

# 115 Homework 8

Due MONDAY November 29

**Question 1** Draw a table showing why  $\phi(7)\phi(3) = \phi(21)$ . Indicate what features apply to any pairs of relatively prime integers  $m$  and  $n$ .

**Question 2** (Rosen 6.3.6) Find the last digit (base 10) of  $3^{999,999}$ .

**Question 3** (Rosen 6.3.10) Show that  $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$  if  $(a, b) = 1$  and  $a, b \in \mathbb{N}$ .

**Question 4** (Rosen 7.1.8) Show  $\nexists n \in \mathbb{N}$  such that  $\phi(n) = 14$ .

**Question 5** (Rosen 7.1.18) If  $m, k \in \mathbb{N}$ , show  $\phi(m^k) = m^{k-1}\phi(m)$ .