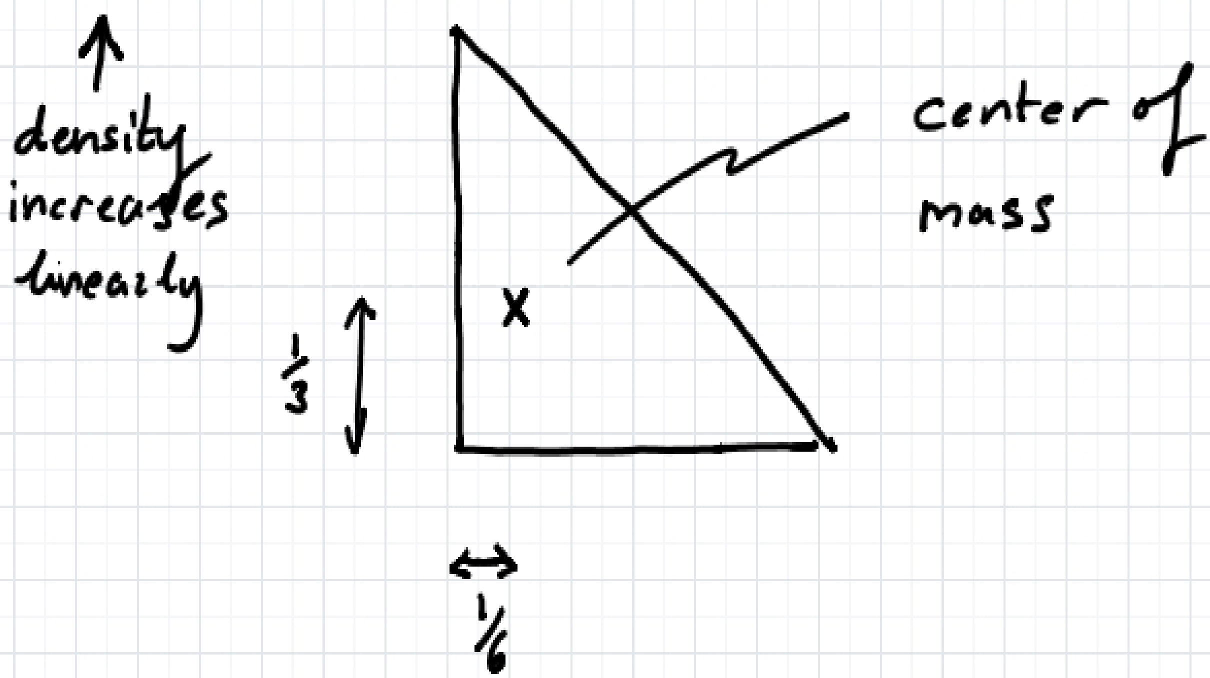
# LECTURE #4

<u>Note</u> Quiz 1 Comes from Ch 15.3

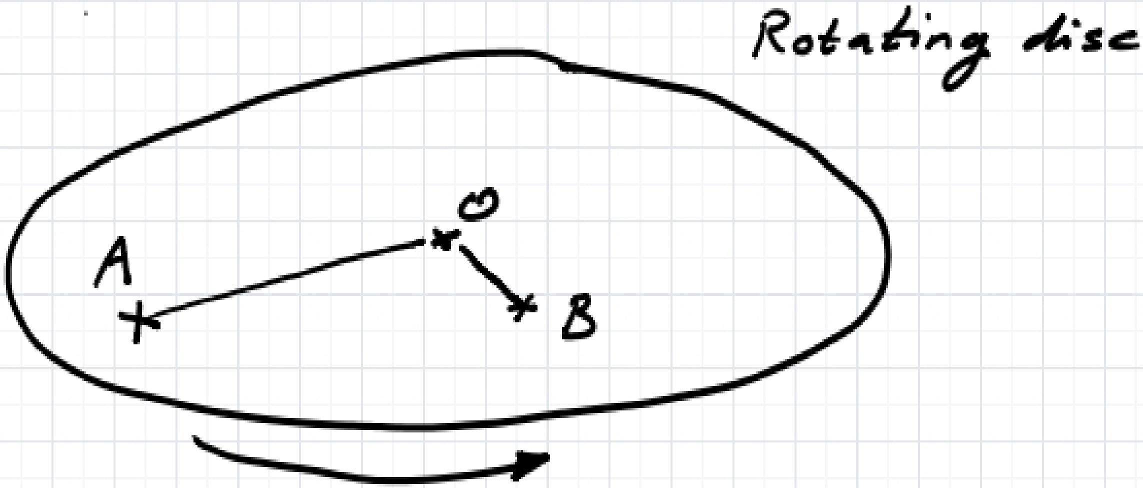
NO CALCULATORS ON QUIZZES

Answer to the airplane wing




Notice answer is <u>independent</u> of the actual size or density of the wing.

# SECOND MOMENTS & MOMENTS OF INERTIA




Easier to rotate a disc when you place a mass $M$ at $B$ than $A$ because

kinetic energy $E = \frac{1}{2} M v^2$

But at a <u>fixed angular velocity</u>

$v \propto r$ = distance from $O$

$\Rightarrow E \propto Mr^2$

This is a second moment

Hence study

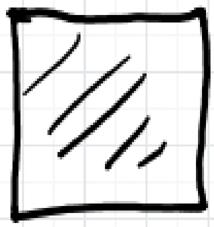
$$I_L = \int r^2 \rho(x,y)\, dA$$

$\rho$ = density

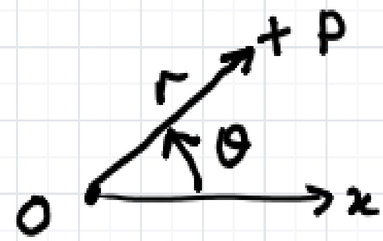
$r$ = distance from axis of rotation $L$

# OTHER COORDINATE SYSTEMS

(Polar Coord.s chapter 15.3)

| Consider | Actual Town | Our Description |
|---|---|---|
| Blockville |  | $-1 \le x, y \le 1$ |
| versus | | |
| Cycleville |  | $-1 \le x \le 1$<br>$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$ |

In <u>polar coordinates</u>,



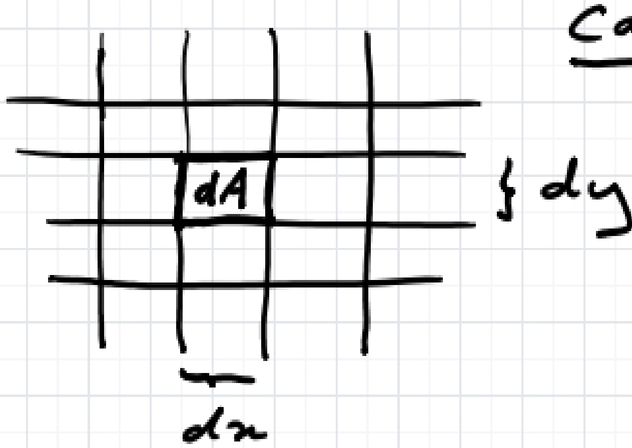

Cycleville has a simpler description

$$0 \le r \le 1$$

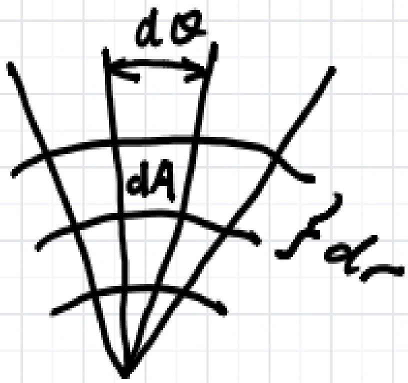
$$0 \le \theta < 2\pi$$

which looks just like Cartesian rectangle case!

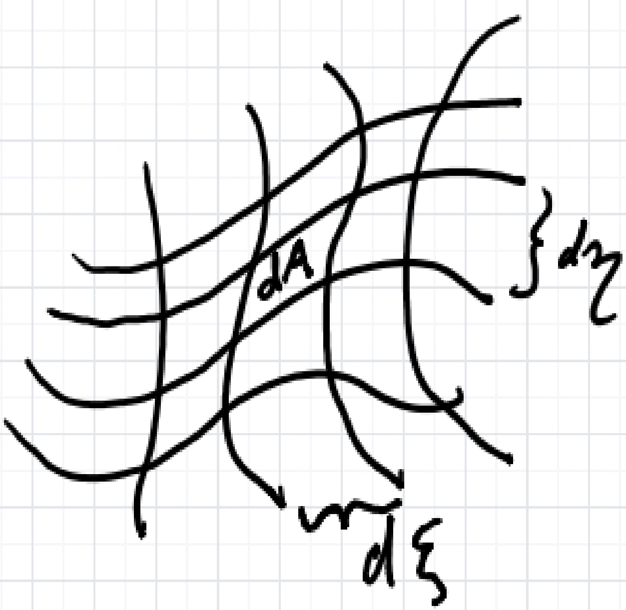
# Infinitesimal areas are <u>different shapes</u> in different coordinates




Cartesian

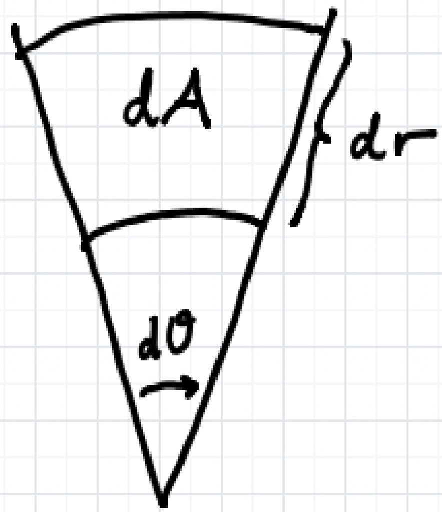



Polar




Curvilinear

# Polar infinitesimal area

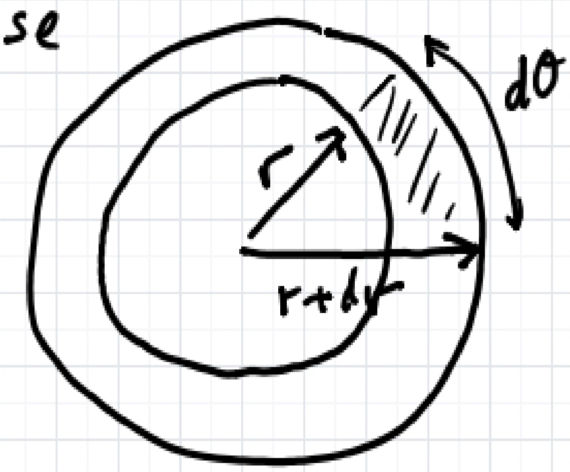



$$dA \neq dr\,d\theta$$

Actually

$$\boxed{dA = \underset{\text{length}}{r}\;\underset{\text{length}}{dr}\;\underset{\text{angle}}{d\theta}} = (\text{length})^2$$

Because




annulus area

$$= \pi(r+dr)^2 - \pi r^2$$

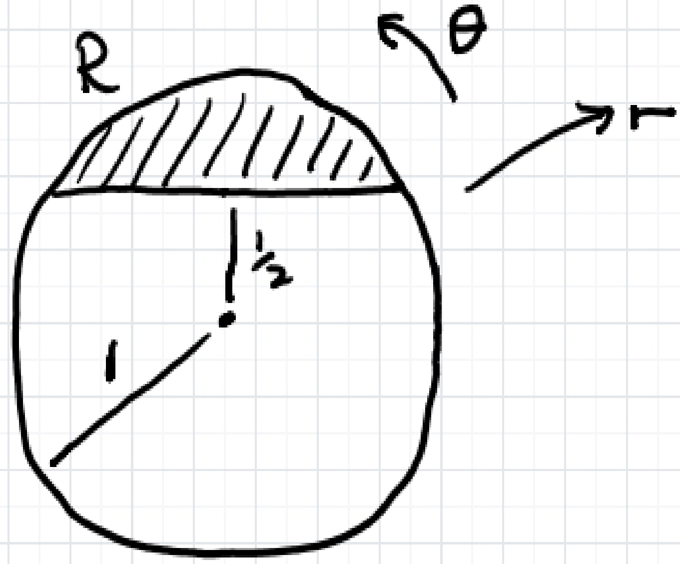
but only get wedge

$$dA = \frac{d\theta}{2\pi}\left[\pi(r+dr)^2 - \pi r^2\right]$$

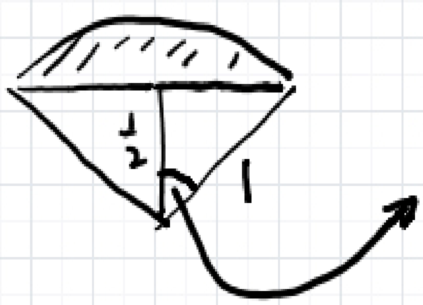
$$= \frac{d\theta}{2}\left[\cancel{r^2} + 2r\,dr + dr^2 - \cancel{r^2}\right]$$

$$\doteq r\,dr\,d\theta$$

**Example** Find the area shown



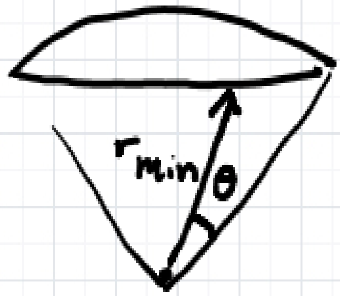

① Sketch (& describe region)




$\arccos \frac{1}{2} = \frac{\pi}{3}$

$\Rightarrow \quad 0 \leq \theta \leq \frac{2\pi}{6}$

Clearly $r \leq 1$ but need to find lower bound:




$$r_{min} \cos\left(\frac{\pi}{3} - \theta\right) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{2\cos\left(\frac{\pi}{3} - \theta\right)} \leq r \leq 1$$

② "Slices" $\theta$ - wedges

③ Set up integral

$$\text{Area} = \int_0^{2\pi/3} d\theta \int_{\frac{1}{2\cos\left(\frac{\pi}{3}-\theta\right)}}^{1} r\,dr$$

$$= \int_0^{2\pi/3} d\theta \left[ \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2\cos\left(\frac{\pi}{3}-\theta\right)}\right)^2 \right]$$

$$= \left[ \frac{\theta}{2} + \frac{1}{8}\tan\left(\frac{\pi}{3}-\theta\right) \right]_0^{2\pi/3}$$

$$= \frac{\pi}{3} + \frac{1}{8}\left[ \tan\left(\frac{-\pi}{3}\right) - \tan\frac{\pi}{3} \right]$$

$$= \frac{\pi}{3} - \frac{\sqrt{3}}{4} \approx .61$$

<u>Exercise</u> Check geometrically by thinking about: