Beyond the unitarity bound in AdS/CFT

Tomás Andrade
in collaboration with
T. Faulkner, J. Jottar, R. Leigh, D. Marolf, C. Uhlemann

October 5th, 2011

Introduction 1

AdS/CFT relates the dynamics of fields in AdS_{d+1} to that of operators in a CFT_d that lives on the boundary. [Maldacena, Witten, Gubser-Klebanov-Polyakov]

Introduction 1

AdS/CFT relates the dynamics of fields in AdS_{d+1} to that of operators in a CFT_d that lives on the boundary. [Maldacena, Witten, Gubser-Klebanov-Polyakov]

In some limit, bulk is classical and the boundary is quantum: Einstein's equations "know" QFT!

Introduction 1

AdS/CFT relates the dynamics of fields in AdS_{d+1} to that of operators in a CFT_d that lives on the boundary. [Maldacena, Witten, Gubser-Klebanov-Polyakov]

In some limit, bulk is classical and the boundary is quantum: Einstein's equations "know" QFT!

Symmetries:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\eta_{ij} dx^i dx^j$$

Isometries of AdS $\Leftrightarrow$ global sym. group of the QFT $SO(d,2)$.

Introduction 1

AdS/CFT relates the dynamics of fields in AdS_{d+1} to that of operators in a CFT_d that lives on the boundary. [Maldacena, Witten, Gubser-Klebanov-Polyakov]

In some limit, bulk is classical and the boundary is quantum: Einstein's equations "know" QFT!

Symmetries:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\eta_{ij} dx^i dx^j$$

Isometries of AdS $\Leftrightarrow$ global sym. group of the QFT $SO(d, 2)$.

Precise dictionary of various quantities in each side

$$e^{-I_{AdS}[\phi^{(0)}]} = \langle e^{\int \phi^{(0)} \mathcal{O}} \rangle_{CFT}$$

Introduction 2

For a scalar field of mass m^2 in AdS_{d+1}, there are in principle two dual operators of dimension

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{m^2 - m_{BF}^2} \quad m_{BF}^2 = -\frac{d^2}{4}$$

Introduction 2

For a scalar field of mass m^2 in AdS_{d+1}, there are in principle two dual operators of dimension

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{m^2 - m_{BF}^2} \quad m_{BF}^2 = -\frac{d^2}{4}$$

BF stability bound: tachyonic masses are allowed as long as $m^2 > m_{BF}^2$.

Introduction 2

For a scalar field of mass m^2 in AdS_{d+1}, there are in principle two dual operators of dimension

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{m^2 - m_{BF}^2} \quad m_{BF}^2 = -\frac{d^2}{4}$$

BF stability bound: tachyonic masses are allowed as long as $m^2 > m_{BF}^2$.

The boundary conditions play a crucial role: they select between Δ_-/Δ_+ (determine the operator content)

Introduction 2

For a scalar field of mass m^2 in AdS_{d+1}, there are in principle two dual operators of dimension

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{m^2 - m_{BF}^2} \quad m_{BF}^2 = -\frac{d^2}{4}$$

BF stability bound: tachyonic masses are allowed as long as $m^2 > m_{BF}^2$.

The boundary conditions play a crucial role: they select between Δ_-/Δ_+ (determine the operator content)

If $m^2 > m_{BF}^2 + 1$ we have $\Delta_- < \frac{d}{2} - 1$ in conflict with unitarity bound for scalar operators, so we expect "something" to go wrong.

Introduction 2

For a scalar field of mass m^2 in AdS_{d+1}, there are in principle two dual operators of dimension

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{m^2 - m_{BF}^2} \quad m_{BF}^2 = -\frac{d^2}{4}$$

BF stability bound: tachyonic masses are allowed as long as $m^2 > m_{BF}^2$.

The boundary conditions play a crucial role: they select between Δ_-/Δ_+ (determine the operator content)

If $m^2 > m_{BF}^2 + 1$ we have $\Delta_- < \frac{d}{2} - 1$ in conflict with unitarity bound for scalar operators, so we expect "something" to go wrong.

But what exactly happens in the bulk if we go below the bound?
arXiv:1105.6337 (DM and TA)

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product
- Spectrum, norms and 2-point function

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product
- Spectrum, norms and 2-point function
- IR deformation

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product
- Spectrum, norms and 2-point function
- IR deformation
- Extension of the results

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product
- Spectrum, norms and 2-point function
- IR deformation
- Extension of the results
- UV modification

Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
- Inner product
- Spectrum, norms and 2-point function
- IR deformation
- Extension of the results
- UV modification
- Conclusions

Set up

Take a scalar field with mass $m^2 = m_{BF}^2 + \nu^2$ with $1 < \nu < 2$ in AdS_{d+1}

$$I_0 = -\frac{1}{2}\int_M \sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2],$$

Set up

Take a scalar field with mass $m^2 = m^2_{BF} + \nu^2$ with $1 < \nu < 2$ in AdS_{d+1}

$$I_0 = -\frac{1}{2}\int_M \sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2],$$

$$\phi = r^{d/2-\nu}(\phi^{(0)} + r^2\phi^{(1)} + r^{2\nu}\phi^{(\nu)} + \ldots) \qquad \phi^{(1)} = \frac{1}{4(\nu-1)}\Box_0\phi^{(0)}$$

Set up

Take a scalar field with mass $m^2 = m_{BF}^2 + \nu^2$ with $1 < \nu < 2$ in AdS_{d+1}

$$I_0 = -\frac{1}{2}\int_M \sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2],$$

$$\phi = r^{d/2-\nu}(\phi^{(0)} + r^2\phi^{(1)} + r^{2\nu}\phi^{(\nu)} + \ldots) \qquad \phi^{(1)} = \frac{1}{4(\nu-1)}\Box_0\phi^{(0)}$$

Impose the (N) boundary condition $\phi^{(\nu)} = 0$, so the dynamical operator has dimension $\Delta_- = d/2 - \nu$.

Set up

Take a scalar field with mass $m^2 = m_{BF}^2 + \nu^2$ with $1 < \nu < 2$ in AdS_{d+1}

$$I_0 = -\frac{1}{2}\int_M \sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2],$$

$$\phi = r^{d/2-\nu}(\phi^{(0)} + r^2\phi^{(1)} + r^{2\nu}\phi^{(\nu)} + \dots) \qquad \phi^{(1)} = \frac{1}{4(\nu-1)}\Box_0\phi^{(0)}$$

Impose the (N) boundary condition $\phi^{(\nu)} = 0$, so the dynamical operator has dimension $\Delta_- = d/2 - \nu$. We take the action to be

$$I_N = I_0 + \int_{\partial M}\sqrt{\gamma}\left[\rho_\mu\partial^\mu\phi\phi - \frac{1}{2}(d/2-\nu)\phi^2 + \frac{1}{4(\nu-1)}\gamma^{ij}\partial_i\phi\partial_j\phi\right],$$

which satisfies $\delta I_N = \int_{\partial M}\phi^{(0)}\delta\phi^{(\nu)}$.

Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{bulk} = -i \int_\Sigma \sqrt{g_\Sigma} n^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2$$

Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{bulk} = -i \int_\Sigma \sqrt{g_\Sigma} n^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2$$

diverges.

Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{bulk} = -i \int_\Sigma \sqrt{g_\Sigma} n^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2$$

diverges. Using the prescription of arXiv:0805.1902 [GC and DM], we can construct a finite and conserved i.p. (valid for $1 < \nu < 2$)

$$(\phi_1, \phi_2)_{ren} = (\phi_1, \phi_2)_{bulk} - \frac{1}{2(\nu - 1)} (\phi_1, \phi_2)_{bndy}$$

Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{bulk} = -i \int_\Sigma \sqrt{g_\Sigma} n^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2$$

diverges. Using the prescription of arXiv:0805.1902 [GC and DM], we can construct a finite and conserved i.p. (valid for $1 < \nu < 2$)

$$(\phi_1, \phi_2)_{ren} = (\phi_1, \phi_2)_{bulk} - \frac{1}{2(\nu - 1)}(\phi_1, \phi_2)_{bndy}$$

Not manifestly positive definite! expect ghosts in the bulk as a result of $\Delta_- < d/2 - 1$.

Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{bulk} = -i \int_\Sigma \sqrt{g_\Sigma} n^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2$$

diverges. Using the prescription of arXiv:0805.1902 [GC and DM], we can construct a finite and conserved i.p. (valid for $1 < \nu < 2$)

$$(\phi_1, \phi_2)_{ren} = (\phi_1, \phi_2)_{bulk} - \frac{1}{2(\nu - 1)}(\phi_1, \phi_2)_{bndy}$$

Not manifestly positive definite! expect ghosts in the bulk as a result of $\Delta_- < d/2 - 1$.

To do: find spectrum, compute norms and look for ghosts. Include D results for comparison.

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m_{bndy}^2 = -k_i k^i$, we have

- $m_{bndy}^2 > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's
- $m^2_{bndy} = 0$: $\psi = r^{d/2\pm\nu}$. For N bc's, becomes normalizable when $\nu > 1$.

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's
- $m^2_{bndy} = 0$: $\psi = r^{d/2\pm\nu}$. For N bc's, becomes normalizable when $\nu > 1$. Pure gauge

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's
- $m^2_{bndy} = 0$: $\psi = r^{d/2\pm\nu}$. For N bc's, becomes normalizable when $\nu > 1$. Pure gauge

For N bc's and $\nu > 1$ do not find negative norms...how is this consistent?

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's
- $m^2_{bndy} = 0$: $\psi = r^{d/2\pm\nu}$. For N bc's, becomes normalizable when $\nu > 1$. Pure gauge

For N bc's and $\nu > 1$ do not find negative norms...how is this consistent?

The operator is not gauge invariant, so UB does not apply.

Spectrum and norms

Look for solutions of the form $\phi = e^{ik\cdot x}\psi(r)$. Defining $m^2_{bndy} = -k_i k^i$, we have

- $m^2_{bndy} > 0$: p.w. normalizable at PP, $(\phi, \phi) > 0$ for D/N even when $\nu > 1$
- $m^2_{bndy} < 0$: ruled out for D/N bc's. In principle allowed for "mixed" bc's
- $m^2_{bndy} = 0$: $\psi = r^{d/2\pm\nu}$. For N bc's, becomes normalizable when $\nu > 1$. Pure gauge

For N bc's and $\nu > 1$ do not find negative norms...how is this consistent?

The operator is not gauge invariant, so UB does not apply.

Let's take a closer look at the theory and investigate the 2-point function.

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Fixing the gauge we obtain

$$(u_1, u_2)_N = C_\nu^2 \delta^{(d)}(k_1^i - k_2^i)(m_{bndy,1})^{2\nu}$$

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Fixing the gauge we obtain

$$(u_1, u_2)_N = C_\nu^2 \delta^{(d)}(k_1^i - k_2^i)(m_{bndy,1})^{2\nu}$$

The commutator is given by $[a, a^\dagger] = (u,u)^{-1}$.

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Fixing the gauge we obtain

$$(u_1, u_2)_N = C_\nu^2 \delta^{(d)}(k_1^i - k_2^i)(m_{bndy,1})^{2\nu}$$

The commutator is given by $[a, a^\dagger] = (u,u)^{-1}$. Using this we find

$$\langle \phi(x_1, r_1)\phi(x_2, r_2)\rangle = C_\nu^{-2} \int_{\omega \geq |\vec{k}|} d\omega d^{d-1}\vec{k} e^{ik\cdot(x_1 - x_2)} \frac{\psi_N(r_1,k)\psi_N(r_2,k)}{(\omega^2 - |\vec{k}|^2)^\nu}$$

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Fixing the gauge we obtain

$$(u_1, u_2)_N = C_\nu^2 \delta^{(d)}(k_1^i - k_2^i)(m_{bndy,1})^{2\nu}$$

The commutator is given by $[a, a^\dagger] = (u,u)^{-1}$. Using this we find

$$\langle \phi(x_1, r_1)\phi(x_2, r_2)\rangle = C_\nu^{-2} \int_{\omega \geq |\vec{k}|} d\omega d^{d-1}\vec{k} e^{ik\cdot(x_1 - x_2)} \frac{\psi_N(r_1,k)\psi_N(r_2,k)}{(\omega^2 - |\vec{k}|^2)^\nu}$$

Diverges near $m_{bndy} = 0$ ("IR") for $\nu > 1$, so the theory does not exist.

Neumann 2-point function

We expand the field as

$$\phi(x,r) = \int_{V^+} d^d k [a^\dagger(k) u_k(x,r) + a(k) u_k^*(x,r)] \qquad u = e^{ik\cdot x}\psi$$

Fixing the gauge we obtain

$$(u_1, u_2)_N = C_\nu^2 \delta^{(d)}(k_1^i - k_2^i)(m_{bndy,1})^{2\nu}$$

The commutator is given by $[a, a^\dagger] = (u,u)^{-1}$. Using this we find

$$\langle \phi(x_1,r_1)\phi(x_2,r_2)\rangle = C_\nu^{-2} \int_{\omega \geq |\vec{k}|} d\omega d^{d-1}\vec{k} e^{ik\cdot(x_1-x_2)} \frac{\psi_N(r_1,k)\psi_N(r_2,k)}{(\omega^2 - |\vec{k}|^2)^\nu}$$

Diverges near $m_{bndy} = 0$ ("IR") for $\nu > 1$, so the theory does not exist.

Try to fix it adding terms that are relevant in the IR.

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions.

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions. We consider

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}} [\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2],$$

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions. We consider

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}} [\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2],$$

which requires

$$\phi^{(\nu)} + \kappa \Box_0 \phi^{(0)} - \lambda \phi^{(0)} = 0.$$

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions. We consider

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}} [\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2],$$

which requires

$$\phi^{(\nu)} + \kappa \Box_0 \phi^{(0)} - \lambda \phi^{(0)} = 0.$$

κ and λ are dimensionful and the theory flows to Δ_+ in the IR, i.e.

$$(\phi, \phi)_{def} \approx (m_{bndy})^{-2\nu} \quad \text{for } m_{bndy} \approx 0$$

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions. We consider

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}} [\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2],$$

which requires

$$\phi^{(\nu)} + \kappa \Box_0 \phi^{(0)} - \lambda \phi^{(0)} = 0.$$

κ and λ are dimensionful and the theory flows to Δ_+ in the IR, i.e.

$$(\phi, \phi)_{def} \approx (m_{bndy})^{-2\nu} \quad \text{for } m_{bndy} \approx 0$$

This solves the IR issue, but...

Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions. We consider

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}}[\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2],$$

which requires

$$\phi^{(\nu)} + \kappa \Box_0 \phi^{(0)} - \lambda \phi^{(0)} = 0.$$

κ and λ are dimensionful and the theory flows to Δ_+ in the IR, i.e.

$$(\phi, \phi)_{def} \approx (m_{bndy})^{-2\nu} \quad \text{for } m_{bndy} \approx 0$$

This solves the IR issue, but...

now we do find tachyon-ghosts for all values of κ, λ.

Extensions

Neumann boundary conditions are pathological for

Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts

Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts
- Maxwell fields: odd d 2-pt function ill-defined; even d ghosts. Exception: AdS_4 (analog of scalar in BF window)

Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts
- Maxwell fields: odd d 2-pt function ill-defined; even d ghosts. Exception: AdS_4 (analog of scalar in BF window)
- Gravitons: odd d 2-pt function ill-defined; even d ghosts. [GC and DM]

Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts
- Maxwell fields: odd d 2-pt function ill-defined; even d ghosts. Exception: AdS_4 (analog of scalar in BF window)
- Gravitons: odd d 2-pt function ill-defined; even d ghosts. [GC and DM]
- Maxwell-Chern-Simons AdS_3, even when bulk Ω can be used [with JJ and RL].

Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts
- Maxwell fields: odd d 2-pt function ill-defined; even d ghosts. Exception: AdS_4 (analog of scalar in BF window)
- Gravitons: odd d 2-pt function ill-defined; even d ghosts. [GC and DM]
- Maxwell-Chern-Simons AdS_3, even when bulk Ω can be used [with JJ and RL].
- Holographic $CFT_{(A)dS}$: find ghosts (with CU)

Can we get rid of the ghosts? (with TF and DM)

Try adding UV modification in addition to the IR deformation.

Can we get rid of the ghosts? (with TF and DM)

Try adding UV modification in addition to the IR deformation. We consider a UV wall as a model of a "physical" UV cut-off (e.g. provided by a domain wall)

Can we get rid of the ghosts? (with TF and DM)

Try adding UV modification in addition to the IR deformation. We consider a UV wall as a model of a "physical" UV cut-off (e.g. provided by a domain wall)

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} \sqrt{\gamma} r_0^{2\nu} [\frac{\kappa}{r_0^2} (\partial\phi)^2 + \lambda\phi^2]$$

where ∂M corresponds to $r = r_0$.

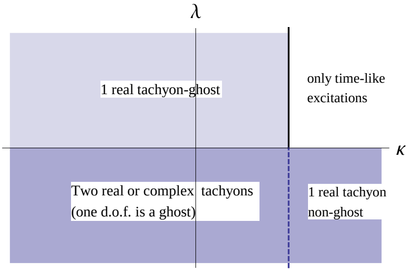
Can we get rid of the ghosts? (with TF and DM)

Try adding UV modification in addition to the IR deformation. We consider a UV wall as a model of a "physical" UV cut-off (e.g. provided by a domain wall)

$$I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} \sqrt{\gamma} r_0^{2\nu} [\frac{\kappa}{r_0^2} (\partial \phi)^2 + \lambda \phi^2]$$

where ∂M corresponds to $r = r_0$.

$$\kappa > \frac{1}{4\nu(\nu-1) r_0^{2(\nu-1)}}$$



Conclusions

Bulk theories with holographic duals that violate the unitarity bound are indeed pathological.

Conclusions

Bulk theories with holographic duals that violate the unitarity bound are indeed pathological.

The specific pathologies depend on where the CFT lives.

Conclusions

Bulk theories with holographic duals that violate the unitarity bound are indeed pathological.

The specific pathologies depend on where the CFT lives.

It seems that it is possible to remove the ghosts by adding a "UV cut-off"

Conclusions

Bulk theories with holographic duals that violate the unitarity bound are indeed pathological.

The specific pathologies depend on where the CFT lives.

It seems that it is possible to remove the ghosts by adding a "UV cut-off"

It would be interesting to find a physical realization of the cut-off.

Thank you!

Extra Slides

Maxwell fields $\nabla_\mu F^{\mu\nu} = 0$

In the radial gauge, for $d > 2$ (there are logs in even d)

$$A_i = A_i^{(0)} + r^{d-2} A_i^{(d-2)} + \ldots, \qquad \partial^i A_i^{(d-2)} = 0$$

$$A^{(0)} \Leftrightarrow \text{gauge field (res. } U(1)) \; ; \qquad A^{(d-2)} \Leftrightarrow \; U(1) \text{ current}$$

N bc's allow $A^{(0)}$ to fluctuate. Gauge invariant operator $F_{ij}^{(0)}$, $\Delta_F = 2$. $\Delta_{UB} = \max(d-2, 2)$.

Conflict with UB for $d > 4$: find ghosts (even d) or IR divergence (odd d)

$d = 4$: $F^{(0)}$ saturates UB. But $\partial^i F_{ij}^{(0)} \neq 0$ and ghosts appear.

$d = 3$: $F^{(0)}$ dual to $j = \star F^{(0)}$, which saturates UB. But! $dj = 0$ (Bianchi), so no ghosts are expected and indeed they do not arise.

$d = 2$: Neumann allows j to fluctuate, so satisfies "UB" (conformal sym. is lost). Find ghosts. Implications for HSC.

Gravitons $G_{\mu\nu} = \Lambda g_{\mu\nu}$

(Most of this is in arXiv:0805.1902 [GC and DM]).

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}(g^{(0)}_{ij} + r^d g^{(d)}_{ij})dx^i dx^j$$

$$g^{(0)} \Leftrightarrow \text{metric in the CFT} \; ; \qquad g^{(d)}_{ij} \sim T_{ij}$$

N bc's allow $g^{(0)}$ to fluctuate and bndy diff are gauge. Gauge invariant operator transverse part of $R^{(0)}_{ij}$, which has $\Delta_R = 2$ for $d > 2$. In this case $\Delta_{UB} = d$.

Conflict with UB for $d > 2$: find ghosts (even d) or IR divergence (odd d)

For $d = 2$, no obvious conflict with UB but still find ghosts.

MCS in AdS_3

$$I_0 = -\frac{1}{4}\int_M d^3x\sqrt{g}(F^2 + \alpha\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda})$$

For $0 < \alpha < 1$, AdS asymptotics are preserved. In addition, it turns out that Ω can be take to be simply the bulk expression.

The asymptotic expansion reads

$$A_i = A_i^{(0)} + r^{-\alpha}A_i^{(-)} + r^{\alpha}A_i^{(+)}$$

where $F_{ij}^{(0)} = 0$, $A_v^{(+)} = 0$, $A_u^{(-)} = 0$.

$A_i^{(-)}$ is a vector operator of dimension $\Delta_- = 1 - \alpha < \Delta_{UB} = 1$. Accordingly, find ghosts for bc's that allow $A^{(-)}$ to fluctuate.