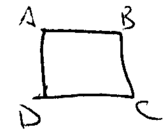
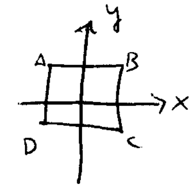
Handout 2

Math 150A
October '06

Cosets, Conjugation, Cycle Notation for S_n

① Consider the square  in the plane 

and the map $T : D_4 \to \mathbb{R}^2$
$$g \mapsto g(A)$$

(Is T a homomorphism?)

a.) What is $T(r)$? What is $\{T(g) \mid g \in D_4\}$?

Recall, $r = \text{rot } \frac{\pi}{2}$, $f = $ flip horizontal 

b.) What is $T^{-1}(A)$?

$T^{-1}(B)$? $T^{-1}(C)$? $T^{-1}(D)$?

c.) Are any of your answers to part b.) a subgroup of D_4?

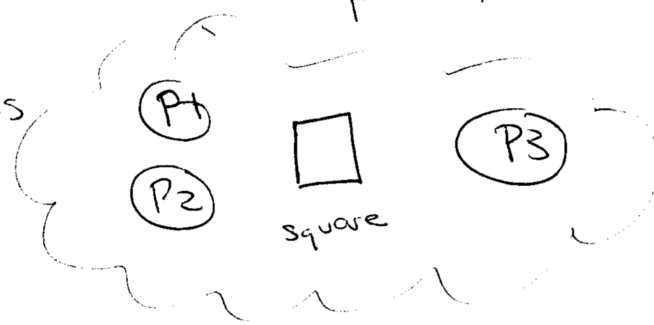
d.) For your answer(s) to part c.), compute its left cosets.
(If the answer to c) is subgroup H, compute D_4/H)

e.) Compare d.) to b.)

(2) Conjugation.

Get into groups of 3 or more.
Let's say your group is P1 P2 P3
"person 1" ....

Stand as follows

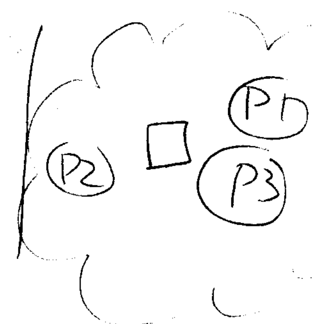


"starting position"

a.) Have P1 execute $r \in D_4$ to the square.

What motion does P3 see executed?
What does P2 see?

b.) Now have P1 walk to join P3



have P1 execute $r \in D_4$,
and now have P1 walk back
to starting position.

What motion does P3 see executed?
" " P2 " " ?

c.) In starting position, have P1 execute

$$(r^2 f)\, r\, (r^2 f)^{-1}$$

recalling $r^2 f =$  the vertical flip.

2c cont.
What does P2 see executed? P3?

d.) How does c) relate to a.) and b.)?

e.) Using the ideas above
analyze

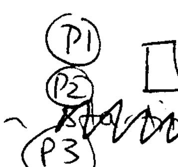
$$f\,(\mathrm{rot}\,\Theta)\,f^{-1}$$

where Θ is <u>any</u> angle (rotated counterclockwise).

3

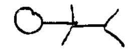
a.) What is rfr^{-1}? (which motion?)
$r(rf)r^{-1}$?

b.) Similar to above, have (in ~~starting~~ position)



P1 execute f.
What does P2 see?

c.) Now have P1 turn his/her head
"counterclockwise" $\frac{\pi}{2}$



and execute f (from his/her POV of what horizontal means).

What does P2 see?

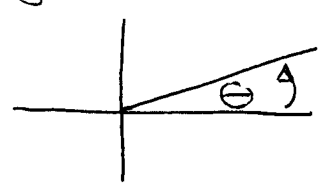
3 d. Compare a) & c).

Note, if P1 rotates head $\pi/2$, it's like P1 staying upright and rotating square $-\pi/2$ (doing r^{-1}).

e.) Using the ideas above, analyze

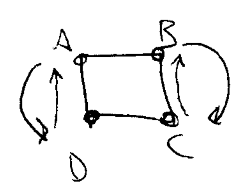
$$F = \text{rot}(\theta)\ f\ \text{rot}(\theta)^{-1}$$
$$= \text{rot}(\theta)\ f\ \text{rot}(-\theta)$$

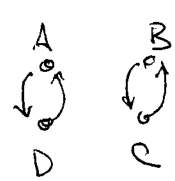
Where θ is any angle.

It is a flip. If 

this is θ, draw the line that F flips over on this).
(Recall, f flips over the x-axis )

4 Cycles

We could also depict f by 

or 

.

Replace A goes to D goes to A by (AD).
Note (AD) = (DA). Then f = (AD)(BC)

(4) con't

Also $f = (DA)(BC) = (BC)(AD)$ etc. . .

Similarly, we depict $r =$ 

as $(A\,D\,C\,B)$

Let's call this "cycle notation" for r. (wrt labelling 

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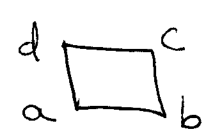
a) compute the cycle notation for frf^{-1}.

b) Suppose we had relabelled the square as 

Compute cycle notation wrt (with respect to) this labelling.

c.) Suppose $r(A)$ for where r sends vertex A.
i.e. $r(A) = D$. And similarly $f(A) = D$.

Rewrite $(\,f(A)\ f(D)\ f(C)\ f(B)\,)$.

Compare to a.), b).

5 a.) Now compute rfr^{-1}, in cycle notation wrt .

b.) Compute cycle notation for f wrt the labelling 

c.) Rewrite
$$(r(A)\ r(D))(r(B)\ r(C))$$

Compare to a.), b.).

6 Think about the following:

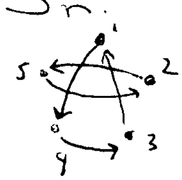
a) If σ is any permutation of the letters A, B, C, D

then
$$\sigma \circ r \circ \sigma^{-1} \text{ has cycle notation } (\sigma(A)\ \sigma(D)\ \sigma(C)\ \sigma(B)).$$

(First, note it is true for $\sigma = \text{identity}$)

b.) $\sigma \circ f \circ \sigma^{-1}$ has cycle notation $(\sigma(A)\ \sigma(D))(\sigma(B)\ \sigma(C))$.

7 We can do this for S_n.

ex/ If $n=5$ $\quad p: \begin{array}{l} 1 \mapsto 4 \\ 2 \mapsto 5 \\ 3 \mapsto 1 \\ 4 \mapsto 3 \\ 5 \mapsto 2 \end{array}$  $p = (143)(25)$

a.) Write cycle notation for q: $\begin{array}{l} 1 \mapsto 5 \\ 2 \mapsto 3 \\ 3 \mapsto 2 \\ 4 \mapsto 4 \\ 5 \mapsto 1 \end{array}$. What is qpq^{-1}?