Homework 7

Question 1 Let $f(z)$ be analytic in a domain containing the contour Γ_w encircling w. Prove

$$f''(w) = \frac{1}{\pi i} \oint_{\Gamma_w} \frac{f(z)dz}{(z-w)^3} \,.$$

Explain the significance of this result.

Question 2 Define carefully the function $f(z) = z^\alpha$ where $\alpha \in \mathbb{C}$. Indicate your choice of any branch cuts. Use your definition to compute

(i) i^i,

(ii) $\left[\frac{e.(1+i)}{\sqrt{2}}\right]^{4\pi i}$.

Question 3 Consider the integral

$$I = -\frac{\pi i}{4} \oint_\Gamma \frac{dz \cos z}{z(z-\pi/2)(z-\pi)} \,.$$

Suggest contours Γ which return the result (i) $I = 0$, (ii) $I = 1$, (iii) $I = -1$ (iv) $I = -7$. Your answers can be pictures and Γ need not be simple. Briefly discuss the uniqueness of your solutions.

Question 4 Let $z = x + iy$. Which of the following functions are entire?

(i) $f(x, y) = x - iy$,

(ii) $f(x, y) = x^2 + y^2$,

(iii) $f(x, y) = x^3 + 3ix^2y - 2xy^2 - iy^3$,

(iv) $f(z) = \sinh(z)$,

(v) $f(z) = \exp(-1/z^2)$.

Include a **brief** explanation of your answers.

Question 5 *Essay question:* Find out what p-adic numbers are. Describe your findings in one page or less.