115 Homework 5

Due Friday November 5

Question 1 (Rosen 4.1.3) For which $m \in \mathbb{N}$ are the following true

(a) $27 \equiv 5 \bmod m$

(b) $1000 \equiv 1 \bmod m$

(c) $1331 \equiv 0 \bmod m$

Why?

Question 2 Compute $5^{127} \bmod 7$. Express your answer as the least positive residue and show your working.

Question 3 (Rosen 4.1.22) Use induction to show $4^n \equiv 1 + 3n \bmod 9$ for $n \in \mathbb{N}$.

Question 4 (Rosen 4.1.38) Coconuts! 5 shipwrecked men and 1 monkey collect a big pile of coconuts which they plan to divide equally the next morning. However, during the night, each man in turn wakes up, divides the pile in 5 equal parts with one leftover coconut which he gives to the monkey and then steals one of the 5 parts. In the morning, the 5 sleepy men divide the remaining coconuts into 5 equal piles and again 1 coconut remains for the monkey. What is the minimum possible number of coconuts in the original pile?

Question 5 (Rosen 4.2.2abc) Find all solutions to the linear congruences

(a) $3x \equiv 2 \bmod 7$

(b) $6x \equiv 3 \bmod 9$

(c) $17x \equiv 14 \bmod 21$

Question 6 (Rosen 4.2.12) Show that if a' and b' are inverses of a and b modulo m, respectively, then $a'b'$ is an inverse of ab modulo m.