185A Homework 6

Question 1 Use Liouville's theorem to prove that there is no (non-trivial) entire function mapping the entire complex plane to the inside of the unit disc.

Question 2 Let $\gamma : [0, 2\pi] \to \mathbb{C}$ be a circle of radius r around 0, that is, $\gamma(t) = re^{it}$. Calculate the winding number of the curve $f \circ \gamma$ with respect to 0, where $f(z)$ is a polynomial with complex coefficients which does not have any zeroes on the circle $|z| = r$.
[Hint: You may use that $f(z)$ is a product of linear polynomials.]

Question 3 Using the Hessian, show that a non-constant harmonic function can have neither maxima nor minima, but rather, only saddle points.

Question 4 In one page or less, summarize the main results/theorems studied to date. (Include a one line synopsis of their proof in each case.)