LECTURE #2

Office hours: Mondays 10 – 12

Grade distribution:

Quizzes 20%

Midterms 20% + 20%

Final 40%

First quiz next Tuesday

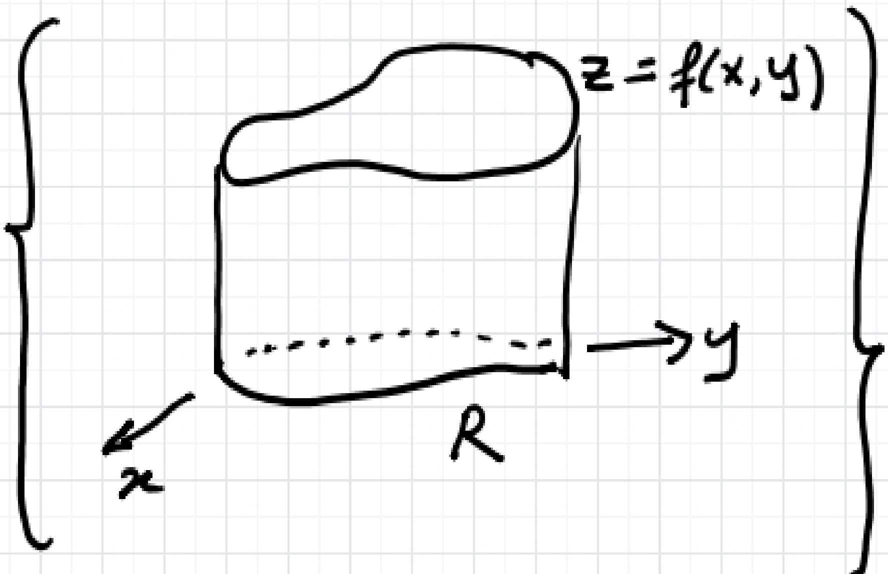
Ch 15.1, 15.2, 15.3

One of the homework problems!

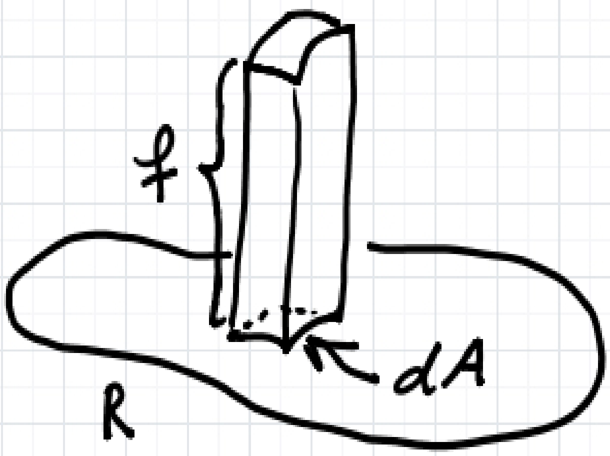
Advice: Do all the homeworks, they will appear again in Quizzes, Midterms & the Final

PROBLEM Find the volume under a surface $z = f(x,y)$ over a region R in xy-plane:

VOLUME



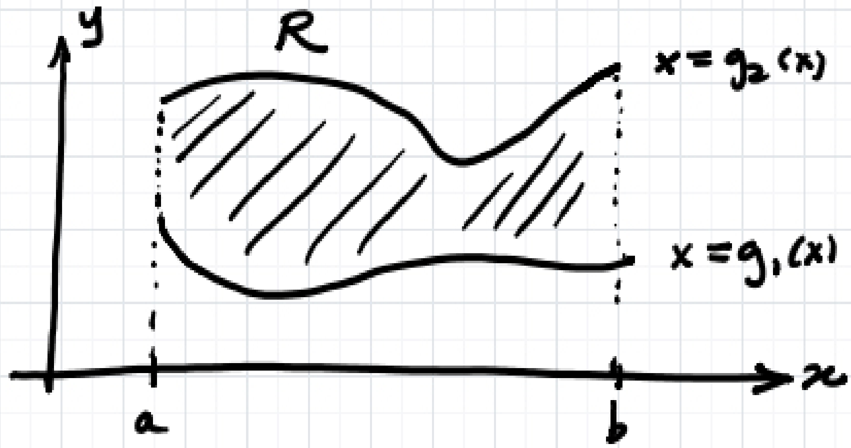
$= \int_R$

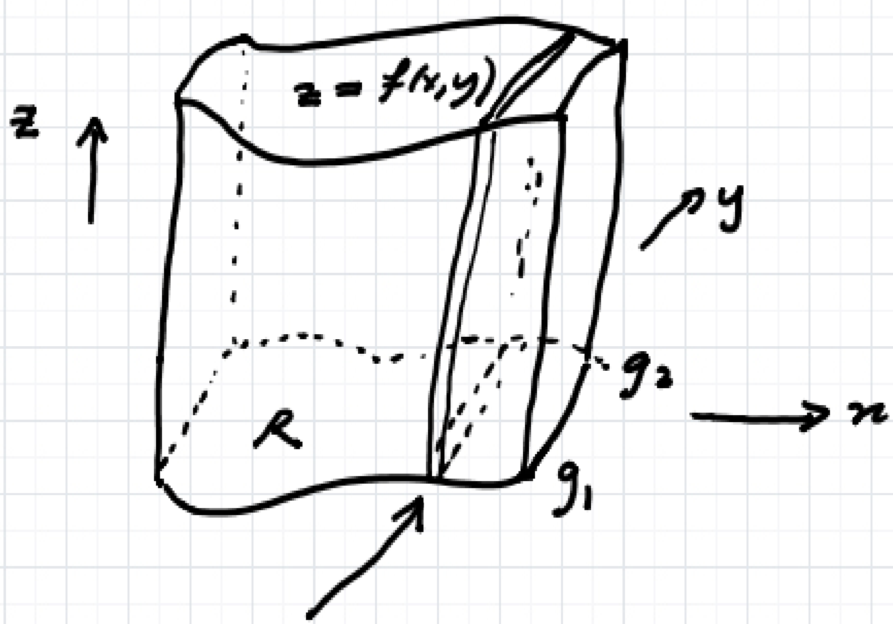


$= \int_R dA\, f$

Example $R = \{(x,y) : a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$

a region bounded by curves





slice thickness dx

Volume of slice at position x $\quad dx \int_{g_1(x)}^{g_2(x)} dy\, f(x,y)$

Now add slices!

Fubini's Theorem

$f : R \longrightarrow \mathbb{R}$ continuous

$R = \{(x,y) : a \leq x \leq b,\ g_1(x) \leq y \leq g_2(y)\}$

then

$$\boxed{\int_R dA\, f = \int_a^b dx \int_{g_1(x)}^{g_2(x)} dy\, f(x,y)}$$

Adding Slices

* The double integral, or integral over a region R bounded by curves is an iterated integral.

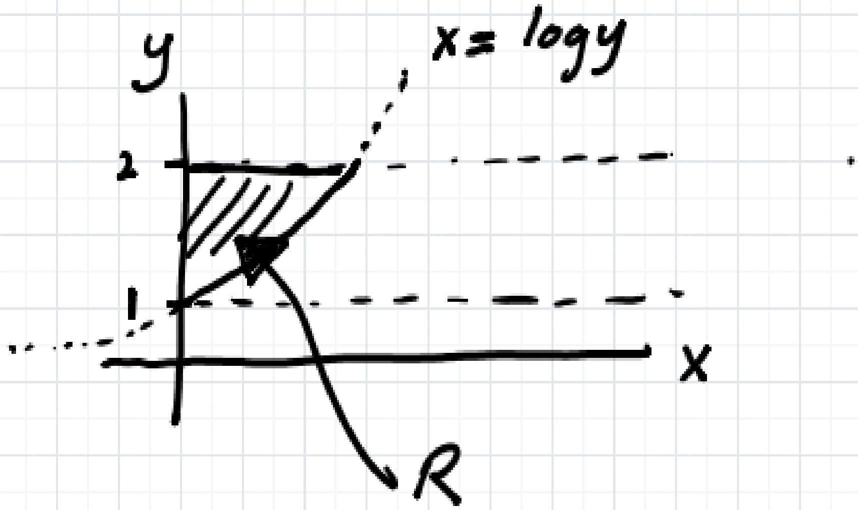
* WARNING

$$\int_R dA\, f = \int_{g_1(x)}^{g_2(x)} dy \int_a^b dx\, f(x,y) \text{ is NONSENSE!}$$

Example $f(x,y) = e^x \log y$

$R = \{1 \le y \le 2,\ 0 \le x \le \log y\}$, find $\int_R dA\, f$

① Sketch R



② Choose how to slice! Here use dy slices

③ Set up iterated integral

$$\int_R dA\, f = \int_1^2 dy \int_0^{\log y} dx\, e^x \log y = \int_1^2 dy \left[\log y\, e^x\right]_0^{\log y}$$

$$= \int_1^2 dy \log y \left[e^{\log y} - 1\right] = \int_1^2 dy \left[y \log y - \log y\right]$$

$$= \frac{1}{4}$$

Applications: Areas, Moments & Center of Mass (15.2)

AREAS If R is a bounded region

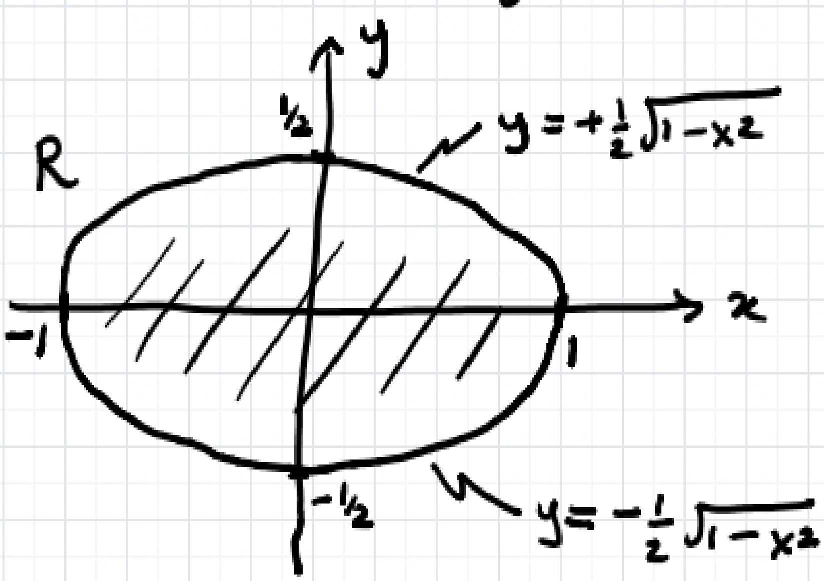
then

$$\boxed{\int_R dA = \text{Area of } R}$$

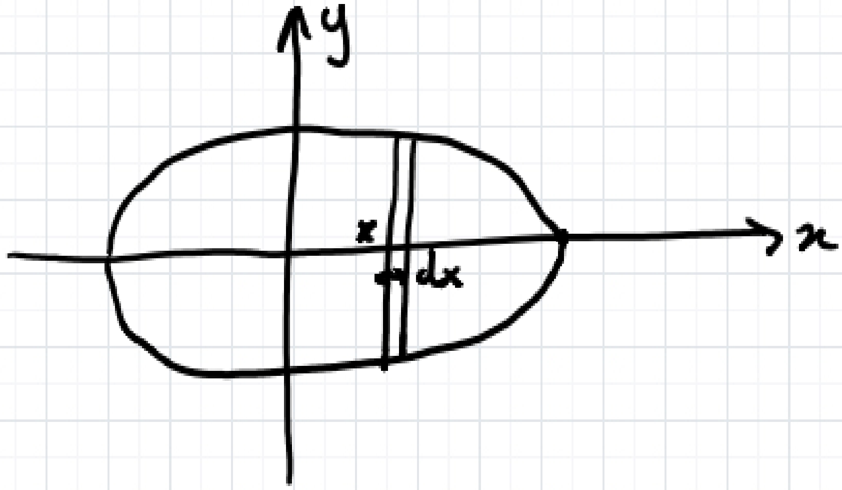
Example Find the area of the ellipse

$$-1 \leq x \leq 1 \quad , \quad -\frac{1}{2}\sqrt{1-x^2} \leq y \leq \frac{1}{2}\sqrt{1-x^2}$$

Step ① Sketch the region



Step 2 Choose slicing x-slices:



Step 3 Iterated integral

$$\text{Area} = \int_{-1}^{1} dx \int_{-\frac{1}{2}\sqrt{1-x^2}}^{+\frac{1}{2}\sqrt{1-x^2}} dy$$

controls which slice

integration limits determine width of slice at x

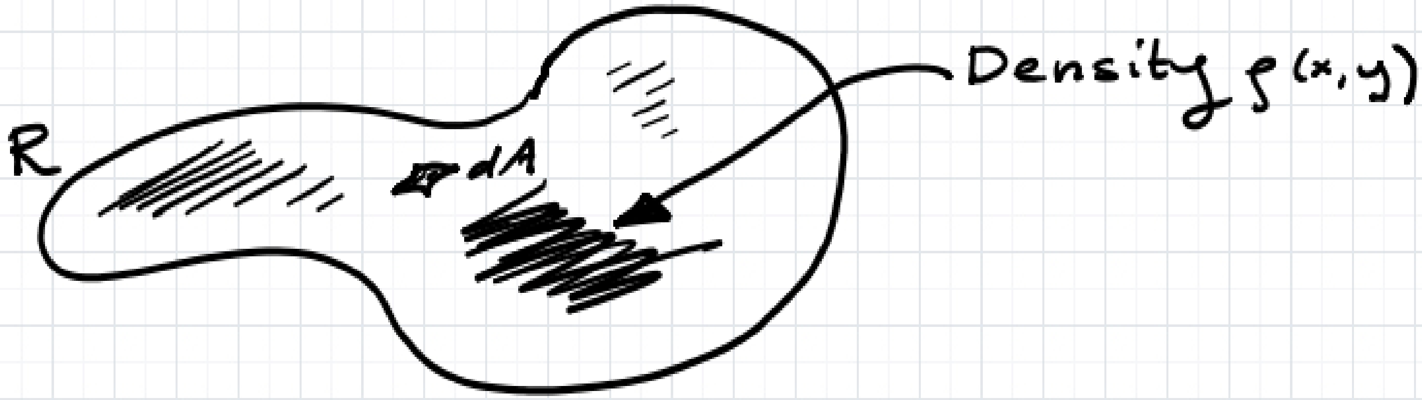
$$= \int_{-1}^{1} dx \left(\frac{1}{2}\sqrt{1-x^2} - \frac{-1}{2}\sqrt{1-x^2} \right)$$

$$= \int_{-1}^{1} dx \sqrt{1-x^2} = \text{Area}\Big($$ 

$$\Big)$$

$$= \frac{\pi}{2} \qquad \left[\text{c.f. Area} = \pi \, r_{max} \, r_{min}\right]$$

Thin LAMINAE



Mass of infinitesimal area dA

$$\rho(x,y)\,dA$$

$\Rightarrow$ Total mass $\boxed{M = \int_R \rho \, dA}$

First moments

$$\begin{cases} M_x = \int_R x\rho \, dA \\ M_y = \int_R y\rho \, dA \end{cases}$$

Center of Mass

$$(\bar{x}, \bar{y}) = \left(\frac{M_x}{M}, \frac{M_y}{M}\right)$$