LECTURE # 3

MIDTERM 1 : FRIDAY APRIL 17

Nb: Homeworks are NOT collected, they are assessed in quizzes.

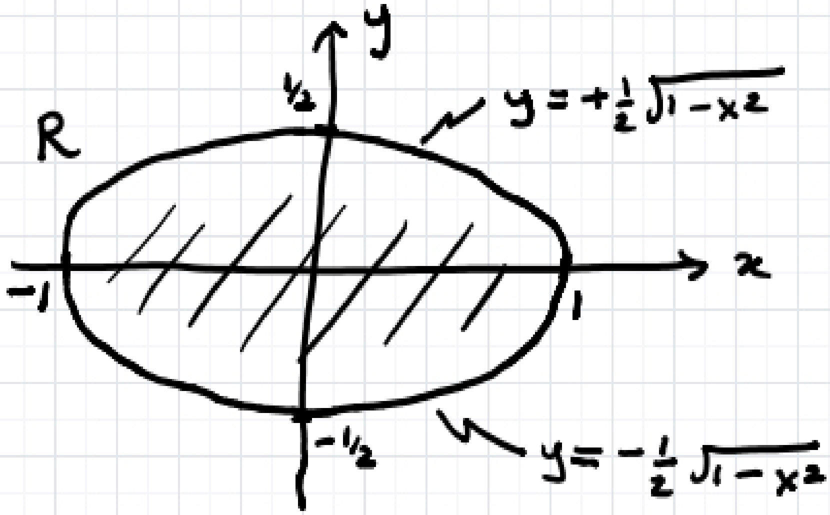
AREAS

$$\boxed{\int_R dA = \text{Area of } R}$$

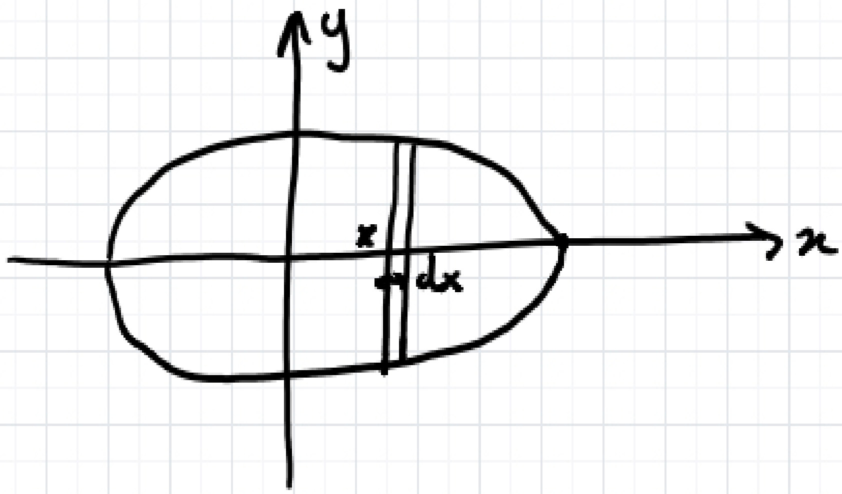
Example Find the area of the ellipse

$-1 \leq x \leq 1$, $-\frac{1}{2}\sqrt{1-x^2} \leq y \leq \frac{1}{2}\sqrt{1-x^2}$

Step ① Sketch the region



Step 2 Choose slicing <u>x-slices:</u>



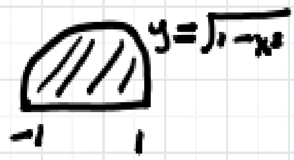
Step 3 Iterated integral

$$\text{Area} = \int_{-1}^{1} dx \int_{-\frac{1}{2}\sqrt{1-x^2}}^{+\frac{1}{2}\sqrt{1-x^2}} dy$$

controls which slice

integration limits determine width of slice at x

$$= \int_{-1}^{1} dx \left(\frac{1}{2}\sqrt{1-x^2} - \frac{-1}{2}\sqrt{1-x^2} \right)$$

$$= \int_{-1}^{1} dx \sqrt{1-x^2} = \text{Area}\Big($$  $$\Big)$$

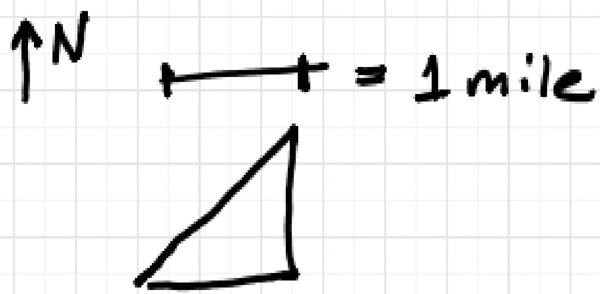
$$= \frac{\pi}{2} \qquad \left[\text{c.f. Area} = \pi \, r_{max} \, r_{min} \right]$$

AVERAGE VALUES

$$\boxed{\bar{f} \equiv \langle f \rangle = \frac{\int_R f\, dA}{\int_R dA}}$$

The average value of f in R is the integral of f over R divided by the area of R

<u>Example</u> Angleville



The temperature in Angleville is 30° in the East and increases steadily to 50° in the West. What is the average temperature in Angleville?

① Describe region:

$0 \leq x \leq 1$

$0 \leq y \leq x$

② Describe function:

Temperature $T = 50 - 20x$

③ Slice region: x-slices

④ Set up integral

$$\int_R dA\, T = \int_0^1 dx \int_0^x dy\, (50 - 20x)$$

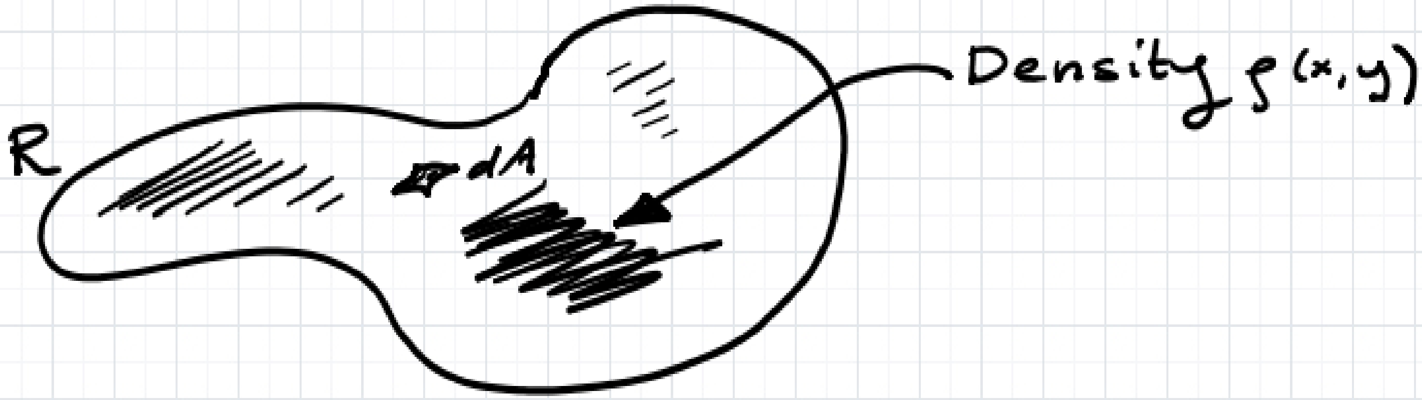
$$= \int_0^1 dx \left[50y - \frac{20y^2}{2} \right]_0^x = \int_0^1 dx \left[50x - 10x^2 \right]$$

$$= \left[\frac{50x^2}{2} - \frac{10x^3}{3} \right]_0^1 = 25 - \frac{10}{3} = \frac{65}{3} \text{ mile}^2\ {}^\circ C$$

⑤ Solve: Area $= \frac{1}{2}$ mile^2

$$\text{Av. Temp.} = \cancel{\frac{65/3}{1/2}} = \frac{130}{3} = 43\tfrac{1}{3}\ {}^\circ C$$

Thin LAMINAE



Mass of infinitesimal area dA

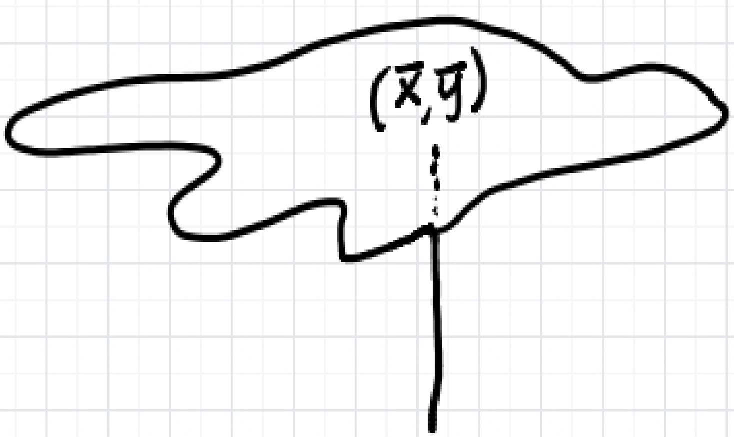
$$\rho(x,y)\,dA$$

$\Rightarrow$ Total mass $\boxed{M = \int_R \rho\, dA}$

First moments

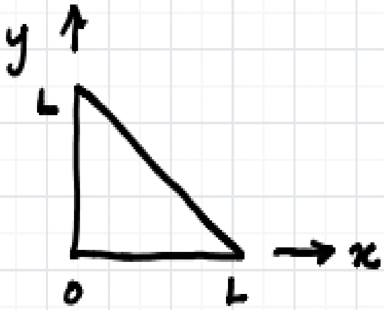
$$\begin{cases} M_x = \int_R x\rho\, dA \\ M_y = \int_R y\rho\, dA \end{cases}$$

Center of Mass $(\bar{x}, \bar{y}) = \left(\frac{M_x}{M}, \frac{M_y}{M}\right)$



Example The density of the airplane wing depicted below

whose density increases linearly from zero to ρ_F lb/sq in. from front to rear. Find its center of mass

① 

$$0 \le y \le L$$
$$0 \le x \le y$$

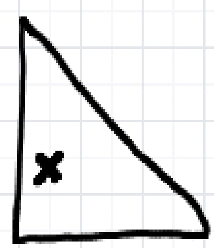
② $\rho(x,y) = \frac{\rho_F}{L} y$ density ③ y - slices

④ $$M_x = \int_0^L dy \, \frac{\rho_F}{L} y \int_0^y x \, dx \qquad M_y = \int_0^L dy \, \frac{\rho_F}{L} y^2 \int_0^y dx$$

$$= \frac{1}{6} \rho_F L^2 \qquad\qquad = \frac{1}{3} \rho_F L^2$$

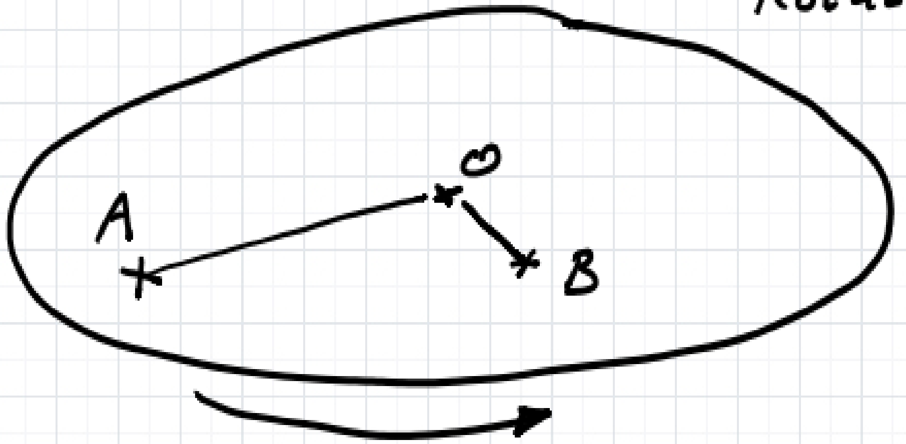
$$M = \int_0^L dy \, \frac{\rho_F}{L} y \int_0^y dx = \frac{1}{2} \rho_F L^3$$

$$\Rightarrow (\bar{x}, \bar{y}) = \left(\frac{L}{3}, \frac{L}{6}\right)$$



SECOND MOMENTS & MOMENTS OF INERTIA

Rotating disc



Easier to rotate a disc when you place a mass M at B than A because

kinetic energy $E = \frac{1}{2} M v^2$

But at a <u>fixed angular velocity</u>

$v \propto r$ = distance from O

$\Rightarrow E \propto M r^2$

This is a second moment

Hence study

$$I_L = \int r^2 \rho(x,y)\, dA$$

ρ = density
r = distance from axis of rotation L