185A Homework 3

Question 1 Check that the following functions are harmonic (in some domain) and compute their harmonic conjugates

(i) $u = 2x(1-y)$
(ii) $u = 2x - x^3 + 3xy^2$
(iii) $u = \sinh x \, \sin y$
(iv) $u = y/(x^2+y^2)$

In each case use your answer to write down a function (in terms of z) analytic in some domain.

Question 2 Find a suitable branch cut for the function $f(z) = \sqrt{\frac{z-1}{z+1}}$. What can you say about this function for $|z|$ large?

Question 3 Let $\log$ denote the principal branch of the logarithm. Evaluate

(i) $\log(i)$ and $\log(i-1)$
(ii) $\log[i(i-1)] - \log(i) - \log(i-1)$

Question 4 Differentiate and give the appropriate region of analyticity for each of the following

(i) $\log(z+1)$
(ii) $z^{(1+i)}$
(iii) $\sqrt{z^2-2}$

Question 5 Prove that any function both analytic and antianalytic in any domain is constant.

Question 6 For what values of z is $\log z^2 = 2\log z$ if the principal branch of the logarithm is used on both sides of the equation?

Question 7 Consider the analytic function $z \mapsto z^2$. Show that the angle between lines that do not intersect at the origin is preserved by this mapping.