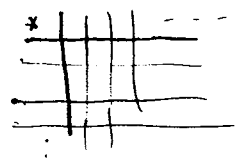
ABCD , BCDA , ADCB , ...

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\begin{pmatrix} & \end{pmatrix}$

nothing



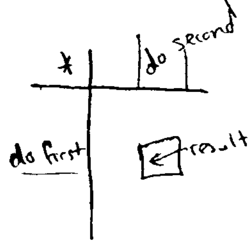
e.) Make the "multiplication table" (a larger version of



)

where you label the rows/columns consistently with your favorite of the 3 ways from part d.).
In class, we will agree upon an order so you can compare answers with neighors.

Also



f.) Note each column of your table should be a complete list of movements/words/ matrices.

Also, each motion has a unique motion that undoes it. (Check this).

Also, if you used matrices, compare your table to the one you get from matrix multiplication.

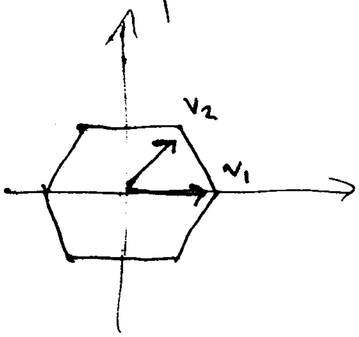
g.) List the determinants of all the matrices. What do all the matrices with det = -1 have in common (or really, their related motions)? For det = +1 ?

h.) Can you find any smaller subsets of the matrices closed under multiplication?

i) Can only one matrix and all its powers generate all 8 ?
Can you find two so that all their products generate all 8? Which two?

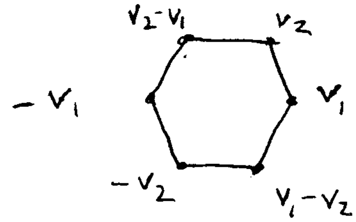
g) Do a.) - i.) for a regular hexagon.

For part d.) use the basis



where $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} \cos \pi/3 \\ \sin \pi/3 \end{pmatrix}$

Notice then all vertices are given as linear combos of v_1, v_2 via

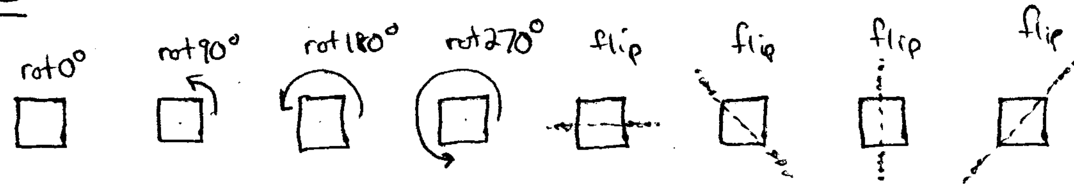


Recall the matrix of T is $\begin{pmatrix} x & z \\ y & w \end{pmatrix}$ if $Tv_1 = xv_1 + yv_2$, $Tv_2 = zv_1 + wv_2$.

Comments on HW0 (MAT 150A - F03)

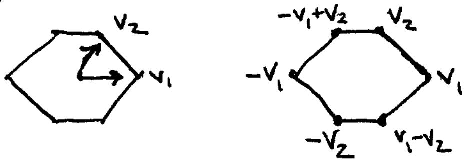
Square

rot0°	rot90°	rot180°	rot270°	flip	flip	flip	flip

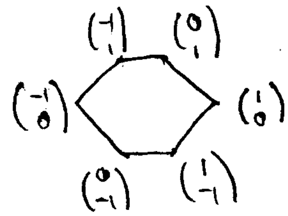


$$\begin{pmatrix}1&0\\0&1\end{pmatrix}\quad \begin{pmatrix}0&-1\\1&0\end{pmatrix}\quad \begin{pmatrix}-1&\\&-1\end{pmatrix}\quad \begin{pmatrix}0&1\\-1&0\end{pmatrix}\quad \begin{pmatrix}1&0\\0&-1\end{pmatrix}\quad \begin{pmatrix}0&-1\\-1&0\end{pmatrix}\quad \begin{pmatrix}-1&0\\0&1\end{pmatrix}\quad \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

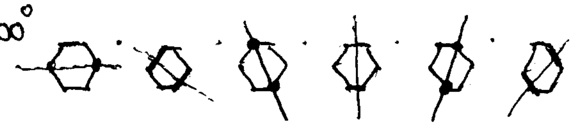
hexagon



in $\{\underline{v_1}, \underline{v_2}\}$ coordinates



rot0°   rot60°   rot120°   rot180°   rot240°   rot300°



$$\begin{pmatrix}1&\\&1\end{pmatrix}\quad \begin{pmatrix}0&-1\\1&1\end{pmatrix}\quad \begin{pmatrix}-1&-1\\1&0\end{pmatrix}\quad \begin{pmatrix}-1&0\\0&-1\end{pmatrix}\quad \begin{pmatrix}0&1\\-1&-1\end{pmatrix}\quad \begin{pmatrix}1&1\\-1&0\end{pmatrix}\quad \begin{pmatrix}1&1\\0&-1\end{pmatrix}\quad \begin{pmatrix}1&0\\-1&-1\end{pmatrix}\quad \begin{pmatrix}0&-1\\-1&0\end{pmatrix}\quad \begin{pmatrix}-1&-1\\0&1\end{pmatrix}\quad \begin{pmatrix}-1&0\\1&1\end{pmatrix}\quad \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$\boxed{\text{Linear Algebra POV}}$

Why? Given a point in the plane $\underline{w}$ we'll write $[\underline{w}]_v = \begin{pmatrix}a\\b\end{pmatrix}$ if $\underline{w} = a\underline{v_1} + b\underline{v_2}$. Given a linear transformation T, write $[T]_v$ for the matrix such that

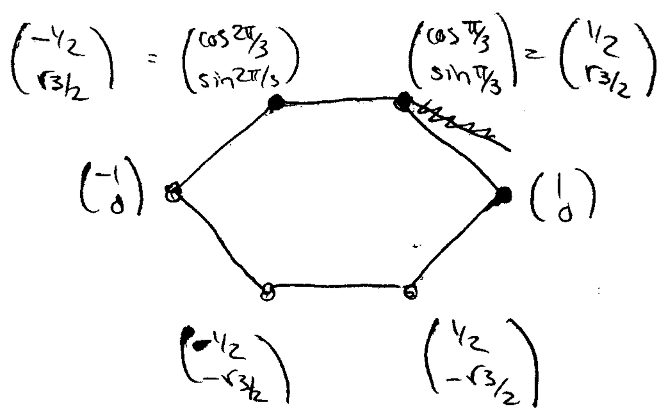
$$[T]_v \begin{pmatrix}a\\b\end{pmatrix} = [T(\underline{w})]_v .$$ Since $T(a\underline{v_1}+b\underline{v_2}) = aT(\underline{v_1}) + bT(\underline{v_2})$

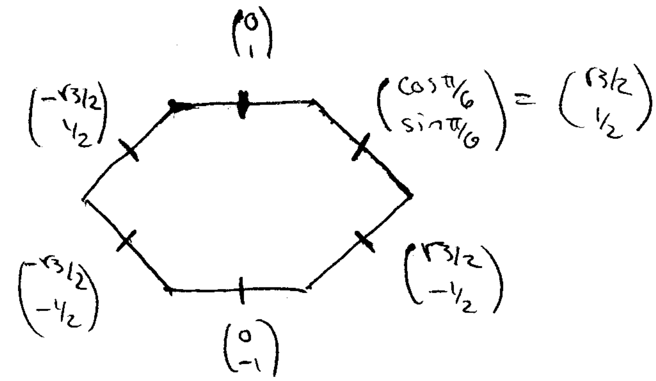
we need only compute $T(v_1)$ and $T(v_2)$.

Then since $[\underline{v_1}]_v = \begin{pmatrix}1\\0\end{pmatrix}$ and $[\underline{v_2}]_v = \begin{pmatrix}0\\1\end{pmatrix}$ and $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$; $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}b\\d\end{pmatrix}$

We just need to see where T sends $\underline{v_1}$ and $\underline{v_2}$, and write it as the columns in $\{\underline{v_1}, \underline{v_2}\}$ coordinates.

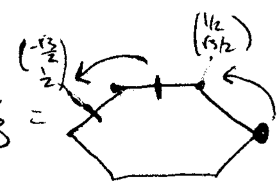
If you do this in the standard basis then you need to write the coordinates of all vertices <u>and</u> midpoints of edges





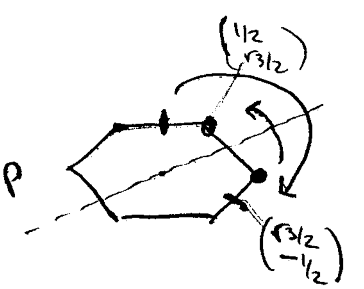
Then the matrix of T has (in the standard basis) columns $\left[T\begin{pmatrix} 1 \\ 0 \end{pmatrix} \middle| T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$

So for ex: $\text{rot}\,60^\circ = \text{rot}\,\pi/3 =$



$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$

And flip



$= \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}.$

You can check $\det(\text{rot}) = 1$, $\det(\text{flip}) = -1$.

How messy compared to the other basis!

One can also go back and forth btwn bases conjugating by $A = \begin{bmatrix} 1 & 1/2 \\ 0 & \sqrt{3}/2 \end{bmatrix}$

(Verify: $A \overset{\text{rot}60^\circ}{\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}} A^{-1} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$; and $A \overset{\text{flip}}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} A^{-1} = \overset{\text{flip}}{\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}}$)