115 Homework 3

Due Friday October 22

Question 1 Show that a matrix with integer entries can have determinant 1 only if the greatest common divisor of every row and column is also 1.

Question 2 (Rosen 3.2.20) Show that $(a_1, \ldots, a_n)$ is the least positive integer linear combination $m_1 a_1 + \cdots m_n a_n$.

Question 3 (Rosen 3.3.4a,c) Use the (extended) Euclidean algorithm to compute $(51, 87)$ and $(981, 1234)$ and express your answers as linear combinations.

Question 4* (Rosen 3.3.21,22) *The Game of Euclid*
Two players start with a pair of positive integers $\{x, y\}$, $(x \geq y)$. They take turns replacing $\{x, y\} \mapsto \{\max(x - ty, y), \min(x - ty, y)\}$ where $x - ty \geq 0$. The game is won by moving to a pair with one vanishing element. Show:

(i) The game always ends and at $\{(x, y), 0\}$ to boot!

(ii) The player starting can always win if $x = y$ or $x > y(1 + \sqrt{5})/2$, otherwise the second player can always win.

Question 5 (Rosen 3.4.8) Show that every positive integer is the product of possibly a square and a "square-free" integer (no factor other than 1 appears more than once).

Question 6 Develop and prove an algorithm for writing $(a, b) = ma + nb$. Feel free to use Rosen Theorem 3.13.