Lecture #25

Review Session Times

Tuesday June 9

10:30 or 1:00 or 3:30 ??

(poll students!)

Final Exam

Thursday June 11

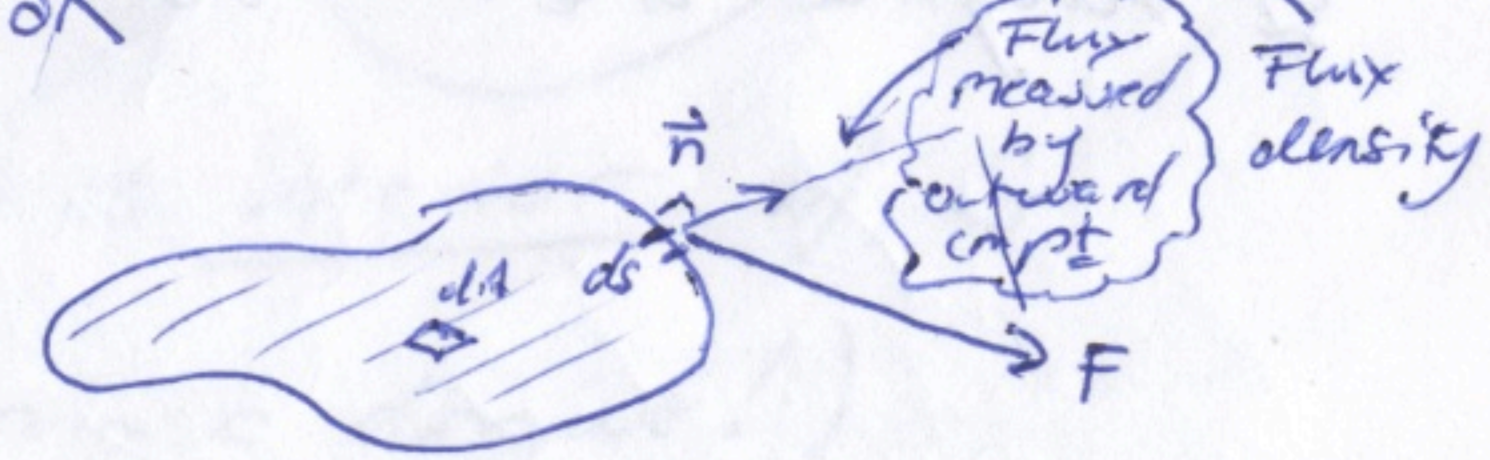
1:00pm

This ROOM!

The Divergence Theorem 16.8

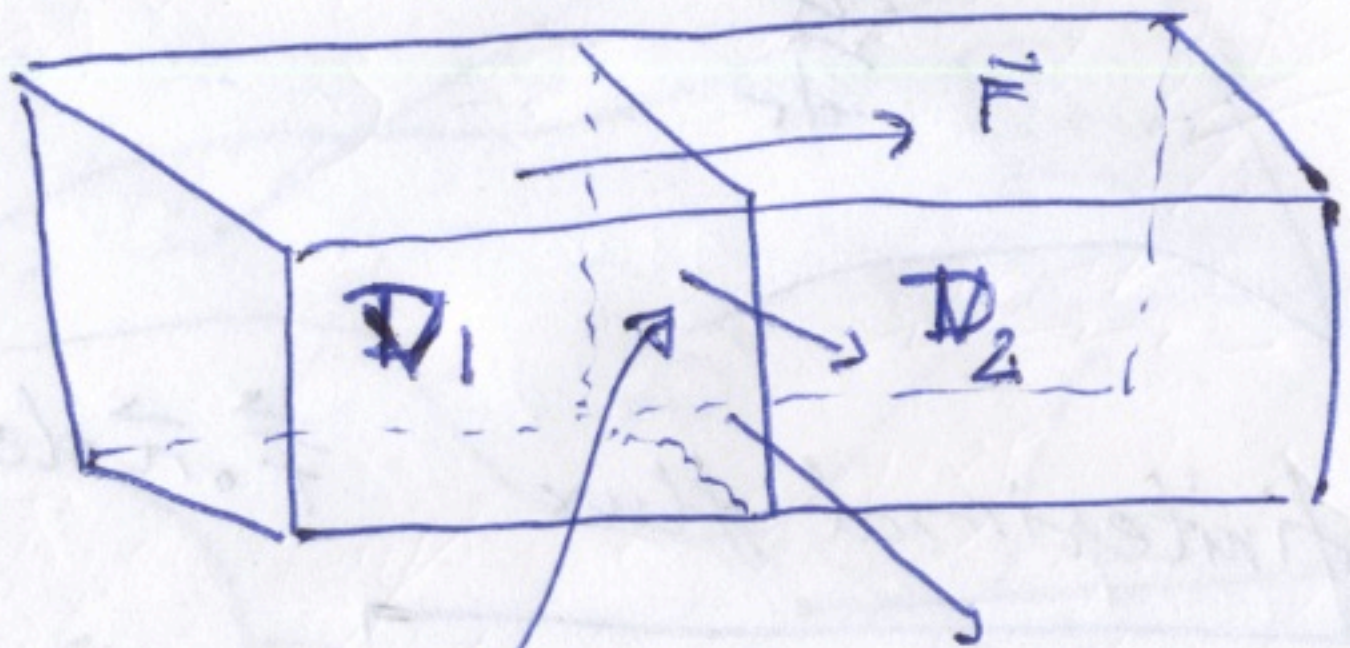
Recall Green's theorem (flux form)

$$\oint_{C=\partial R} \vec{F}\cdot\vec{n}\,ds = \int_R dA\, \operatorname{div}\vec{F}$$



Consider two adjacent cubes D_1, D_2

$$D = D_1 \cup D_2$$



At this interface all flux leaving D_1 enters D_2

Hence we expect

$$\text{FLUX} = \int_{S=\partial D} d\sigma\, \vec{F}\cdot\vec{n} =$$

$$= \int_D dV \,(\text{flux density})$$

infinitesimal volume

flux per volume

Again

Flux density = $\operatorname{div} \vec{F}$

See 16.8 for a sketch of a proof.

$$\Rightarrow \boxed{\int_{S=\partial D} d\sigma\, \vec{F}\cdot\vec{n} = \int_D dV \operatorname{div}\vec{F}}$$

DIVERGENCE THEOREM

Example

$$\vec{F} = xi + yj + zk$$

$$S = \text{unit sphere } x^2+y^2+z^2=1$$

Flux leaving S:

Surface integral

Notice $\vec{F} = \vec{n}$ on S

$$\Rightarrow \vec{F}\cdot\vec{n}\, d\sigma = d\sigma$$

$$\Rightarrow \text{FLUX} = \int_S \vec{F}\cdot\vec{n}\, d\sigma = \int d\sigma = 4\pi$$

Volume integral

$$\operatorname{div} \vec{F} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot(xi+yj+kz)$$

$$= i.i\frac{\partial x}{\partial x} + j.j\frac{\partial y}{\partial y} + k.k\frac{\partial z}{\partial z} = 3$$

$$\text{FLUX} = \int_D dV \operatorname{div} \vec{F} = \int_D dV \cdot 3$$

D

$$= 3 \text{ (volume of sphere)} = 3 \cdot \frac{4\pi}{3} = 4\pi \checkmark$$

<u>WORKS</u>

<u>Notice</u> we can use this as a method for computing volumes (sometimes)

GAUSS' LAW

<u>MAXWELL'S EQUATIONS</u>

$$\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0}$$

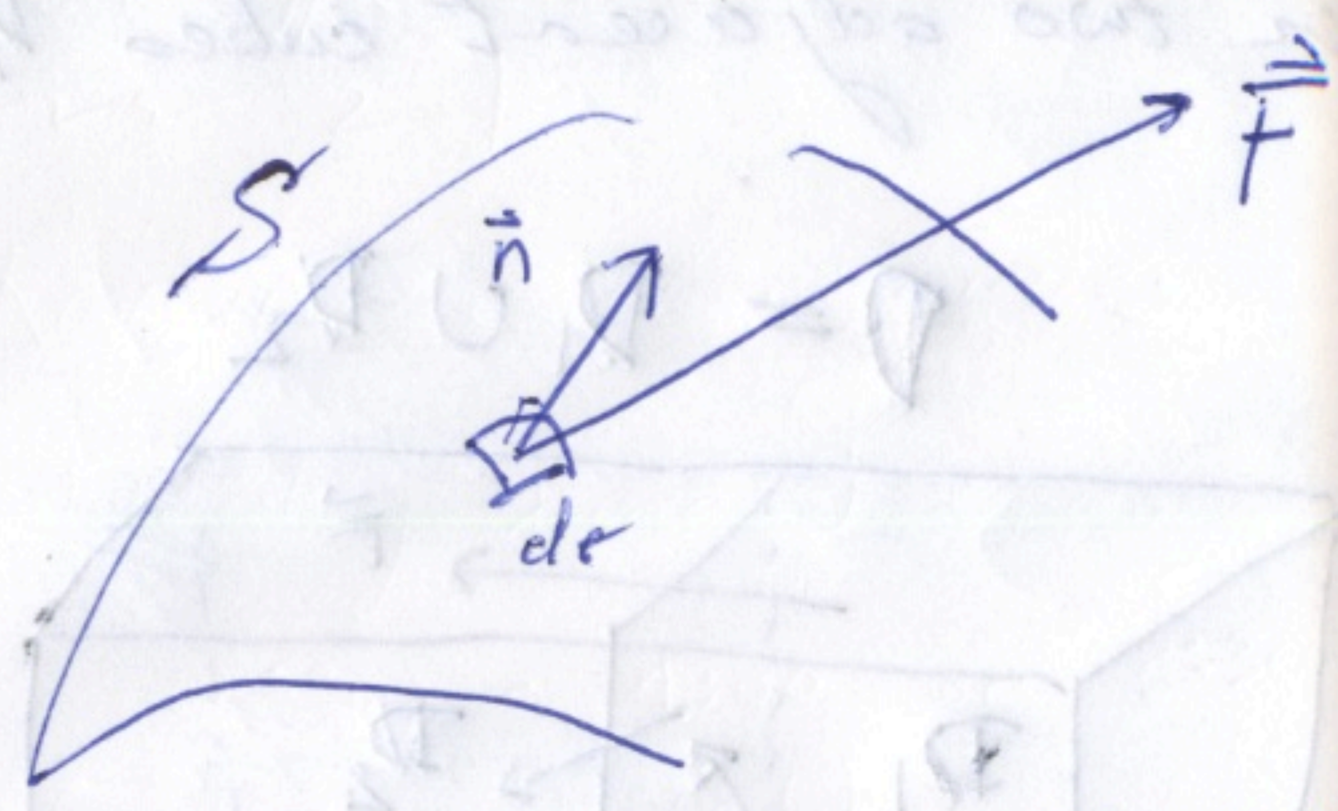
ρ — charge density; ε_0 — permitivity

$$\operatorname{div} \vec{B} = 0$$

$$\operatorname{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad t = \text{time}$$

$$\operatorname{curl} \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

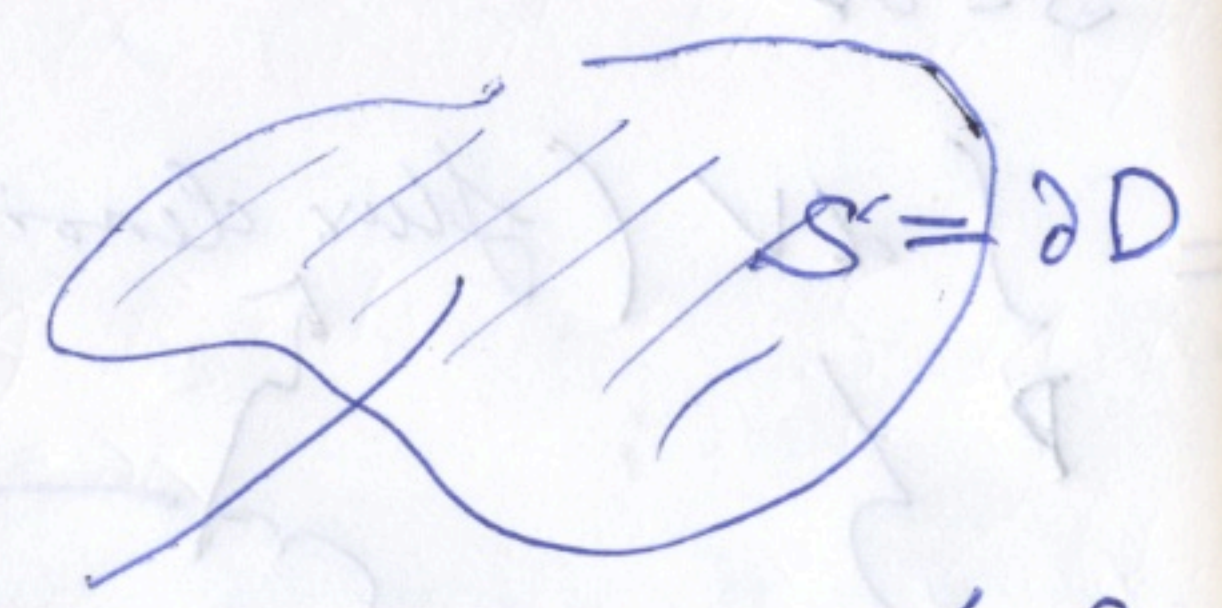
μ_0 — permeability; $\vec{J}$ — current density



Infinitesimal flux $\quad \vec{F} \cdot \vec{n}\, d\sigma$

$$\boxed{\text{FLUX} = \int_S d\sigma\, \vec{F} \cdot \vec{n}}$$

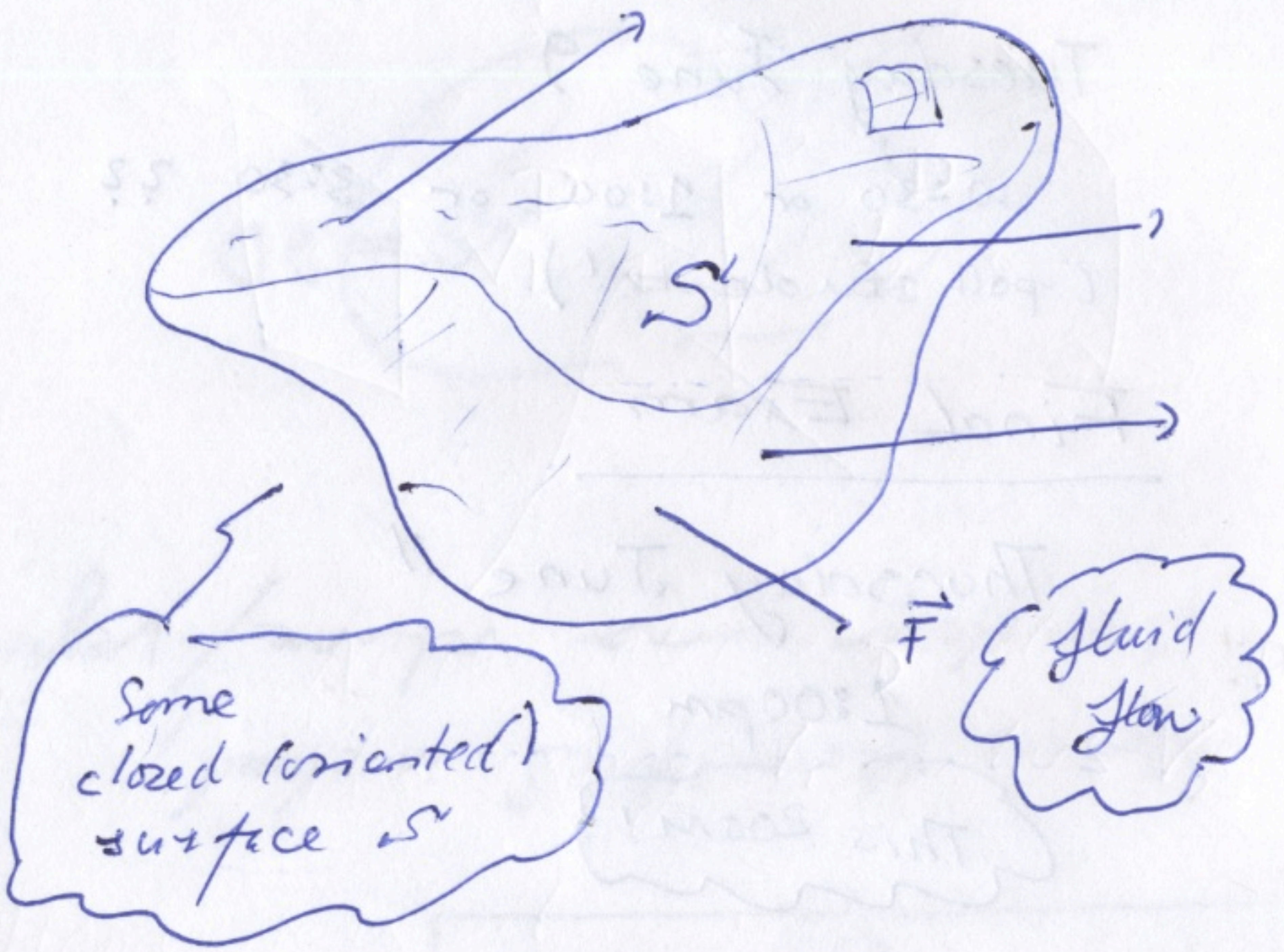
Notice that if S is closed, it encloses some volume/region



D is <u>interior</u> of S

(like jam in a donut!)

Now consider fluid flow — in 3-dimensional space



AT WHAT RATE DOES FLUD LEAVE S ?

Consider small piece of surface area $d\sigma$

CLAIM FLUX OF $\vec{E}$ ACROSS ANY SURFACE

= TOTAL CHARGE IN VOLUME CONTAINED IN THAT VOLUME.

<u>Proof</u>

Total charge in region D is

$$Q = \int_D dV \rho$$ ← charge density

$$= \varepsilon_0 \int_D \operatorname{div} \vec{E} -$$

Maxwell

$$= \varepsilon_0 \int_{S=\partial D} \vec{E} \cdot \vec{n} \, ds$$

Divergence theorem

$$= \varepsilon_0 \times \text{FLUX}$$

Holds for any choice of surface S <u>QED</u>